Dictionaries, Matrices, and Trees

Today will cover...

- Announcements
- Dictionary type
- Matrix abstraction
- Tree abstraction
- Tree processing

Review: Layers of abstraction

Primitive Representation	`1 2 3 True False` `(..,..) [..,..]`

Data abstraction	`make_rat() numer() denom()`
	 `add_rat() mul_rat()`
	 `print_rat() equal_rat()`

User program	`exact_harmonic_number()`

Each layer only uses the layer above it.

Review: Python types

Type	Examples
Integers	`0 -1 0xFF 0b1101`
Booleans	`True False`
Functions	`def f(x)... lambda x: ...`
Strings	`"pear""I say, \"hello!\""`
Tuples	`(1, 10) ("Oh", "hi", 11)`
Ranges	`range(11) range(1, 6)`
Lists	`[] ["apples", "bananas"]` `[x**3 for x in range(2)]`

Dictionaries

A `dict` is a mutable mapping of key-value pairs

```
states = {
         "CA": "California",
         "DE": "Delaware",
         "NY": "New York",
         "TX": "Texas",
         "WY": "Wyoming"
}
```

Queries:

```
>>> len(states)
```

```
>>> "CA" in states
```

```
>>> "ZZ" in states
```

Dictionaries

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```
>>> len(states)
5
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```
>>> "CA" in states
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```
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```
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True
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```
>>> "ZZ" in states
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}
```

Queries:

```
>>> len(states)
5
```

```
>>> "CA" in states
True
```

```
>>> "ZZ" in states
False
```

Dictionary selection

```
words = {
        "más": "more",
        "otro": "other",
        "agua": "water"
}
```

Select a value:

```
>>> words["otro"]
```

```
>>> first_word = "agua"
>>> words[first_word]
```

```
>>> words["pavo"]
```

```
>>> words.get("pavo", "🤔")
```

Dictionary selection

```
words = {
        "más": "more",
        "otro": "other",
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}
```

Select a value:

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>>> words["otro"]
'other'
```

```
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>>> words["pavo"]
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>>> words.get("pavo", "🤔")
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'water'
```

```
>>> words["pavo"]
KeyError: pavo
```

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>>> words.get("pavo", "🤔")
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Dictionary selection

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Select a value:

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>>> first_word = "agua"
>>> words[first_word]
'water'
```

```
>>> words["pavo"]
KeyError: pavo
```

```
>>> words.get("pavo", "🤔")
'🤔'
```

Dictionary mutation

Create an empty dict:

```
users = {}
```

Add values:

```
users["profpamela"] = "b3stp@ssEvErDontHackMe"
```

Change values:

```
users["profpamela"] += "itsLongerSoItsMoreSecure!!"
```

```
>>> users["profpamela"]
```

Dictionary mutation

Create an empty dict:

```
users = {}
```

Add values:

```
users["profpamela"] = "b3stp@ssEvErDontHackMe"
```

Change values:

```
users["profpamela"] += "itsLongerSoItsMoreSecure!!"
```

```
>>> users["profpamela"]
'b3stp@ssEvErDontHackMeitsLongerSoItsMoreSecure!!'
```

Dictionary rules

- A key **cannot** be a list or dictionary (or any mutable type)
- All keys in a dictionary are distinct (there can only be one value per key)
- The values can be any type, however!

```
spiders = {
  "smeringopus": {
          "name": "Pale Daddy Long-leg",
          "length": 7
  },
  "holocnemus pluchei": {
          "name": "Marbled cellar spider",
          "length": (5, 7)
  }
}
```

Dictionary iteration

```
insects = {"spiders": 8, "centipedes": 100, "bees": 6}
for name in insects:
    print(insects[name])
```

...is the same as:

```
for name in list(insects):
    print(insects[name])
```

What will be the order of items?

Dictionary iteration

```
insects = {"spiders": 8, "centipedes": 100, "bees": 6}
for name in insects:
    print(insects[name])
```

...is the same as:

```
for name in list(insects):
    print(insects[name])
```

What will be the order of items?

