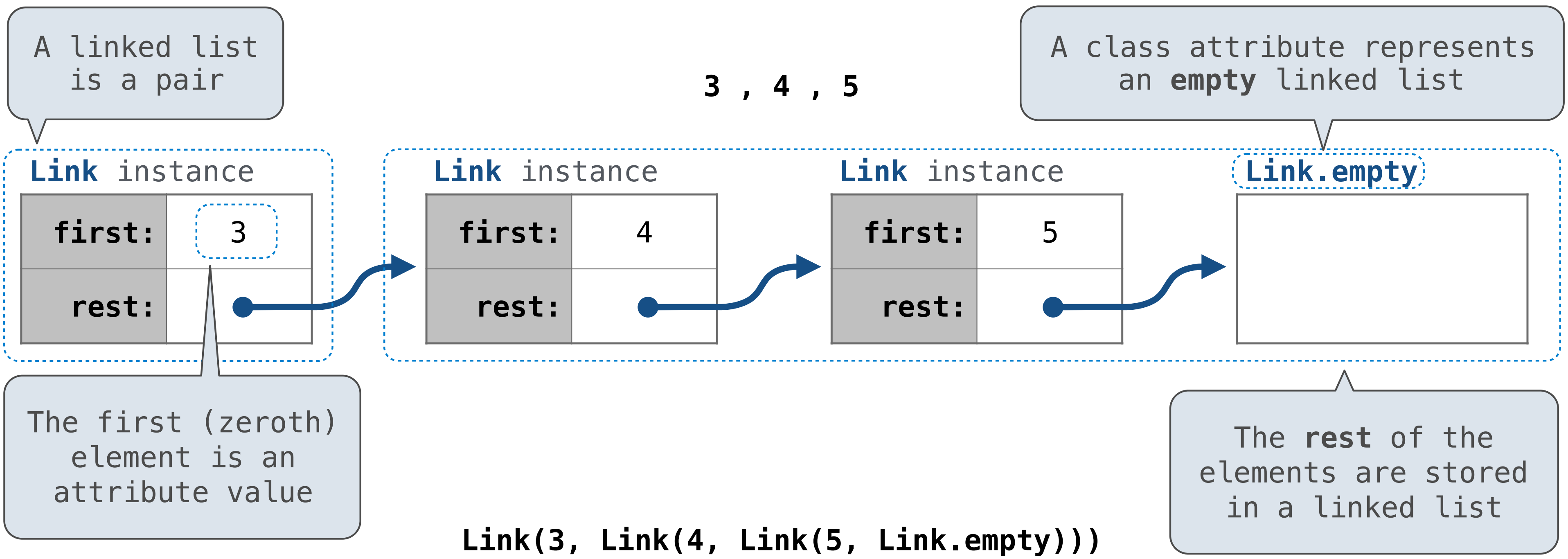
Linked Lists

Linked List Structure

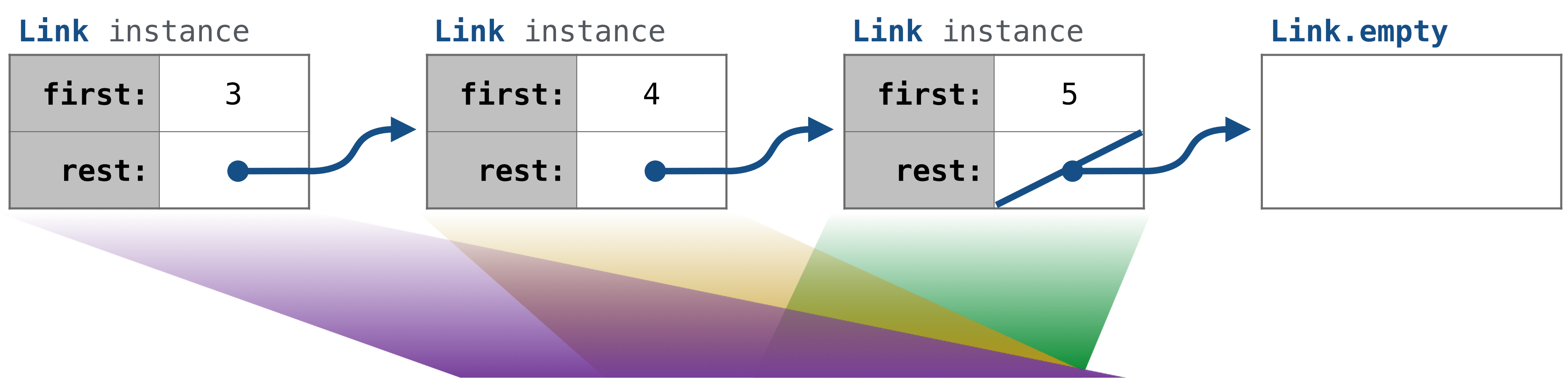
A linked list is either empty **or** a first value and the rest of the linked list



Linked List Structure

A linked list is either empty **or** a first value and the rest of the linked list

3 , 4 , 5



`Link(3, Link(4, Link(5, Link.empty)))`

Linked List Class

Linked list class: attributes are passed to `__init__`

```
class Link:

    empty = ()

    def __init__(self, first, rest=empty):
        assert rest is Link.empty or isinstance(rest, Link)
        self.first = first
        self.rest = rest
```

Some zero-length sequence

Returns whether rest is a Link

help(isinstance): Return whether an object is an instance of a class or of a subclass thereof.

