CS 70 Discrete Mathematics and Probability Theory
Spring 2021

DIS 11A

1 Geometric and Poisson

Let $X \sim \text{Geo}(p)$ and $Y \sim \text{Poisson}(\lambda)$ be independent. random variables. Compute $\mathbb{P}(X > Y)$. Your final answer should not have summations.

2 Vegas

On the planet Vegas, everyone carries a coin. Many people are honest and carry a fair coin (heads on one side and tails on the other), but a fraction p of them cheat and carry a trick coin with heads on both sides. You want to estimate p with the following experiment: you pick a random sample of n people and ask each one to flip his or her coin. Assume that each person is independently likely to carry a fair or a trick coin.

1. Given the results of your experiment, how should you estimate p?
   (*Hint:* Construct an (unbiased) estimator for p such that $E[\hat{p}] = p$.)

2. How many people do you need to ask to be 95% sure that your answer is off by at most 0.05?

3 LLSE

We have two bags of balls. The fractions of red balls and blue balls in bag A are $2/3$ and $1/3$ respectively. The fractions of red balls and blue balls in bag B are $1/2$ and $1/2$ respectively. Someone gives you one of the bags (unmarked) uniformly at random. You then draw 6 balls from that same bag with replacement. Let X_i be the indicator random variable that ball i is red. Now, let us define $X = \sum_{1 \leq i \leq 3} X_i$ and $Y = \sum_{4 \leq i \leq 6} X_i$. Find $L(Y \mid X)$. *Hint*: Recall that

$$L(Y \mid X) = \mathbb{E}(Y) + \frac{\text{cov}(X,Y)}{\text{Var}(X)}\big(X - \mathbb{E}(X)\big).$$

Also remember that covariance is bilinear.