EECS16A DIS 1B

We'll start @ Berkeley Time

Reminder: HW1 Due 9/4, if watching 1A/1B, fill out checkoff form

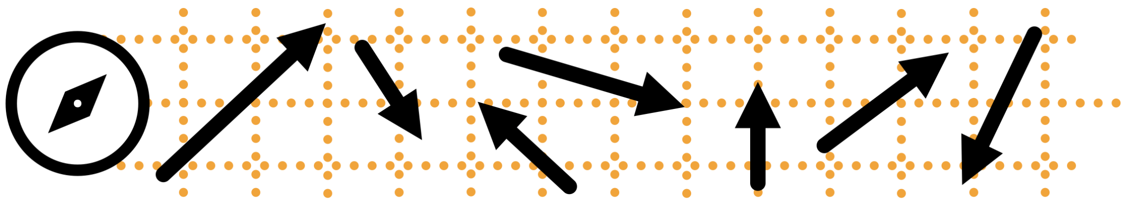
Goals for today's discussion

1. Get familiar with vector notation
2. Know how to plot and add vectors
3. Apply Gaussian Elimination to solving systems
4. Know how to identify # of solutions and write them.

EECS 16A Fall 2020 — Designing Information Devices and Systems I

Discussion 1B

1. Vectors



A vector is an ordered list of numbers. For instance, a point on a plane (x,y) is a vector! We label vectors using an arrow overhead $\vec{v}$, and since vectors can live in ANY dimension of space we'll need to leave our notation general $(x,y) \rightarrow \vec{v} = (v_1, v_2, \ldots)$. Below are few more examples (the left-most form is the general definition):

$$\vec{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \in \mathbb{R}^n \qquad \vec{a} = \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix} \in \mathbb{R}^3 \qquad \vec{b} = \begin{bmatrix} 2.4 \\ 5.3 \end{bmatrix} \in \mathbb{R}^2$$

Just to unpack this a bit more, $\vec{b} \in \mathbb{R}^3$ in english means "vector $\vec{b}$ lives in 3-Dimensional space".

- The $\in$ symbol literally means "in"
- The $\mathbb{R}$ stands for "real numbers" (FUN FACT: $\mathbb{Z}$ means "integers" like $-2, 4, 0, \ldots$)
- The exponent $\mathbb{R}^{n \leftarrow}$ indicates the dimension of space, or the amount of numbers in the vector.

One last thing: it is standard to write vectors in column-form, like seen with $\vec{a}, \vec{b}, \vec{x}$ above. We call these *column vectors*, in contrast to horizontally written vectors which we call *row vectors*.

Okay, let's dig into a few examples:

(a) Which of the following vectors live in $\mathbb{R}^2$ space?

$$i. \begin{bmatrix} 3 \\ 6 \end{bmatrix} \qquad ii. \begin{bmatrix} 5 \\ 0 \\ 3 \\ 5 \end{bmatrix} \qquad iii. \begin{bmatrix} -4.76 \\ 1.32 \\ 0.01 \end{bmatrix} \qquad iv. \begin{bmatrix} -20 \\ 100 \end{bmatrix}$$

Yes $\begin{bmatrix} 3 \\ 6 \end{bmatrix} \in \mathbb{R}^2$

because it has 2 entries/ 2 rows/ 2 #'s

No, $\begin{bmatrix} 5 \\ 0 \\ 3 \\ 5 \end{bmatrix} \notin \mathbb{R}^2$ has 4 entries

No, not in $\mathbb{R}^2$

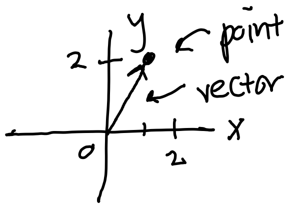
Yes

$\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \notin \mathbb{R}^2$?