Link(3, Link(4, Link(5)))

(Demo)

Property Methods

Property Methods

In some cases, we want the value of instance attributes to be computed on demand

For example, if we want to access the second element of a linked list

```
>>> s = Link(3, Link(4, Link(5)))
>>> s.second
4
>>> s.second = 6
>>> s.second
6
>>> s
Link(3, Link(6, Link(5)))
```

No method calls!

The @property decorator on a method designates that it will be called whenever it is looked up on an instance

A @<attribute>.setter decorator on a method designates that it will be called whenever that attribute is assigned. <attribute> must be an existing property method.

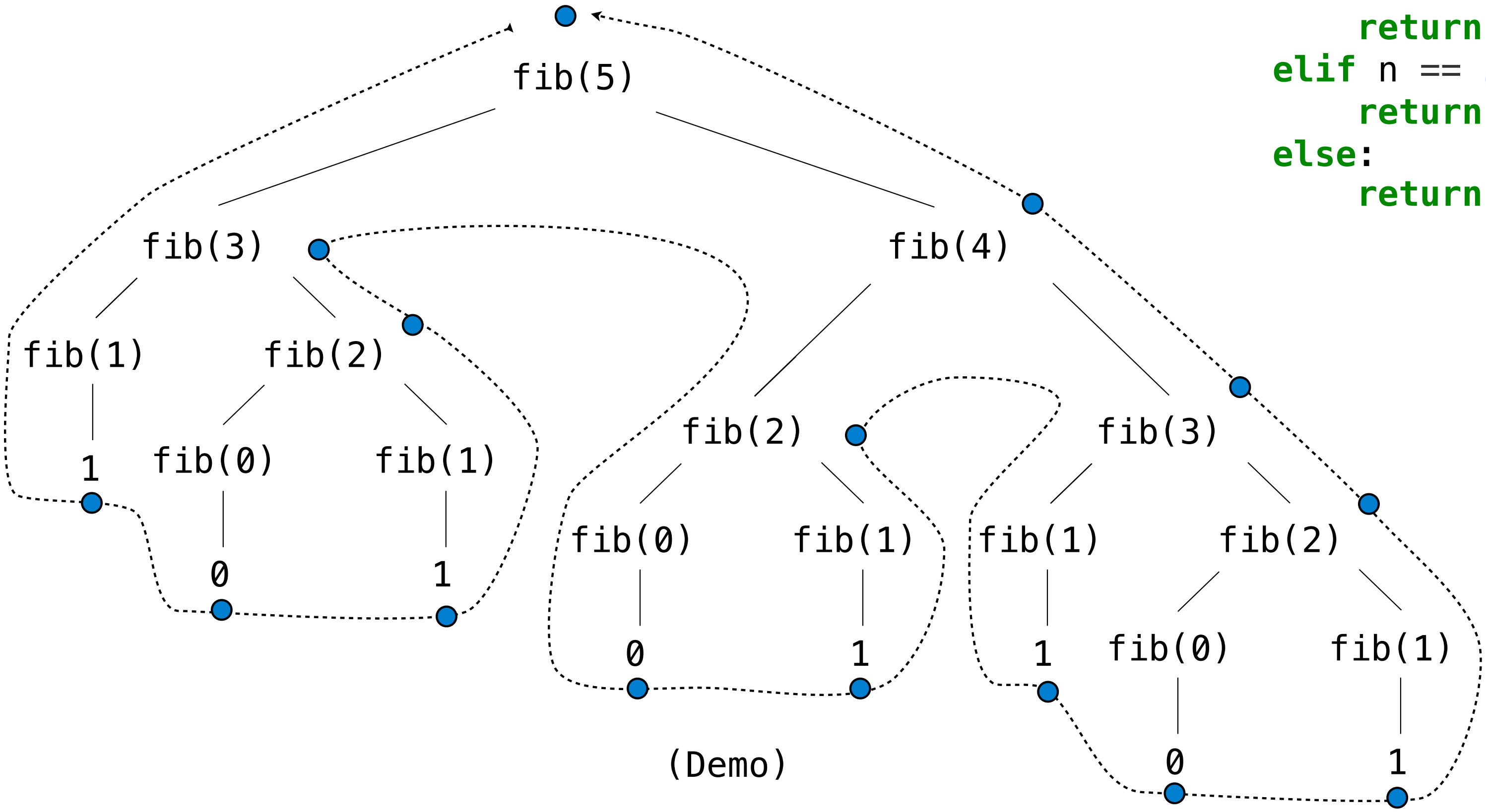
(Demo)

Tree Recursion Efficiency

Recursive Computation of the Fibonacci Sequence

Our first example of tree recursion:

```python
def fib(n):
    if n == 0:
        return 0
    elif n == 1:
        return 1
    else:
        return fib(n-2) + fib(n-1)
```



(Demo)