```
8 100 6
```

Keys are iterated over in the order they are first added.

Nested data

Lists of lists	`[ [1, 2], [3, 4] ]`
Lists of tuples	`[ (1, 2), (3, 4) ]`
Tuples of tuples	`( (1, 2), (3, 4) )`
Dicts of tuples	`{"x": (1, 2), "y": (3, 4)}`
Dicts of lists	`{"heights": [89, 97], "ages": [6, 8]}`

...what else?!

Nested data

Lists of lists	`[ [1, 2], [3, 4] ]`
Lists of tuples	`[ (1, 2), (3, 4) ]`
Tuples of tuples	`( (1, 2), (3, 4) )`
Dicts of tuples	`{"x": (1, 2), "y": (3, 4)}`
Dicts of lists	`{"heights": [89, 97], "ages": [6, 8]}`

...what else?! Dicts of dicts, Lists of dicts, etc.

Next up: more abstractions

Matrices

Consider a matrix (two-dimensional table) like this:

1	2	0	4
0	1	3	-1
0	0	1	8

That matrix has three **rows** and four **columns**, with integer values in each location.

Matrices: Data abstraction

We want this constructor, selector, and mutator:

`matrix(rows, cols)`	Returns a ROWS x COLS matrix with all values set to 0
`value(matrix, row, col)`	Returns value of MATRIX at (ROW, COL)
`set_value(matrix, row, col, val)`	Sets value of MATRIX at (ROW, COL) to VAL

How could we implement? Answer the poll!

Matrices: Implementation A

A list of lists, row-major order:

```
[ [1,2,0,4], [0,1,3,-1], [0,0,1,8] ]
```

```
def matrix(rows, cols):
        return [ [0 for col in range(cols)] for row in range(rows) ]

def value(matrix, row, col):
        return matrix[row][col]

def set_value(matrix, row, col, val):
        matrix[row][col] = val
```

```
m = matrix(3, 4)
set_value(m, 0, 0, 1)
set_value(m, 0, 1, 2)
set_value(m, 0, 3, 4)
```

Matrices: Implementation B

A list of lists, column-major order:

```
[ [1,0,0], [2,1,0], [0,3,1], [4,-1,8] ]
```

```
def matrix(rows, cols):
        return [ [0 for row in range(rows)] for col in range(cols) ]

def value(matrix, row, col):
        return matrix[col][row]

def set_value(matrix, row, col, val):
        matrix[col][row] = val
```

```
m = matrix(3, 4)
set_value(m, 0, 0, 1)
set_value(m, 0, 1, 2)
set_value(m, 0, 3, 4)
```

Matrices: Implementation C

A tuple of lists, row-major order:

```
( [1,0,0], [2,1,0], [0,3,1], [4, -1,8] )
```

```
def matrix(rows, cols):
        return tuple( [0 for col in range(cols)] for row in range(rows) )

def value(matrix, row, col):
        return matrix[row][col]

def set_value(matrix, row, col, val):
        matrix[row][col] = val
```

```
m = matrix(3, 4)
set_value(m, 0, 0, 1)
set_value(m, 0, 1, 2)
set_value(m, 0, 3, 4)
```

Matrices: Implementation D

A list of tuples?

```
[ (1,2,0,4), (0,1,3,-1), (0,0,1,8) ]
```

```
def matrix(rows, cols):
    return [ tuple(0 for col in range(cols))
        for row in range(rows) ]

def value(matrix, row, col):
    return matrix[row][col]

def set_value(matrix, row, col, val):
    matrix[row][col] = val
```

```
m = matrix(3, 4)
set_value(m, 0, 0, 1)
set_value(m, 0, 1, 2)
set_value(m, 0, 3, 4)
```

Matrices: Implementation D ❌

A list of tuples?

`[ (1,2,0,4), (0,1,3,-1), (0,0,1,8) ]`

```
def matrix(rows, cols):
    return [ tuple(0 for col in range(cols))
        for row in range(rows) ]

def value(matrix, row, col):
    return matrix[row][col]

def set_value(matrix, row, col, val):
    matrix[row][col] = val
```

```
m = matrix(3, 4)
set_value(m, 0, 0, 1) ❌
set_value(m, 0, 1, 2) ❌
set_value(m, 0, 3, 4) ❌
```

Matrices: Implementation D2

A list of tuples?

```
[ (1,2,0,4), (0,1,3,-1), (0,0,1,8) ]
```

```
def matrix(rows, cols):
    return [ tuple(0 for col in range(cols))
        for row in range(rows) ]

def value(matrix, row, col):
    return matrix[row][col]

def set_value(matrix, row, col, val):
    matrix[row] = matrix[row][:col] + (val,) + matrix[row][col+1:]
```

