CS61B Lectures #28

s on sorting by comparison

counting, radix sorts

ay: *DS(IJ)*, Chapter 8; Next topic: Chapter 9.

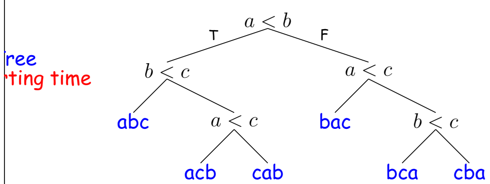
Better than N lg N?

at *if all you can do to keys is compare them,* then sorting $N \lg N)$.

there are $N!$ possible ways the input data could be

our program must be prepared to do $N!$ different com- data-moving operations.

here must be $N!$ possible combinations of outcomes of sts in your program, since those determine what move where (we're assuming that comparisons are 2-way).



Necessary Choices

-test goes two ways, number of possible different out- if-tests is 2^k.

nough tests so that $2^k \geq N!$, which means $k \in \Omega(\lg N!)$.

g's approximation,

$$\sqrt{2\pi N}\left(\frac{N}{e}\right)^N\left(1+\Theta\left(\frac{1}{N}\right)\right),$$

$$1/2(\lg 2\pi + \lg N) + N\lg N - N\lg e + \lg\left(1+\Theta\left(\frac{1}{N}\right)\right)$$

$$\Theta(N \lg N)$$

that k, the worst-case number of tests needed to sort omparison sorting, is in $\Omega(N \lg N)$: there must be cases eed (some multiple of) $N \lg N$ comparisons to sort N

Beyond Comparison: Distribution

can do more than compare keys?

, how can we sort a set of N integer keys whose values) to kN, for some small constant k?

ie: put the integers into N buckets, with an integer p ket $\lfloor p/k \rfloor$.

ys per bucket, so catenate and use insertion sort, which ast.

$= 10:$

```
 10  13  4   2   19  17  0   9
s:
  2 |  4 |   |  9  | 10 | 13 | 14 |  17 |  19  |
```

n sort is fast. Putting in buckets takes time $\Theta(N)$, and t takes $\Theta(kN)$. When k is fixed (constant), we have he $\Theta(N)$.

Distribution Counting

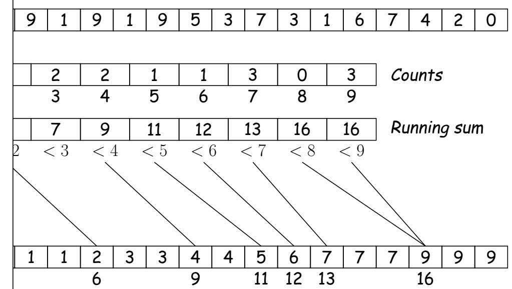
nique: *count* the number of items < 1, < 2, etc.

ems with value $< p$, then in sorted order, the j^{th} item must be item $\#M_p + j$.

r *linear-time* algorithm.

Distribution Counting Example

tems are between 0 and 9 as in this example:



gives # occurrences of each key.

" gives cumulative count of keys $<$ each value. . .

s us where to put each key:

tance of key k goes into slot m, where m is the number ces that are $< k$.

istribution Counting Example (II)

9	1	9	1	9	5	3	7	3	1	6	7	4	2	0

2	2	1	1	3	0	3
3	4	5	6	7	8	9

Counts

7	9	11	12	13	16	16
3	4	5	6	7	8	9

Running sum of Counts

7	9	11	12	14	16	16
3	4	5	6	7	8	9

Next positions

									7					
		6			9			12			15			18

Output

istribution Counting Example (II)

9	1	9	1	9	5	3	7	3	1	6	7	4	2	0

2	2	1	1	3	0	3
3	4	5	6	7	8	9

Counts

7	9	11	12	13	16	16
3	4	5	6	7	8	9

Running sum of Counts

7	10	11	12	14	16	16
3	4	5	6	7	8	9

Next positions

					4				7					
		6			9			12			15			18

Output

istribution Counting Example (II)

9	1	9	1	9	5	3	7	3	1	6	7	4	2	0

2	2	1	1	3	0	3
3	4	5	6	7	8	9

Counts

7	9	11	12	13	16	16
3	4	5	6	7	8	9

Running sum of Counts

7	10	11	12	14	16	17
3	4	5	6	7	8	9

Next positions

					4				7			9		
		6			9			12			15			18

Output

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9	1	9	1	9	5	3	7	3	1	6	7	4	2	0