http://en.wikipedia.org/wiki/File:Fibonacci.jpg

Memoization

Memoization

Idea: Remember the results that have been computed before

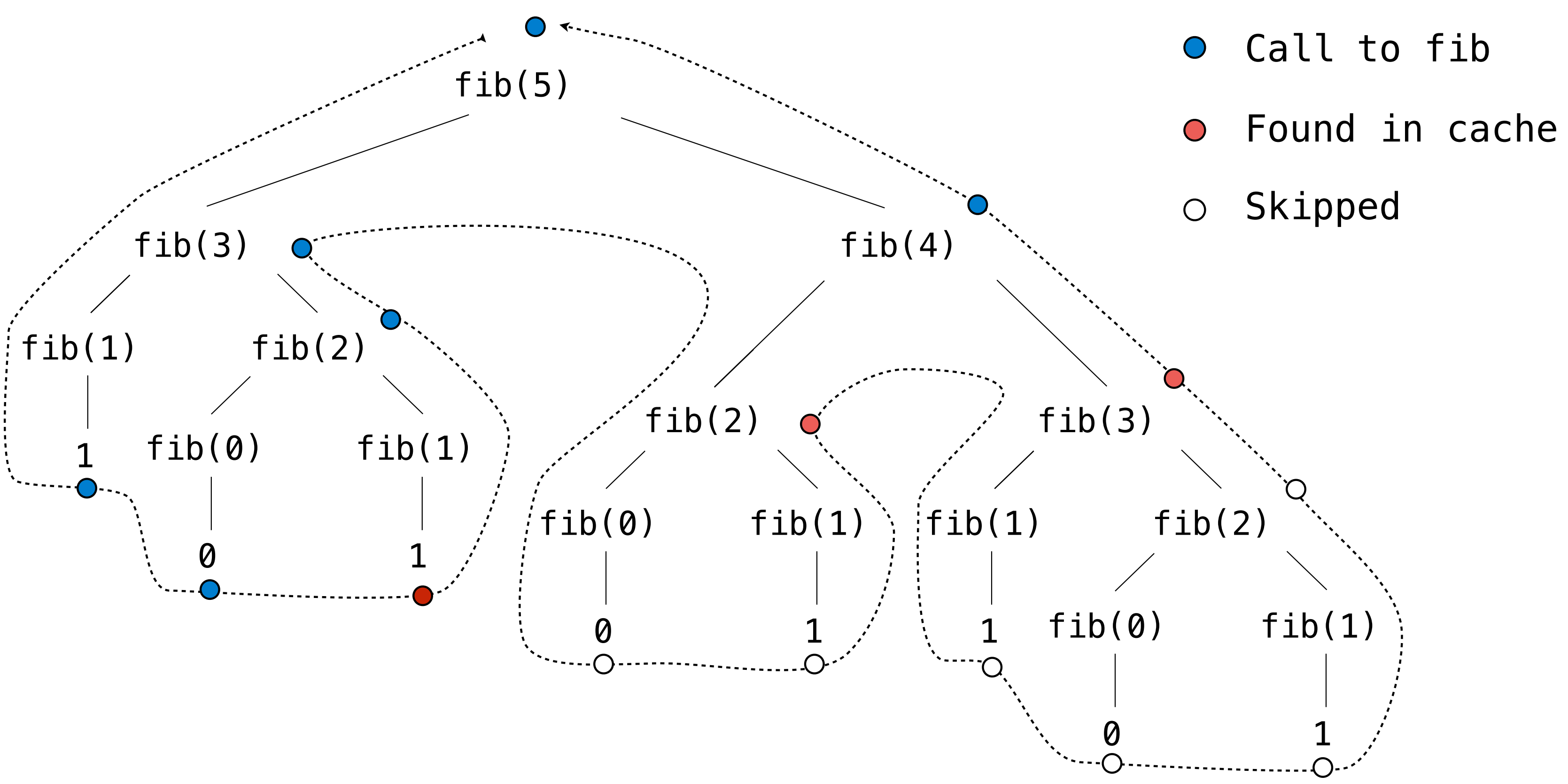
```
def memo(f):
    cache = {}
    def memoized(n):
        if n not in cache:
            cache[n] = f(n)
        return cache[n]
    return memoized
```

Keys are arguments that map to return values

Same behavior as f, if f is a pure function

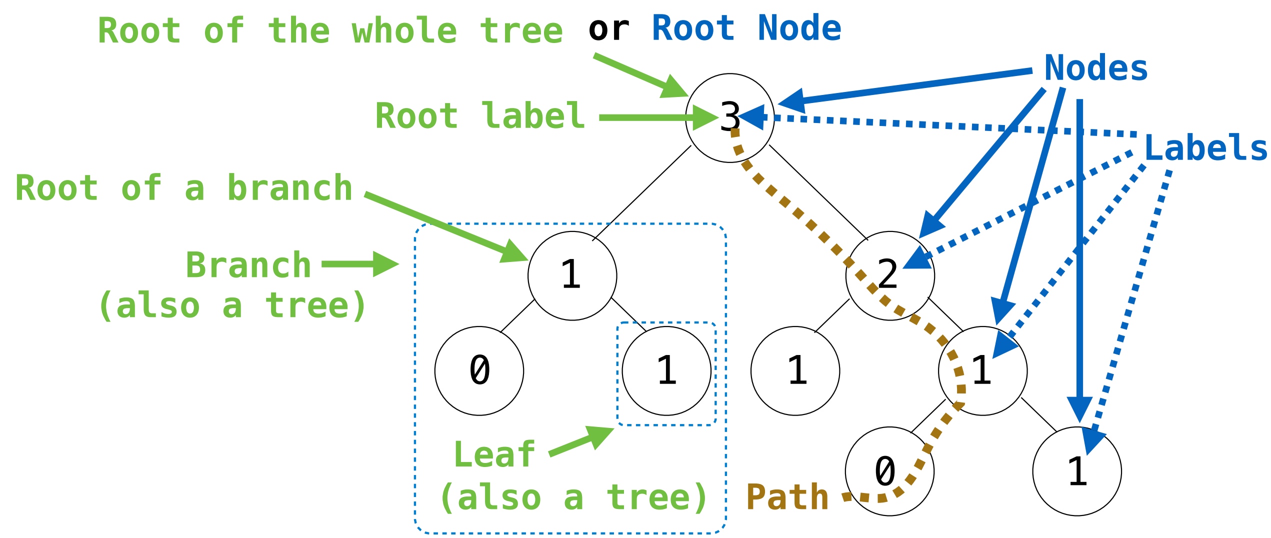
(Demo)