No the 0 entries still count

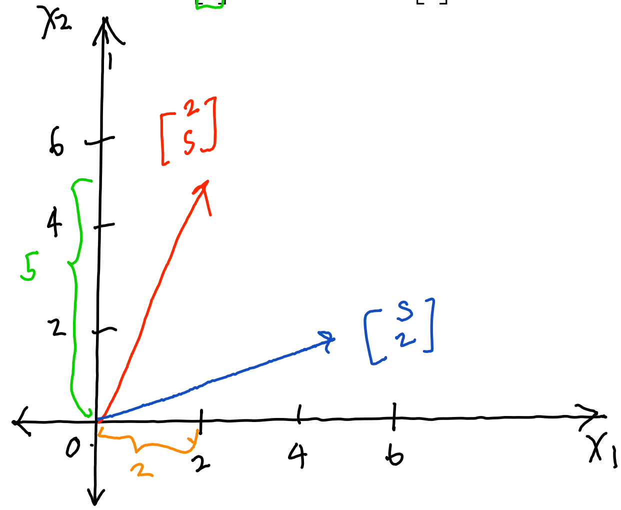
(b) Graphically show the vectors (either in a sketch with axes, or a plot on a computer):

i. $\begin{bmatrix} \boxed{2} \\ \boxed{5} \end{bmatrix}$ ii. $\begin{bmatrix} 5 \\ 2 \end{bmatrix}$



point (1,2)

vector $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$



$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

(c) Compute the sum $\vec{a}+\vec{b}=\vec{c}$ from the vectors below, and then graphically sketch or plot these vectors. (show them in a way that forms a triangle; also is there only one possible triangle?)

$\vec{a} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $\vec{b} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$

Algebraic (numberswise)

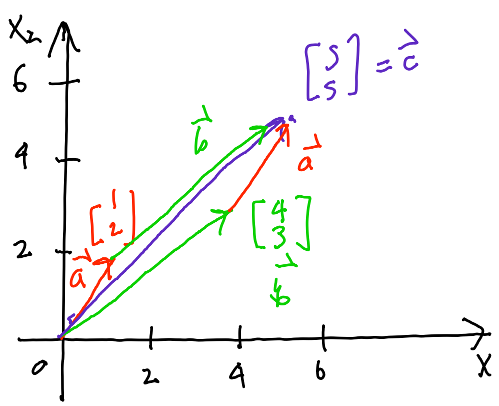
$\vec{a}+\vec{b} = \begin{bmatrix} \boxed{1} \\ \boxed{2} \end{bmatrix} + \begin{bmatrix} \boxed{4} \\ \boxed{3} \end{bmatrix} = \begin{bmatrix} \boxed{1+4} \\ \boxed{2+3} \end{bmatrix}$

$\vec{c} = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$

Can we add a vector from $\mathbb{R}^2$ and a vector from $\mathbb{R}^3$? No

$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ ← no third entry in first vector

Graphically (geometrically)



2. Solving Systems of Equations A system of linear equations can either have one solution, an infinite number of solutions, or no solution at all. For the following systems of equations, state whether there is a unique solution, no solution, or an infinite number of solutions. If there are an infinite number of solutions give one possible solution.

(a) Solve the following system. How many solutions does it have?

$$\begin{aligned} x + y &= 4 \\ x - y &= 2 \end{aligned}$$

$$\begin{cases} x + y = 4 \\ x - y = 2 \end{cases}$$

Add second eq to first

$2x + 0 = 6$

$2x = 6$

$x = 3$

Plug in $x=3$ into first eqn.

$3 + y = 4$

$y = 1$

Solution: $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

$x = 3$

$y = 1$

have only 1 solution (unique)

(b) Now write the system in augmented matrix form:

$$x + y = 4 \quad (1)$$
$$x - y = 2 \quad (2)$$

2×3

$$\begin{matrix} \boxed{x} + \boxed{y} = 4 \\ x - y = 2 \end{matrix} \iff \begin{array}{c} \left[\begin{array}{cc|c} \boxed{1} & \boxed{1} & 4 \\ 1 & -1 & 2 \end{array}\right] \\ \begin{array}{ccc} \uparrow & \uparrow & \\ x & y & \end{array} \end{array} \leftarrow \text{augmented matrix}$$

