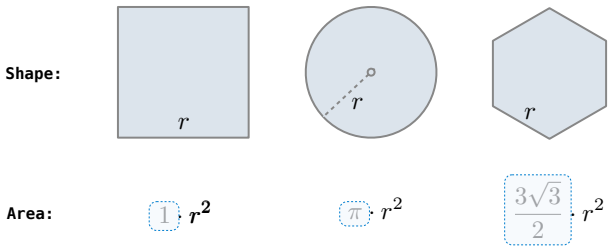
Higher-Order Functions

Generalizing Patterns with Arguments

Regular geometric shapes relate length and area.



Finding common structure allows for shared implementation

(Demo)

Generalizing Over Computational Processes

The common structure among functions may be a computational process, rather than a number.

$$\sum_{k=1}^{5} k = 1 + 2 + 3 + 4 + 5 = 15$$

$$\sum_{k=1}^{5} k^3 = 1^3 + 2^3 + 3^3 + 4^3 + 5^3 = 225$$

$$\sum_{k=1}^{5} \frac{8}{(4k-3)\cdot(4k-1)} = \frac{8}{3} + \frac{8}{35} + \frac{8}{99} + \frac{8}{195} + \frac{8}{323} = 3.04$$

(Demo)

Summation Example

```
def cube(k):
    return pow(k, 3)

def summation(n, term):
    """Sum the first n terms of a sequence.

    >>> summation(5, cube)
    225
    """
    total, k = 0, 1
    while k <= n:
        total, k = total + term(k), k + 1
    return total
```

Function of a single argument (*not called "term"*)

A formal parameter that will be bound to a function

The cube function is passed as an argument value

0 + 1 + 8 + 27 + 64 + 125

The function bound to term gets called here

Functions as Return Values

(Demo)

Locally Defined Functions

Functions defined within other function bodies are bound to names in a local frame

```
def make_adder(n):
    """Return a function that takes one argument k and returns k + n.
    >>> add_three = make_adder(3)
    >>> add_three(4)
    7
    """
    def adder(k):
        return k + n
    return adder
```

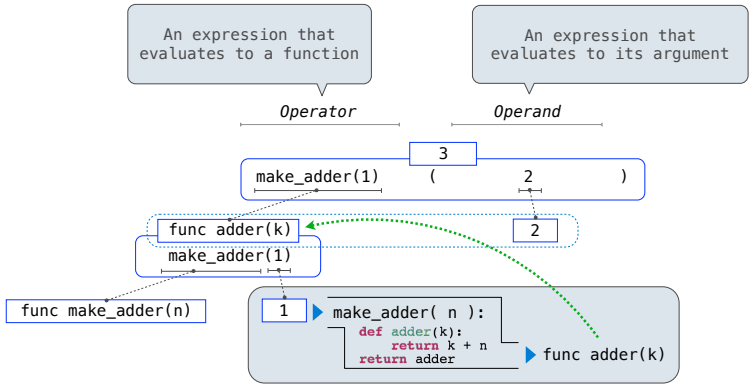
A function that returns a function

The name add_three is bound to a function

A def statement within another def statement

Can refer to names in the enclosing function

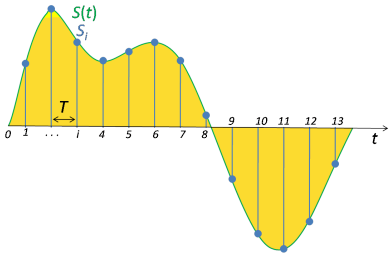
Call Expressions as Operator Expressions



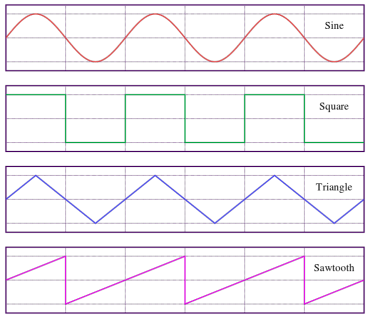
Function Example: Sounds

WAV Files

The Waveform Audio File Format encodes a sampled sound wave



A triangle wave is the simple wave form with the most pleasing sound



(Demo)

https://en.wikipedia.org/wiki/Triangle_wave
https://en.wikipedia.org/wiki/Sampling_(signal_processing)

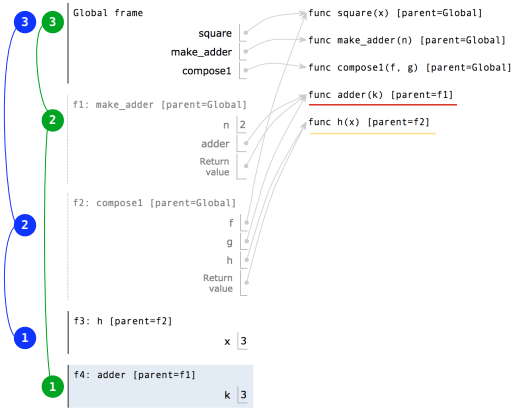
Function Composition

(Demo)

The Environment Diagram for Function Composition

```
1  def square(x):
2      return x * x
3
4  def make_adder(n):
5      def adder(k):
6          return k + n
7      return adder
8
9  def compose1(f, g):
10     def h(x):
11         return f(g(x))
12     return h
13
14 compose1(square, make_adder(2))(3)
```

Return value of make_adder is an argument to compose1



Abstraction

Functional Abstractions

```
def square(x):
    return mul(x, x)
```

```
def sum_squares(x, y):
    return square(x) + square(y)
```

What does sum_squares need to know about square?

- Square takes one argument. **Yes**
- Square has the intrinsic name square. **No**
- Square computes the square of a number. **Yes**
- Square computes the square by calling mul. **No**

```
def square(x):
    return pow(x, 2)
```

```
def square(x):
    return mul(x, x-1) + x
```

If the name "square" were bound to a built-in function, sum_squares would still work identically.

Choosing Names

Names typically don't matter for correctness

but

they matter a lot for composition

From:	To:
true_false	rolled_a_one
d	dice
helper	take_turn
my_int	num_rolls
l, I, O	k, i, m

Names should convey the meaning or purpose of the values to which they are bound.

The type of value bound to the name is best documented in a function's docstring.

Function names typically convey their effect (**print**), their behavior (**triple**), or the value returned (**abs**).

Which Values Deserve a Name

Reasons to add a new name

Repeated compound expressions:

```
if sqrt(square(a) + square(b)) > 1:
    x = x + sqrt(square(a) + square(b))
```

```
hypotenuse = sqrt(square(a) + square(b))
if hypotenuse > 1:
    x = x + hypotenuse
```

PRACTICAL GUIDELINES

Meaningful parts of complex expressions:

```
x1 = (-b + sqrt(square(b) - 4 * a * c)) / (2 * a)
```

```
discriminant = square(b) - 4 * a * c
x1 = (-b + sqrt(discriminant)) / (2 * a)
```

More Naming Tips

- Names can be long if they help document your code:

```
average_age = average(age, students)
```

is preferable to

```
# Compute average age of students
aa = avg(a, st)
```

- Names can be short if they represent generic quantities: counts, arbitrary functions, arguments to mathematical operations, etc.

n, k, i – Usually integers
x, y, z – Usually real numbers
f, g, h – Usually functions