Memoized Tree Recursion



Tree Class

Tree Abstraction (Review)



Recursive description **(wooden trees):**

A **tree** has a **root label** and a list of **branches**

Each **branch** is a **tree**

A **tree** with zero **branches** is called a **leaf**

A **tree** starts at the **root**

Relative description **(family trees):**

Each location in a tree is called a **node**

Each **node** has a **label** that can be any value

One node can be the **parent/child** of another

The top node is the **root node**

People often refer to labels by their locations: "each parent is the sum of its children"

Tree Class

A Tree has a label and a list of branches; each branch is a Tree

```
class Tree:
    def __init__(self, label, branches=[]):
        self.label = label
        for branch in branches:
            assert isinstance(branch, Tree)
        self.branches = list(branches)
```

```
def tree(label, branches=[]):
    for branch in branches:
        assert is_tree(branch)
    return [label] + list(branches)
def label(tree):
    return tree[0]
def branches(tree):
    return tree[1:]
```

```
def fib_tree(n):
    if n == 0 or n == 1:
        return Tree(n)
    else:
        left = fib_tree(n-2)
        right = fib_tree(n-1)
        fib_n = left.label + right.label
        return Tree(fib_n, [left, right])
```

```
def fib_tree(n):
    if n == 0 or n == 1:
        return tree(n)
    else:
        left = fib_tree(n-2)
        right = fib_tree(n-1)
        fib_n = label(left) + label(right)
        return tree(fib_n, [left, right])
```

(Demo)

Measuring Efficiency

Recursive Computation of the Fibonacci Sequence

Our first example of tree recursion:

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    if n == 0:
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        return 1
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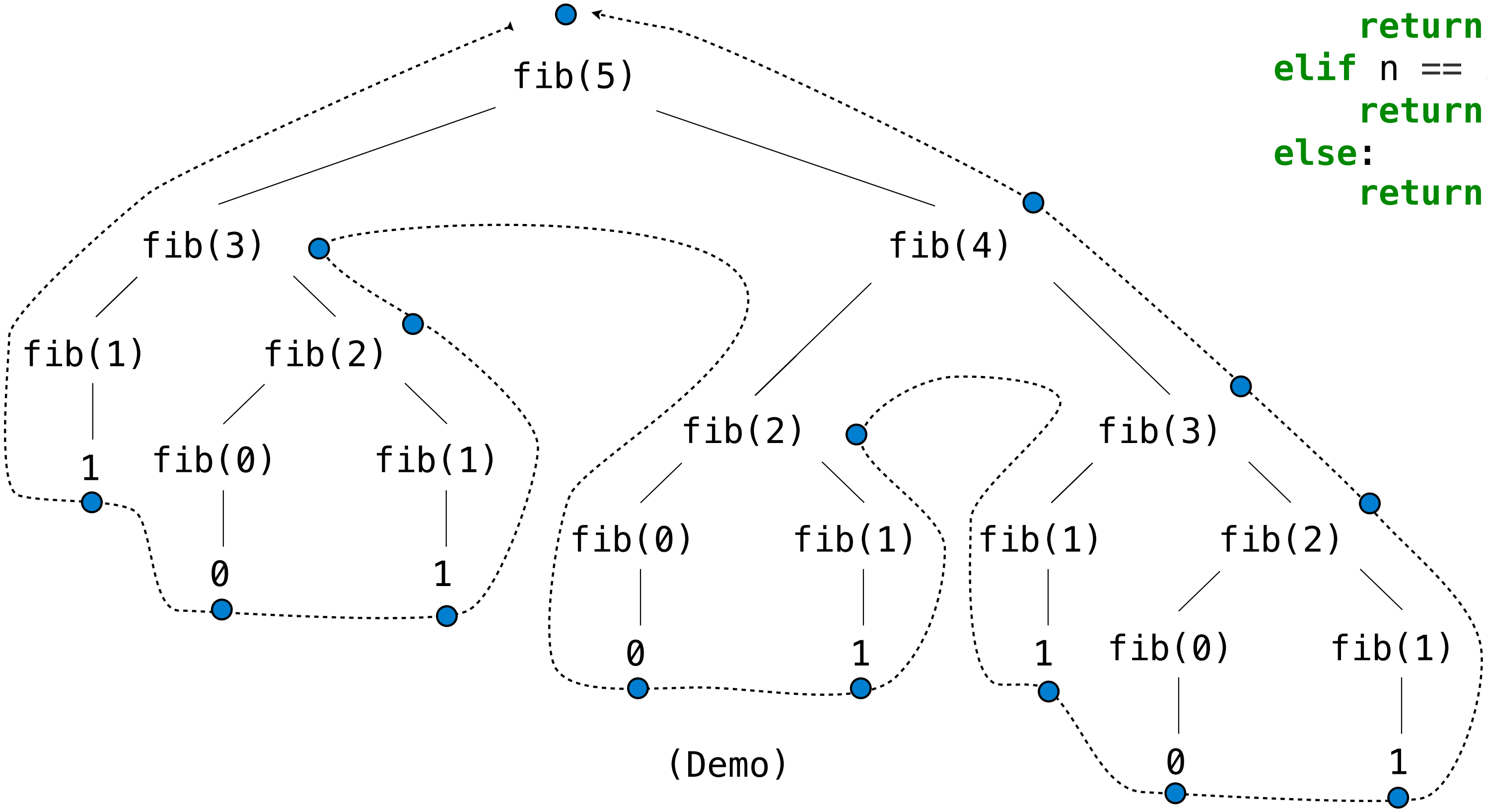