2	2	1	1	3	0	3
3	4	5	6	7	8	9

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		6			9			12			15			18

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2	2	1	1	3	0	3
3	4	5	6	7	8	9

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2	2	1	1	3	0	3
3	4	5	6	7	8	9

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7	9	11	12	13	16	16
3	4	5	6	7	8	9

Running sum of Counts

7	10	11	12	14	16	16
3	4	5	6	7	8	9

Next positions

					4				7					
		6			9			12			15			18

Output

istribution Counting Example (II)

9	1	**9**	1	9	5	3	7	3	1	6	7	4	2	0

2	2	1	1	3	0	3	*Counts*
3	4	5	6	7	8	9	
7	9	11	12	13	16	16	*Running sum of Counts*
3	4	5	6	7	8	9	
7	10	11	12	14	16	18	*Next positions*
3	4	5	6	7	8	9	

					4				7			9	9		Output
		6			9			12			15			18	

istribution Counting Example (II)

9	1	9	1	**9**	5	3	7	3	1	6	7	4	2	0

2	2	1	1	3	0	3	*Counts*
3	4	5	6	7	8	9	
7	9	11	12	13	16	16	*Running sum of Counts*
3	4	5	6	7	8	9	
7	10	11	12	14	16	19	*Next positions*
3	4	5	6	7	8	9	

1					4				7			9	9	9	Output
		6			9			12			15			18	

istribution Counting Example (II)

9	1	9	1	9	5	**3**	7	3	1	6	7	4	2	0

2	2	1	1	3	0	3	*Counts*
3	4	5	6	7	8	9	
7	9	11	12	13	16	16	*Running sum of Counts*
3	4	5	6	7	8	9	
8	10	12	12	14	16	19	*Next positions*
3	4	5	6	7	8	9	

1			3		4		5		7			9	9	9	Output
		6			9			12			15			18	

istribution Counting Example (II)

9	**1**	9	1	9	5	3	7	3	1	6	7	4	2	0

2	2	1	1	3	0	3	*Counts*
3	4	5	6	7	8	9	
7	9	11	12	13	16	16	*Running sum of Counts*
3	4	5	6	7	8	9	
7	10	11	12	14	16	17	*Next positions*
3	4	5	6	7	8	9	

					4				7			9			Output
		6			9			12			15			18	

istribution Counting Example (II)

9	1	9	**1**	9	5	3	7	3	1	6	7	4	2	0

2	2	1	1	3	0	3	*Counts*
3	4	5	6	7	8	9	
7	9	11	12	13	16	16	*Running sum of Counts*
3	4	5	6	7	8	9	
7	10	11	12	14	16	18	*Next positions*
3	4	5	6	7	8	9	

1					4				7			9	9		Output
		6			9			12			15			18	

istribution Counting Example (II)

9	1	9	1	9	**5**	3	7	3	1	6	7	4	2	0

2	2	1	1	3	0	3	*Counts*
3	4	5	6	7	8	9	
7	9	11	12	13	16	16	*Running sum of Counts*
3	4	5	6	7	8	9	
7	10	12	12	14	16	19	*Next positions*
3	4	5	6	7	8	9	

1					4		5		7			9	9	9	Output
		6			9			12			15			18	

istribution Counting Example (II)

9	1	9	1	9	5	3	7	**3**	1	6	7	4	2	0

2	2	1	1	3	0	3	*Counts*
3	4	5	6	7	8	9	

7	9	11	12	13	16	16	*Running sum of Counts*
3	4	5	6	7	8	9	

9	10	12	12	15	16	19	*Next positions*
3	4	5	6	7	8	9	

1			3	3	4		5		7	7		9	9	9	Output
		6			9			12			15			18	

istribution Counting Example (II)

9	1	9	1	9	5	3	7	3	1	**6**	7	4	2	0

2	2	1	1	3	0	3	*Counts*
3	4	5	6	7	8	9	

7	9	11	12	13	16	16	*Running sum of Counts*
3	4	5	6	7	8	9	

9	10	12	13	15	16	19	*Next positions*
3	4	5	6	7	8	9	

1	1		3	3	4		5	6	7	7		9	9	9	Output
		6			9			12			15			18	

istribution Counting Example (II)

9	1	9	1	9	5	3	7	3	1	6	7	**4**	2	0

2	2	1	1	3	0	3	*Counts*
3	4	5	6	7	8	9	

7	9	11	12	13	16	16	*Running sum of Counts*
3	4	5	6	7	8	9	

9	11	12	13	16	16	19	*Next positions*
3	4	5	6	7	8	9	

1	1		3	3	4	4	5	6	7	7	7	9	9	9	Output
		6			9			12			15			18	

istribution Counting Example (II)

