CS61B Lecture #22

Today: Backtracking searches, game trees (*DSIJ*, Section 6.5)

Searching by "Generate and Test"

- We've been considering the problem of searching a set of data stored in some kind of data structure: "Is $x \in S$?"
- But suppose we *don't* have a set S, but know how to recognize what we're after if we find it: "Is there an x such that $P(x)$?"
- If we know how to enumerate all possible candidates, can use approach of *Generate and Test:* test all possibilities in turn.
- Can sometimes be more clever: avoid trying things that won't work, for example.
- What happens if the set of possible candidates is infinite?

Backtracking Search

- Backtracking search is one way to enumerate all possibilities.
- Example: *Knight's Tour*. Find all paths a knight can travel on a chessboard such that it touches every square exactly once and ends up one knight move from where it started.
- In the example below, the numbers indicate position numbers (knight starts at 0).
- Here, knight (N) is stuck; how to handle this?

6							
		5					
4	7						
	10		2				
8	3	0					
N		9		1			

General Recursive Algorithm

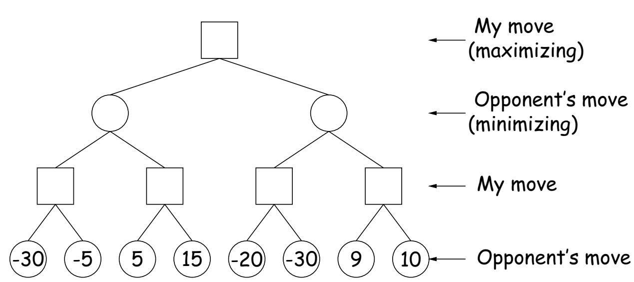
```
/** Append to PATH a sequence of knight moves starting at ROW, COL
 *  that avoids all squares that have been hit already and
 *  that ends up one square away from ENDROW, ENDCOL. B[i][j] is
 *  true iff row i and column j have been hit on PATH so far.
 *  Returns true if it succeeds, else false (with no change to PATH).
 *  Call initially with PATH containing the starting square, and
 *  the starting square (only) marked in B. */

boolean findPath(boolean[][] b, int row, int col,
                 int endRow, int endCol, List path) {
  if (path.size() == 64)   return isKnightMove(row, col, endRow, endCol);
  for (r, c = all possible moves from (row, col)) {
    if (!b[r][c]) {
      b[r][c] = true; // Mark the square
      path.add(new Move(r, c));
      if (findPath(b, r, c, endRow, endCol, path))  return true;
      b[r][c] = false; // Backtrack out of the move.
      path.remove(path.size()-1);
    }
  }
  return false;
}
```


Another Kind of Search: Best Move

- Consider the problem of finding the *best* move in a two-person game.
- One way: assign a *heuristic value* to each possible move and pick highest (aka *static evaluation*). Examples:
  - number of black pieces − number of white pieces in checkers.
  - weighted sum of white piece values − weighted sum of black pieces in chess (Queen=9, Rook=5, etc.)
  - Nearness of pieces to strategic areas (center of board).
- But this is misleading. A move might give us more pieces, but set up a devastating response from the opponent.
- So, for each move, look at *opponent's* possible moves, assume he picks the best one for him, and use that as the value.
- But what if *you* have a great response to his response?
- How do we organize this sensibly?

Game Trees

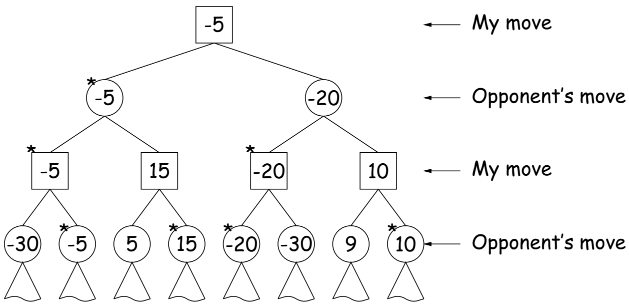
- Think of the space of possible continuations of the game as a tree.
- Each node is a position, each edge a move.



- Suppose numbers at the bottom are the values of those final positions *to me*. Smaller numbers are of more value to *my opponent*.
- What should I move? What value can I get if my opponent plays as well as possible?

Game Trees, Minimax

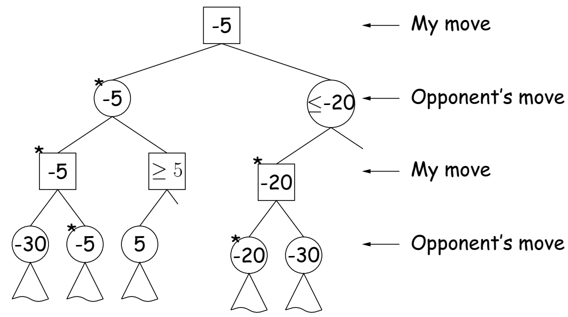
- Think of the space of possible continuations of the game as a tree.
- Each node is a position, each edge a move.