(Demo)

http://en.wikipedia.org/wiki/File:Fibonacci.jpg

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def memo(f):
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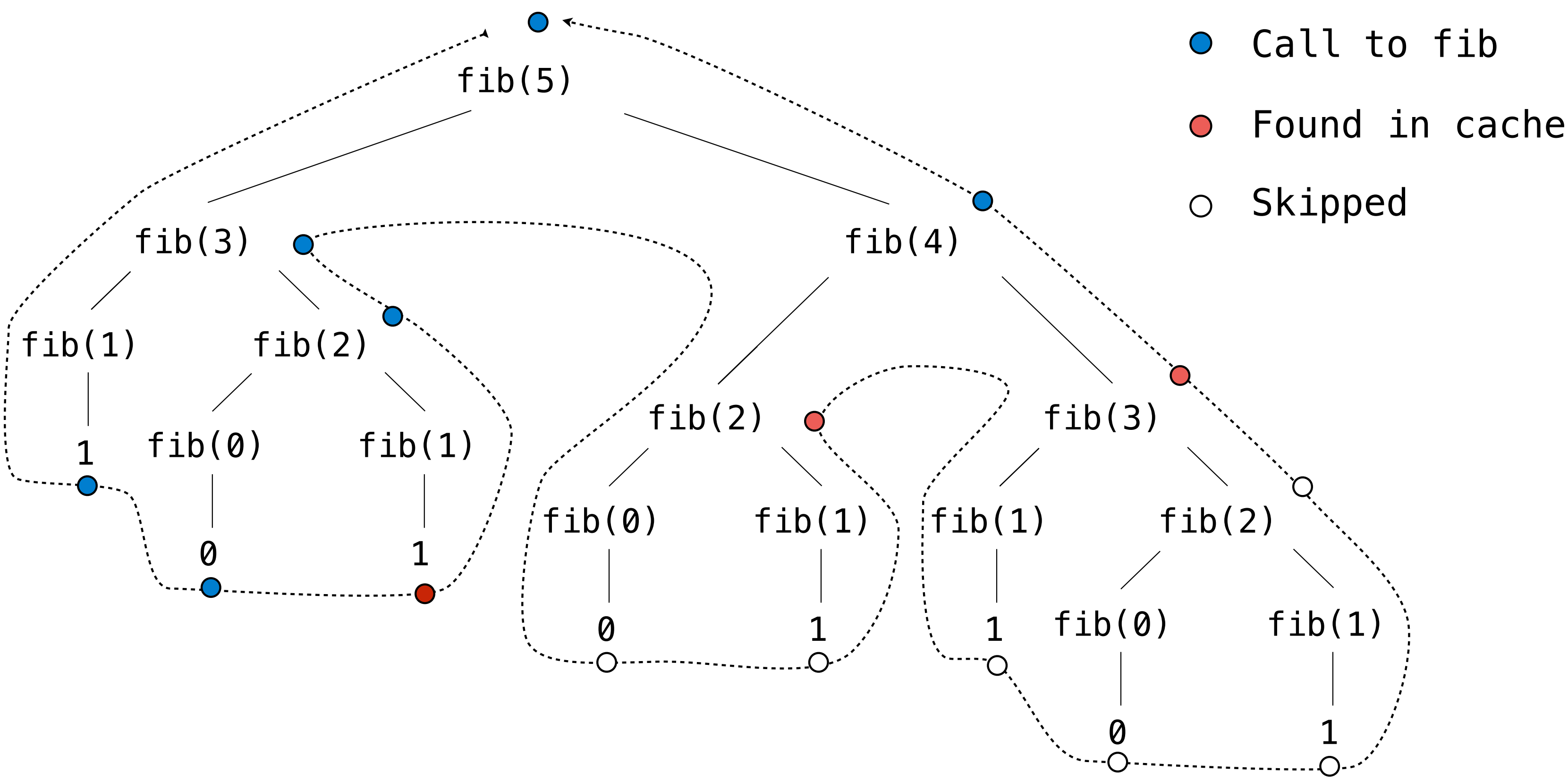
Same behavior as f, if f is a pure function

(Demo)

Memoized Tree Recursion



Exponentiation

Exponentiation

Goal: one more multiplication lets us double the problem size

```
def exp(b, n):
    if n == 0:
        return 1
    else:
        return b * exp(b, n-1)
```

$$b^n = \begin{cases} 1 & \text{if } n = 0 \\ b \cdot b^{n-1} & \text{otherwise} \end{cases}$$

```
def exp_fast(b, n):
    if n == 0:
        return 1
    elif n % 2 == 0:
        return square(exp_fast(b, n//2))
    else:
        return b * exp_fast(b, n-1)

def square(x):
    return x * x
```

$$b^n = \begin{cases} 1 & \text{if } n = 0 \\ (b^{\frac{1}{2}n})^2 & \text{if } n \text{ is even} \\ b \cdot b^{n-1} & \text{if } n \text{ is odd} \end{cases}$$

(Demo)

Exponentiation

Goal: one more multiplication lets us double the problem size

```
def exp(b, n):
    if n == 0:
        return 1
    else:
        return b * exp(b, n-1)
```

Linear time:

- Doubling the input **doubles** the time
- 1024x the input takes 1024x as much time

```
def exp_fast(b, n):
    if n == 0:
        return 1
    elif n % 2 == 0:
        return square(exp_fast(b, n//2))
    else:
        return b * exp_fast(b, n-1)

def square(x):
    return x * x
```

Logarithmic time:

- Doubling the input **increases** the time by a constant C
- 1024x the input increases the time by only 10 times C