9	1	9	1	9	5	3	**7**	3	1	6	7	4	2	0

2	2	1	1	3	0	3	*Counts*
3	4	5	6	7	8	9	

7	9	11	12	13	16	16	*Running sum of Counts*
3	4	5	6	7	8	9	

8	10	12	12	15	16	19	*Next positions*
3	4	5	6	7	8	9	

1			3		4		5		7	7		9	9	9	Output
		6			9			12			15			18	

istribution Counting Example (II)

9	1	9	1	9	5	3	7	3	**1**	6	7	4	2	0

2	2	1	1	3	0	3	*Counts*
3	4	5	6	7	8	9	

7	9	11	12	13	16	16	*Running sum of Counts*
3	4	5	6	7	8	9	

9	10	12	12	15	16	19	*Next positions*
3	4	5	6	7	8	9	

1	1		3	3	4		5		7	7		9	9	9	Output
		6			9			12			15			18	

istribution Counting Example (II)

9	1	9	1	9	5	3	7	3	1	6	**7**	4	2	0

2	2	1	1	3	0	3	*Counts*
3	4	5	6	7	8	9	

7	9	11	12	13	16	16	*Running sum of Counts*
3	4	5	6	7	8	9	

9	10	12	13	16	16	19	*Next positions*
3	4	5	6	7	8	9	

1	1		3	3	4		5	6	7	7	7	9	9	9	Output
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2	2	1	1	3	0	3	*Counts*
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7	9	11	12	13	16	16	*Running sum of Counts*
3	4	5	6	7	8	9	
9	11	12	13	16	16	19	*Next positions*
3	4	5	6	7	8	9	

1	1	2	3	3	4	4	5	6	7	7	7	9	9	9	Output
		6			9			12			15			18	

MSD Radix Sort

omplicated: must keep lists from each step separate

processing 1-element lists

A	posn
cat, cad, con, bat, can, be, let, bet	0
be, bet / cat, cad, con, can / let / set	1
$\star$ be, bet / cat, cad, con, can / let / set	2
be / bet / $\star$ cat, cad, con, can / let / set	1
be / bet / $\star$ cat, cad, can / con / let / set	2
be / bet / cad / can / cat / con / let / set	

And Don't Forget Search Trees

h tree is in sorted order, when read in inorder.

e to really use for sorting [next topic].

e, same performance as heapsort: N insertions in time us $\Theta(N)$ to traverse, gives

$$\Theta(N + N \lg N) = \Theta(N \lg N)$$

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9	1	9	1	9	5	3	7	3	1	6	7	4	2	0

2	2	1	1	3	0	3	*Counts*
3	4	5	6	7	8	9	
7	9	11	12	13	16	16	*Running sum of Counts*
3	4	5	6	7	8	9	
9	11	12	13	16	16	19	*Next positions*
3	4	5	6	7	8	9	

1	1	2	3	3	4	4	5	6	7	7	7	9	9	9	Output
		6			9			12			15			18	

Radix Sort

ys one character at a time.

ribution counting for each digit.

her right to left (LSD radix sort) or left to right (MSD

rt is venerable: used for punched cards.

Initial: set, cat, cad, con, bat, can, be, let, bet

e	cad	can con	bet let bat cat set
'	'd'	'n'	't'

n, can, set, cat, bat, let, bet

Pass 2 (by char #1)

bat cat can cad	bet let set be	con
'a'	'e'	'o'

cad, can, cat, bat, be, set, let, bet, con

Pass 3 (by char #0)

bet be bat	con cat can cad	let	set
'b'	'c'	'l'	's'

bat, be, bet, cad, can, cat, con, let, set

Performance of Radix Sort

akes $\Theta(B)$ time where B is total size of the key data.

ed other sorts as function of #records.

are?

ifferent records, must have keys at least $\Theta(\lg N)$ long

, comparison actually takes time $\Theta(K)$ where K is size st case [why?]

mparisons really means $N(\lg N)^2$ operations.

sort would take $B = N \lg N$ time with minimal-length

r hand, must work to get good constant factors with

Summary

rt: $\Theta(Nk)$ comparisons and moves, where k is maximum
is displaced from final position.

small datasets or almost ordered data sets.

$(N \lg N)$ with good constant factor if data is not patho-
t case $O(N^2)$.

$\Theta(N \lg N)$ guaranteed. Good for external sorting.

eesort with guaranteed balance: $\Theta(N \lg N)$ guaranteed.

istribution sort: $\Theta(B)$ (number of bytes). Also good for
ting.