- Numbers are the values we guess for the positions (larger means better for me). Starred nodes would be chosen.
- I always choose child (next position) with maximum value; opponent chooses minimum value ("Minimax algorithm")

Alpha-Beta Pruning

- We can *prune* this tree as we search it.



- At the '≥ 5' position, I know that the opponent will not choose to move here (since he already has a -5 move).
- At the '≤ -20' position, my opponent knows that I will never choose to move here (since I already have a -5 move).

Cutting off the Search

- If you could traverse game tree to the bottom, you'd be able to force a win (if it's possible).
- Sometimes possible near the end of a game.
- Unfortunately, game trees tend to be either infinite or impossibly large.
- So, we choose a maximum *depth*, and use a heuristic value computed on the position alone (called a *static valuation*) as the value at that depth.
- Or we might use *iterative deepening*, repeating the search at increasing depths until time is up.
- Much more sophisticated searches are possible, however (take CS188).

Overall Search Algorithm

- Depending on whose move it is (maximizing player or minimizing player), we'll search for a move estimated to be optimal in one direction or the other.
- Search will be exhaustive down to a particular depth in the game tree; below that, we guess values.
- Also pass α and β limits:
  - High player does not care about exploring a position further once he knows its value is larger than what the minimizing player knows he can get (β), because the minimizing player will never allow that position to come about.
  - Likewise, minimizing player won't explore a positions whose value is less than what the maximizing player knows he can get (α).
- To start, a maximizing player will find a move with

  `findMax(`*current position*, *search depth* $-\infty$, $+\infty$`)`
- minimizing player:

  `findMin(`*current position*, *search depth* $-\infty$, $+\infty$`)`

Some Pseudocode for Searching (One Level)

- The most basic kind of game-tree search is to assign some heuristic value to any given position, looking at just the next possible move:

```
Move simpleFindMax(Position posn, double alpha, double beta) {
    if (posn.maxPlayerWon())
        return artificial "Move" with value +∞;
    else if (posn.minPlayerWon())
        return artificial "Move" with value −∞;
    Move bestSoFar = artificial "Move" with value −∞;
    for (each M = a legal move for maximizing player from posn) {
         Position next = posn.makeMove(M);
         next.setValue(heuristicEstimate(next));
         if (next.value() >= bestSoFar.value()) {
              bestSoFar = next;
              alpha = max(alpha, next.value());
              if (beta <= alpha) break;
         }
    }
    return bestSoFar;
}
```


One-Level Search for Minimizing Player

```
Move simpleFindMin(Position posn, double alpha, double beta) {
    if (posn.maxPlayerWon())
        return artificial "Move" with value +∞;
    else if (posn.minPlayerWon())
        return artificial "Move" with value −∞;
    Move bestSoFar = artificial "Move" with value +∞;
    for (each M = a legal move for minimizing player from posn) {
         Position next = posn.makeMove(M);
         next.setValue(heuristicEstimate(next));
         if (next.value() <= bestSoFar.value()) {
              bestSoFar = next;
              beta = min(beta, next.value());
              if (beta <= alpha) break;
         }
    }
    return bestSoFar;
}
```

Some Pseudocode for Searching (Maximizing Player)

```
/** Return a best move for maximizing player from POSN, searching
 *  to depth DEPTH.  Any move with value >= BETA is also
 *  "good enough". */
Move findMax(Position posn, int depth, double alpha, double beta) {
    if (depth == 0 || gameOver(posn))
        return simpleFindMax(posn, alpha, beta);
    Move bestSoFar = artificial "Move" with value −∞;
    for (each M = a legal move for maximizing player from posn) {
         Position next = posn.makeMove(M);
         Move response = findMin(next, depth-1, alpha, beta);
         if (response.value() >= bestSoFar.value()) {
              bestSoFar = next;
              next.setValue(response.value());
              alpha = max(alpha, response.value());
              if (beta <= alpha) break;
         }
    }
    return bestSoFar;
}
```


Some Pseudocode for Searching (Minimizing Player)

```
/** Return a best move for minimizing player from POSN, searching
 *  to depth DEPTH.  Any move with value <= ALPHA is also
 *  "good enough". */
Move findMin(Position posn, int depth, double alpha, double beta) {
    if (depth == 0 || gameOver(posn))
        return simpleFindMin(posn, alpha, beta);
    Move bestSoFar = artificial "Move" with value +∞;
    for (each M = a legal move for minimizing player from posn) {
         Position next = posn.makeMove(M);
         Move response = findMax(next, depth-1, alpha, beta);
         if (response.value() <= bestSoFar.value()) {
              bestSoFar = next;
              next.setValue(response.value());
              beta = min(beta, response.value());
              if (beta <= alpha) break;
         }
    }
    return bestSoFar;
}
```