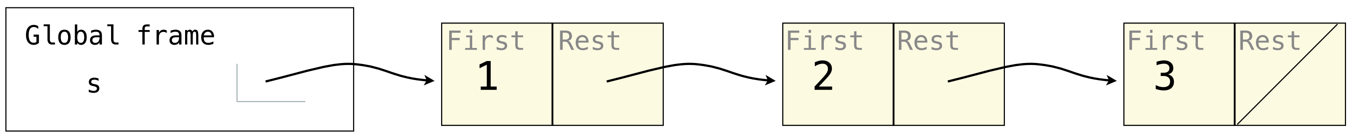
Mutable Linked Lists

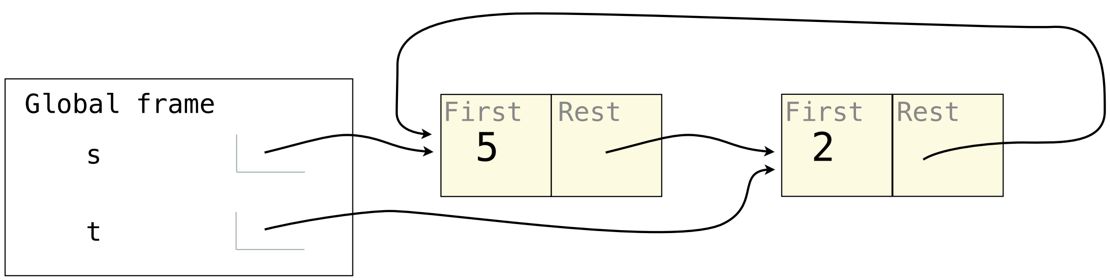
Recursive Lists Can Change

Attribute assignment statements can change first and rest attributes of a Link

The rest of a linked list can contain the linked list as a sub-list

```
>>> s = Link(1, Link(2, Link(3)))
>>> s.first = 5
>>> t = s.rest
>>> t.rest = s
>>> s.first
5
>>> s.rest.rest.rest.rest.rest.first
2
```

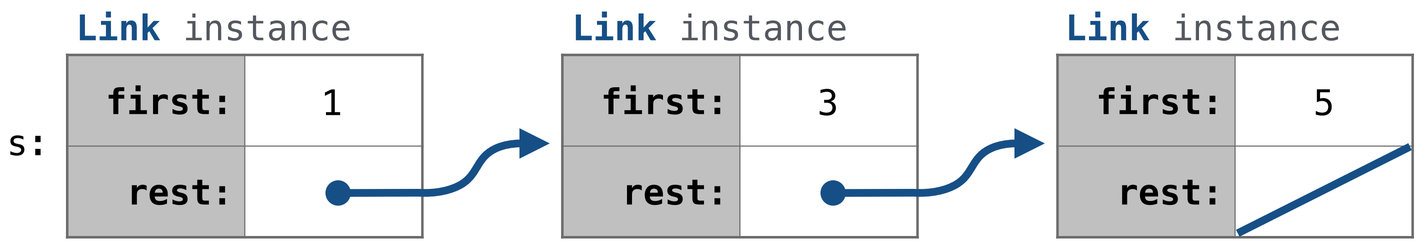




Note: The actual environment diagram is much more complicated.

Linked List Mutation Example

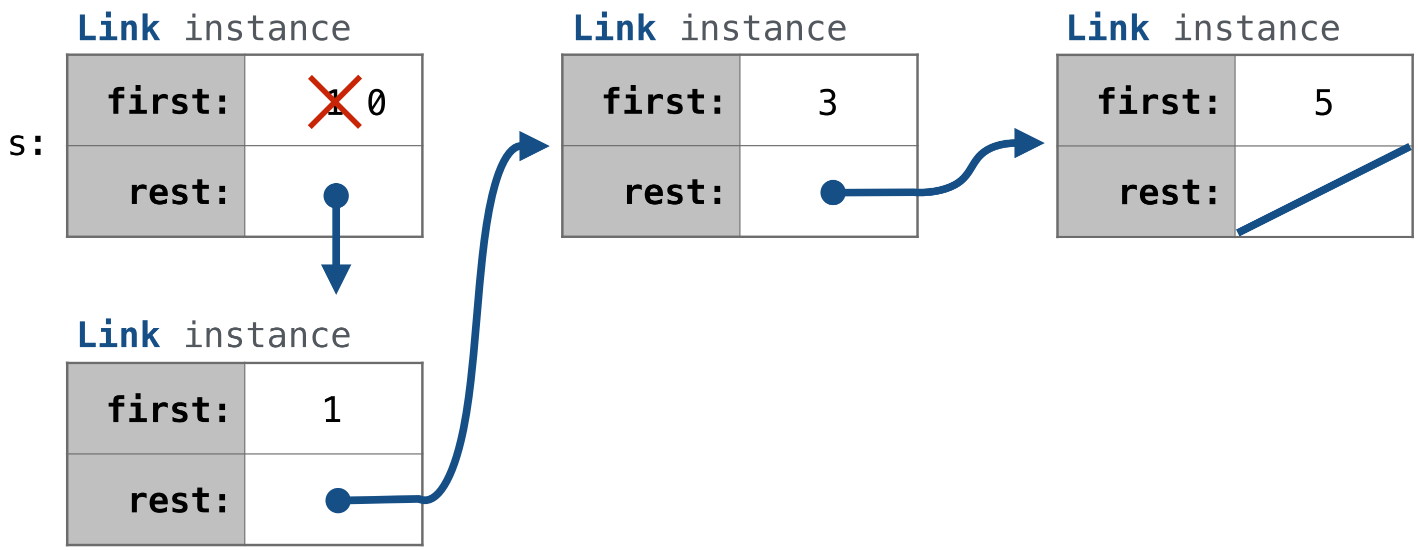
Adding to an Ordered List




```
def add(s, v):
    """Add v to an ordered list s with no repeats, returning modified s."""
    (Note: If v is already in s, then don't modify s, but still return it.)


        add(s, 0)
```

