EECS 16A Designing Information Devices and Systems I
Fall 2020 Discussion 9A

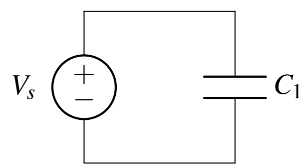
1. **Voltages Across Capacitors**

For the circuits given below, determine the voltage across each capacitor and calculate the charge and energy stored on each capacitor (assume all capacitors start *uncharged*, and then we've let the system reach steady state). We are also given $C_1 = 1\,\mu\text{F}$, $C_2 = 3\,\mu\text{F}$, and $V_s = 1\,\text{V}$.

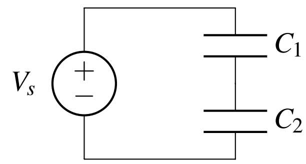
Recall charge has units of Coulombs (C), and capacitance is measured in Farads (F) $= \frac{\text{Coulomb}}{\text{Volt}}$.

It may also help to note metric prefix examples: $3\,\mu\text{F} = 3 \times 10^{-6}\text{F}$.

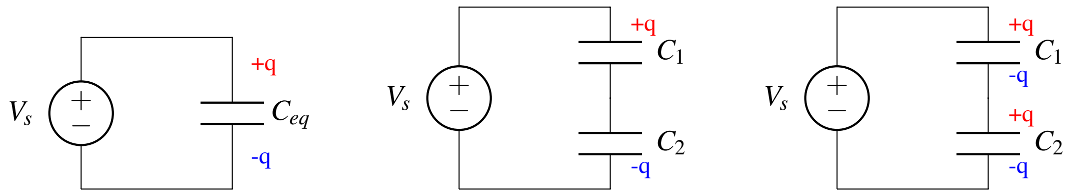
(a)



(b)



Helpful diagrams for considering the charges capacitors linked in series:
(without any initial charges)



Left: Our series capacitors may be modeled as one equivalent capacitor C_{eq}, which after some time is charged up by V_s to have $+q$ on the top plate and $-q$ on the bottom plate.

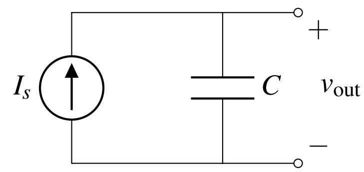
Middle: We return to the 2-capacitor picture, but carry this insight of equivalent charge with us. Now the charge $+q$ is on the top plate *of capacitor* C_1, and $-q$ is on the bottom plate *of capacitor* C_2.

Right: Since capacitor plates have opposite & equal charges, we attain this final right digram.

As another conceptual check, we notice that the node between C_1 and C_2 is isolated from any other connections and should always remain *charge neutral*. From the diagram right we see this is maintained since $(+q)+(-q)=0$.

2. Current Sources And Capacitors

Given the circuit below, find an expression for $v_{out}(t)$ in terms of I_s, C, V_0, and t, where V_0 is the initial voltage across the capacitor at $t = 0$.



Then plot the function $v_{out}(t)$ over time on the graph below for the following conditions detailed below. Use the values $I_s = 1\text{mA}$ and $C = 2\,\mu\text{F}$.

(a) Capacitor is initially uncharged $V_0 = 0$ at $t = 0$.

(b) Capacitor has been charged with $V_0 = +1.5\text{V}$ at $t = 0$.

(c) **Practice:** Swap this capacitor for one with half the capacitance $C = 1\,\mu\text{F}$, which is initially uncharged $V_0 = 0$ at $t = 0$.

HINT: Recall the calculus identity $\int_a^b f'(x)dx = f(b) - f(a)$, where $f'(x) = \frac{df}{dx}$.