(c) Once in augmented matrix form we can use a systematic procedure called Gaussian Elimination to solve the system of equations. See what solution you get using Gaussian elimination.

$$\left[\begin{array}{cc|c} \boxed{1} & 1 & 4 \\ \boxed{1} & -1 & 2 \end{array}\right]$$

① Normalize (Done)

② Add a row to another row (to eliminate)

$R_2 \leftarrow R_2 - R_1$

$$\left[\begin{array}{cc|c} 1 & 1 & 4 \\ 0 & -2 & -2 \end{array}\right]$$

Normalize

$R_2 \leftarrow \frac{R_2}{-2}$

$$\left[\begin{array}{cc|c} 1 & 1 & 4 \\ 0 & 1 & 1 \end{array}\right]$$

upper triangular

$R_1 \leftarrow R_1 - R_2$

$$\left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 1 \end{array}\right]$$

③ Back-substitute

Finished elimination

translate back into eqns

$1 \cdot x + 0 \cdot y = 3$
$0 \cdot x + 1 \cdot y = 1$ $\Rightarrow$ $x = 3$, $y = 1$

If a unique solution every variable will have a 1 in its column

(d) Now consider the system

$$7x + y = 7 \tag{3}$$
$$42x + 6y = 42. \tag{4}$$

How many solutions does it have? Solve it first using any method, then write it as an augmented matrix and try to solve it.

$$\begin{array}{ccc} \left[\begin{array}{cc|c} \boxed{7} & 1 & 7 \\ 42 & 6 & 42 \end{array}\right] \\ x \quad y \end{array}$$

Normalize $R_1 \leftarrow R_1/7$

$$\left[\begin{array}{cc|c} 1 & 1/7 & 1 \\ 42 & 6 & 42 \end{array}\right]$$

Eliminate $R_2 \leftarrow R_2 - 42R_1$

$$\left[\begin{array}{cc|c} 1 & 1/7 & 1 \\ 0 & 0 & 0 \end{array}\right]$$

Stop. No more rows to normalize + eliminate

Translate into equations

$1 \cdot x + \frac{1}{7} \cdot y = 1$

$0 = 0$ $\Leftarrow$ Doesn't tell us about x, y

$y = s$ (any real #)

$x + \frac{1}{7}s = 1$

$x = 1 - \frac{1}{7}s$

$x + \frac{1}{7}y = 1$

$x = 1 - \frac{1}{7}s$

$y = s$

solution to eqns

$$\rightarrow \boxed{\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 - \frac{1}{7}s \\ s \end{bmatrix}}$$

Randomly chose $s = 14$ $\rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 14 \end{bmatrix}$

$7(-1) + 1(14) = 7$ ✓

$42(-1) + 6(14) = 42$ ✓

(e) Now consider the system

$$7x + y = 7 \quad (5)$$
$$42x + 6y = 42 \quad (6)$$
$$7x + y = 6 \quad (7)$$

How many solutions does it have? Solve it first using any method, then write it as an augmented matrix and try to solve it.

$$\left[\begin{array}{cc|c} 7 & 1 & 7 \\ 42 & 6 & 42 \\ 7 & 1 & 6 \end{array}\right]$$

3x3 matrix

Do what we did in part (d)

$$\left[\begin{array}{cc|c} 1 & 1/7 & 1 \\ 0 & 0 & 0 \\ \boxed{7} & 1 & 6 \end{array}\right]$$

Eliminate $R_3 \leftarrow R_3 - 7R_1$

$$\left[\begin{array}{cc|c} 1 & 1/7 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{array}\right]$$

Translate back into eqns

$x + \frac{1}{7}y = 1$

$0 = 0$

$0 = -1 \Leftarrow$ Contradiction

There are no solutions to this system of equations

Summary GE

Contradiction: row says $0 =$ nonzero

Many solns: no contradictions but not all variables are specified (not all variables have $x_i =$ some #)

e.g. $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$ second variable is not specified

Unique soln.: No contradiction all variables have a 1 pivot (all of them are specified)