Adding to an Ordered List

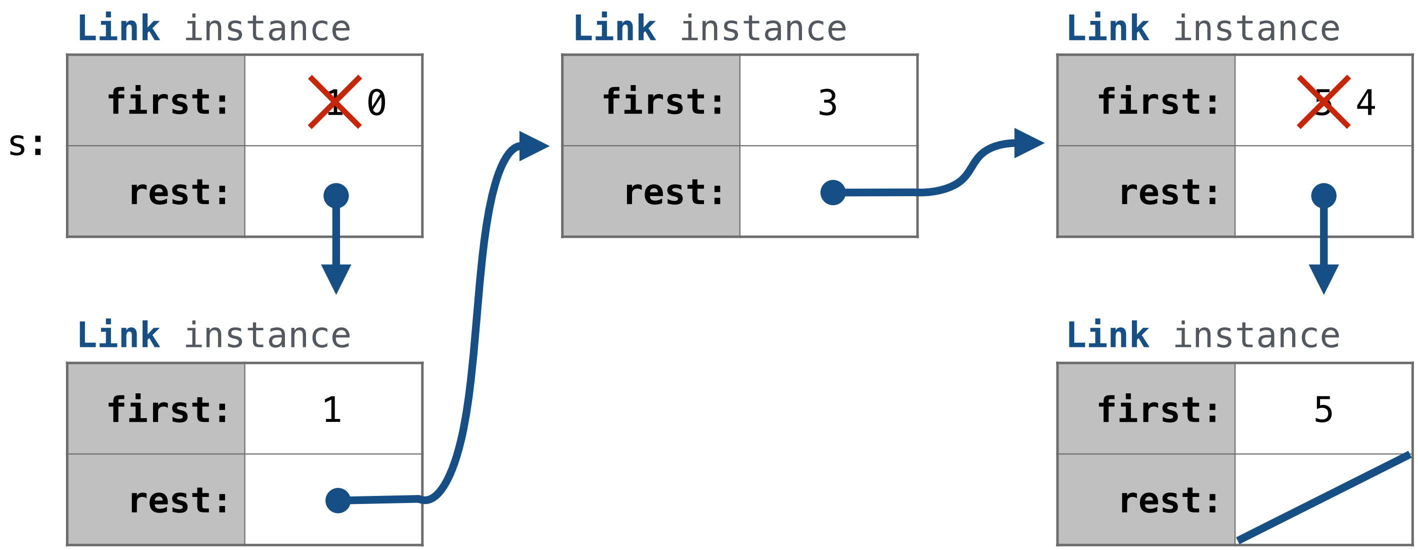



```
def add(s, v):
    """Add v to an ordered list s with no repeats, returning modified s."""
```

(**Note:** If v is already in s, then don't modify s, but still return it.)

```
add(s, 0)      add(s, 3)      add(s, 4)
```

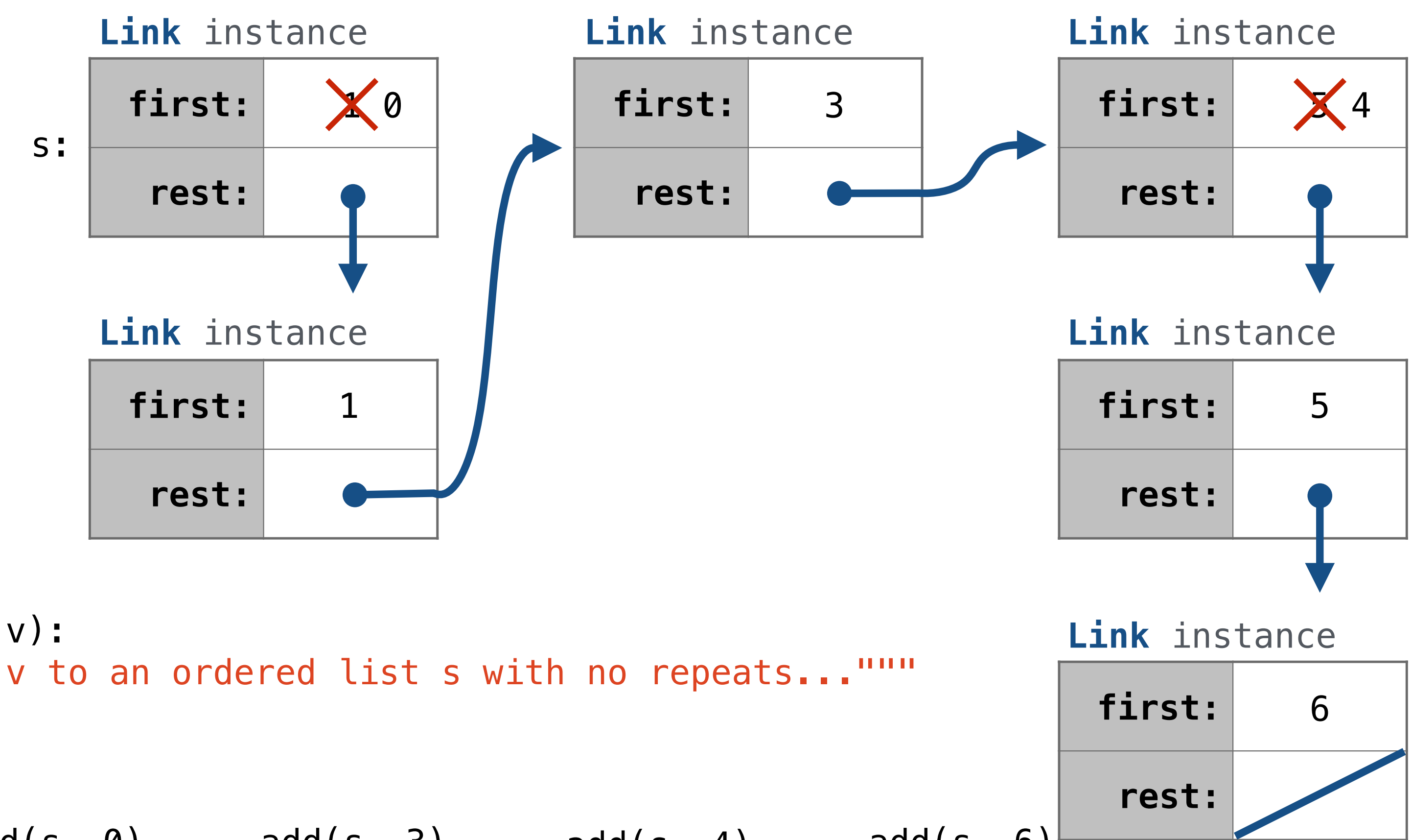
Adding to an Ordered List




```
def add(s, v):
    """Add v to an ordered list s with no repeats..."""
```

