EECS 16A Designing Information Devices and Systems I
Fall 2020 Discussion 14A

1. Polynomial Fitting

Let's try an example. Say we know that the output, y, is a quartic polynomial in x. This means that we know that y and x are related as follows:

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4$$

We're also given the following observations:

x	y
0.0	24.0
0.5	6.61
1.0	0.0
1.5	−0.95
2.0	0.07
2.5	0.73
3.0	−0.12
3.5	−0.83
4.0	−0.04
4.5	6.42

(a) What are the unknowns in this question? What are we trying to solve for?

(b) Can you write an equation corresponding to the first observation (x_0, y_0), in terms of a_0, a_1, a_2, a_3, and a_4? What does this equation look like? Is it linear in the unknowns?

(c) Now, write a system of equations in terms of a_0, a_1, a_2, a_3, and a_4 using *all of the observations*.

(d) Finally, solve for a_0, a_1, a_2, a_3, and a_4 using IPython. You have now found the quartic polynomial that best fits the data!

2. Building a classifier (Final - Fall 2019)

Least squares are often used in practice to classify data. In this scenario, we would like to develop a classifier to classify points based on their distance from the origin.

You are presented with the following data. Each data point $\vec{d}_i^{\,T} = [x_i\, y_i]^T$ has the corresponding label $l_i \in \{-1, 1\}$.

x_i	y_i	l_i
-2	1	-1
-1	1	1
1	1	1
2	1	-1

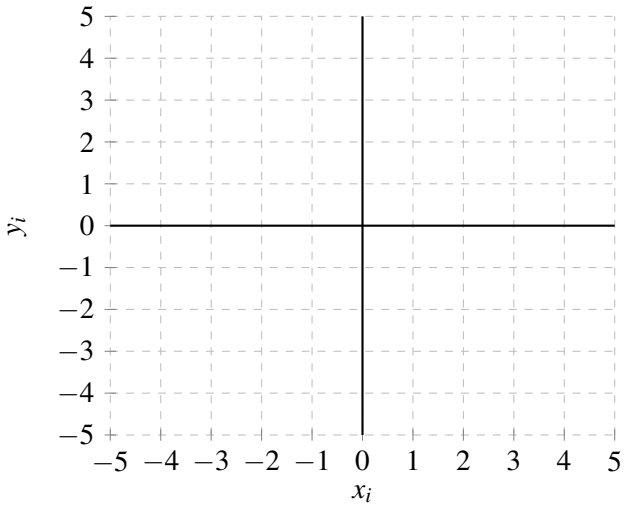
Table 1: *
Labels for data you are classifying

(a) (6 points) You want to build a model to understand the data. You first consider a linear model, i.e. you want to find $\alpha, \beta, \gamma \in \mathbb{R}$ such that $l_i \approx \alpha x_i + \beta y_i + \gamma$.
Set up a least squares problem to solve for α, β and γ. If this problem is solvable, solve it, i.e. find the best values for α, β, γ. If it is not solvable, justify why.

(b) (3 points) **Plot** the data points in the plot below with axes (x_i, y_i). **Is there a straight line such that the data points with a $+1$ label are on one side and data points with a -1 label are on the other side? Answer yes or no, and if yes, draw the line.**

x_i	y_i	l_i
-2	1	-1
-1	1	1
1	1	1
2	1	-1

Table 2: *
Table repeated for your convenience: Labels for data you are classifying



(c) (6 points) You now consider a model with a quadratic term: $l_i \approx \alpha x_i + \beta {x_i}^2$ with $\alpha, \beta \in \mathbb{R}$. *Read the equation carefully!*

Set up a least squares problem to fit the model to the data. If this problem is solvable, solve it, i.e, find the best values for α, β. If it is not solvable, justify why.

x_i	y_i	l_i
-2	1	-1
-1	1	1
1	1	1
2	1	-1

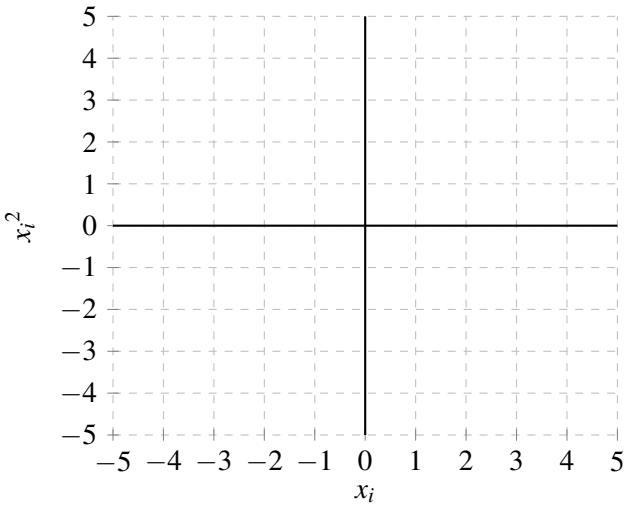
Table 3: *
Table repeated for your convenience: Labels for data you are classifying

(d) (3 points) **Plot** the data points in the plot below with axes (x_i, x_i^2). **Is there a straight line such that the data points with a $+1$ label are on one side and data points with a -1 label are on the other side? Answer yes or no, and if yes, draw the line.**

x_i	y_i	l_i
-2	1	-1
-1	1	1
1	1	1
2	1	-1

Table 4: *

Table repeated for your convenience: Labels for data you are classifying



(e) (4 points) Finally you consider the model: $l_i \approx \alpha x_i + \beta {x_i}^2 + \gamma$, where $\alpha, \beta, \gamma \in \mathbb{R}$. Independent of the work you have done so far, **would you expect this model or the model in part (c) (i.e. $l_i \approx \alpha x_i + \beta {x_i}^2$) to have a smaller error in fitting the data? Explain why.**

x_i	y_i	l_i
-2	1	-1
-1	1	1
1	1	1
2	1	-1

Table 5: *
Table repeated for your convenience: Labels for data you are classifying