add(s, 0)　　add(s, 3)　　add(s, 4)　　add(s, 6)

Adding to an Ordered List




```
def add(s, v):
    """Add v to an ordered list s with no repeats..."""
```

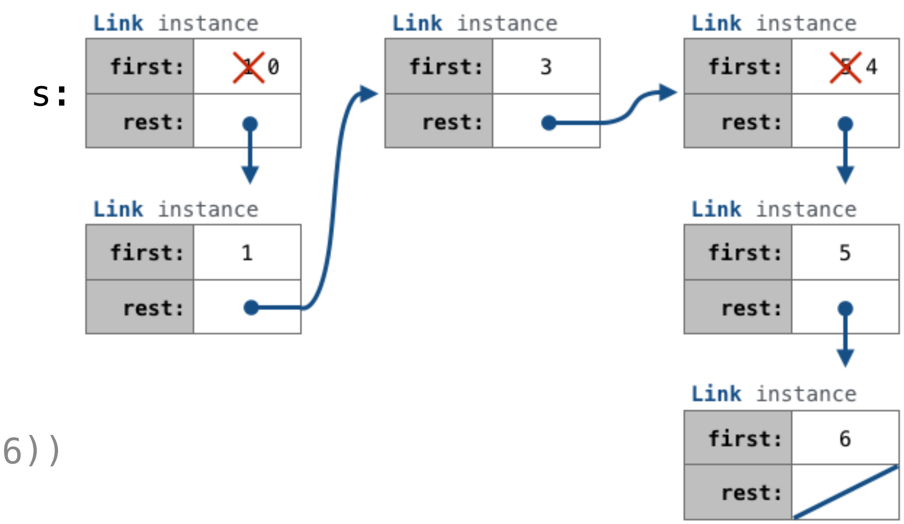
`add(s, 0)` `add(s, 3)` `add(s, 4)` `add(s, 6)`

Adding to a Set Represented as an Ordered List

```
def add(s, v):
    """Add v to a set s, returning modified s."""

    >>> s = Link(1, Link(3, Link(5)))
    >>> add(s, 0)
    Link(0, Link(1, Link(3, Link(5))))
    >>> add(s, 3)
    Link(0, Link(1, Link(3, Link(5))))
    >>> add(s, 4)
    Link(0, Link(1, Link(3, Link(4, Link(5)))))
    >>> add(s, 6)
    Link(0, Link(1, Link(3, Link(4, Link(5, Link(6))
    """

    assert s is not Link.empty
    if s.first > v:
        s.first, s.rest = ____v____ , ____Link(s.first, s.rest)____
    elif s.first < v and empty(s.rest):
        s.rest = ____Link(v)____
    elif s.first < v:
        ____add(s.rest, v)____
    return s
```

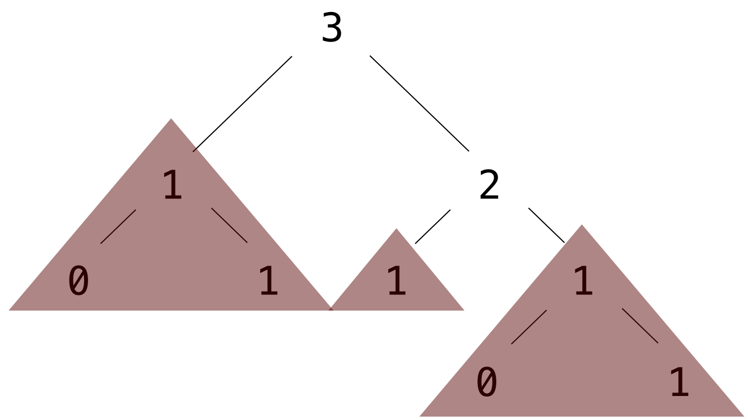


Tree Mutation

Example: Pruning Trees

Removing subtrees from a tree is called *pruning*

Prune branches before recursive processing




```
def prune(t, n):
    """Prune all sub-trees whose label is n."""
    t.branches = [___b___ for b in t.branches if ___b.label != n___]
    for b in t.branches:
        prune(___b___, ___n___)
```