```
m = matrix(3, 4)
set_value(m, 0, 0, 1)
set_value(m, 0, 1, 2)
set_value(m, 0, 3, 4)
```

Designing an implementation

Which implementation was your favorite?

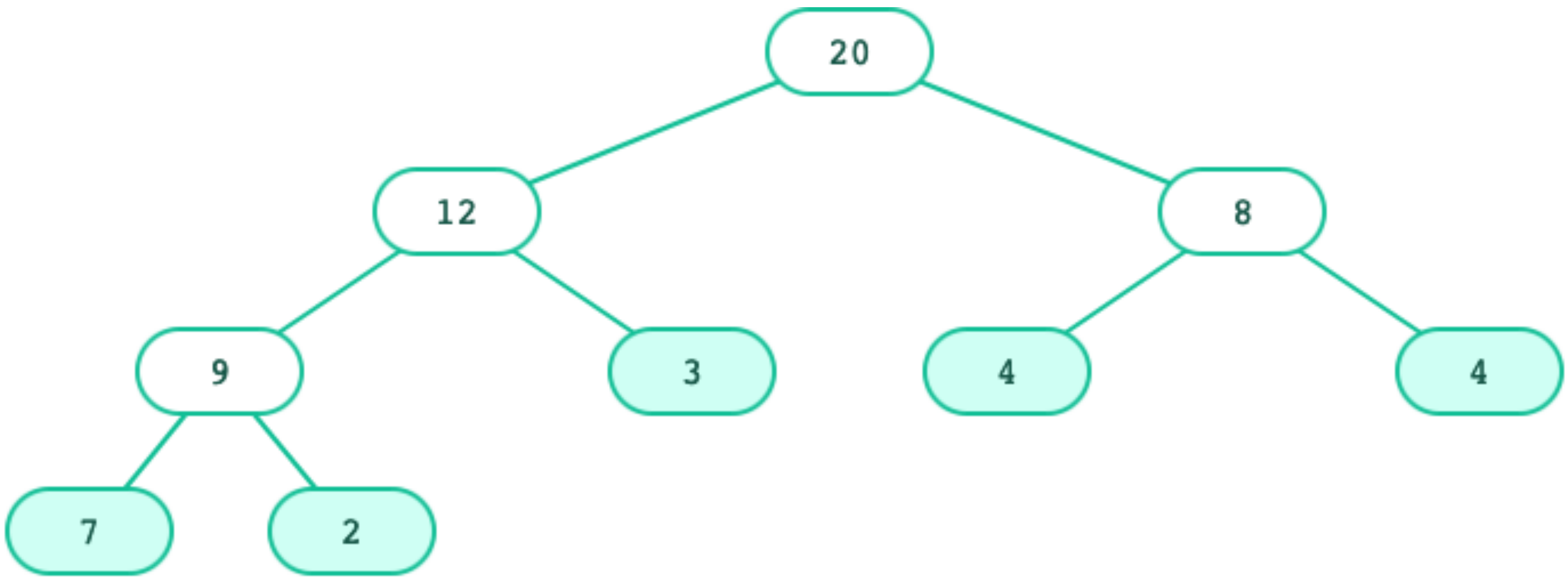
Answer the poll!

When might you use a tuple?

When might you use a list?

When might you use a dict?

Trees



- Each oval is a **node**
- Top node is the **root**
- Each node is itself the root of another tree (called a **subtree**); the nodes immediately below are its **children**
- Nodes without children are **leaves**; others are **inner nodes**
- Each node generally has a **label**

Trees: Data abstraction

We want this constructor and selectors:

`tree(label, children)`	Returns a tree with given LABEL at its root, whose children are CHILDREN
`label(tree)`	Returns the label of root node of TREE
`children(tree)`	Returns the children of TREE (each a tree).
`is_leaf(tree)`	Returns true if TREE is a leaf node.

How could we implement? Answer the poll!

Trees: Implementation A

A list of label + list for each tree/subtree:

```
[20,[12,[9,[7],[2]],[3]],[8,[4],[4]]]
```

```
def tree(label, children=[]):
        return [label] + children

def label(tree):
        return tree[0]

def children(tree):
        return tree[1:]

def is_leaf(tree):
        return len(children(tree)) == 0
```

```
t = tree(20, [tree(12,
               [tree(9,
                  [tree(7), tree(2)]),
                tree(3)]),
              tree(8,
                [tree(4), tree(4)])])
```

Trees: Implementation B

A number-list tuple for each tree/subtree:

`(20,[(12,[(9,[(7,[]),(2, [])]),(3, [])]),(8,[(4,[]),(4, [])])])`

```
def tree(label, children=[]):
  return (label, children)

def label(tree):
  return tree[0]

def children(tree):
  return tree[1]
```

```
t = tree(20, [tree(12,
               [tree(9,
                 [tree(7), tree(2)]),
                tree(3)]),
              tree(8,
               [tree(4), tree(4)])])
```

Trees: Implementation C

A dictionary for each tree/subtree:

```
{'l':20,'c':[{'l':12,'c':[{'l':9,'c':[{'l':7,'c': []},{'l':2,'c':[]}]},{'l':3,'c':[]}]},

{'l':8,'c':[{'l':4,'c':[]},{'l':4,'c':[]}]}]}
```

```
def tree(label, children=[]):
    return {"l": label, "c": children}

def label(tree):
    return tree["l"]

def children(tree):
        return tree["c"]
```

```
t = tree(20, [tree(12,
               [tree(9,
                 [tree(7), tree(2)]),
                tree(3)]),
              tree(8,
                [tree(4), tree(4)])])
```

Tree processing

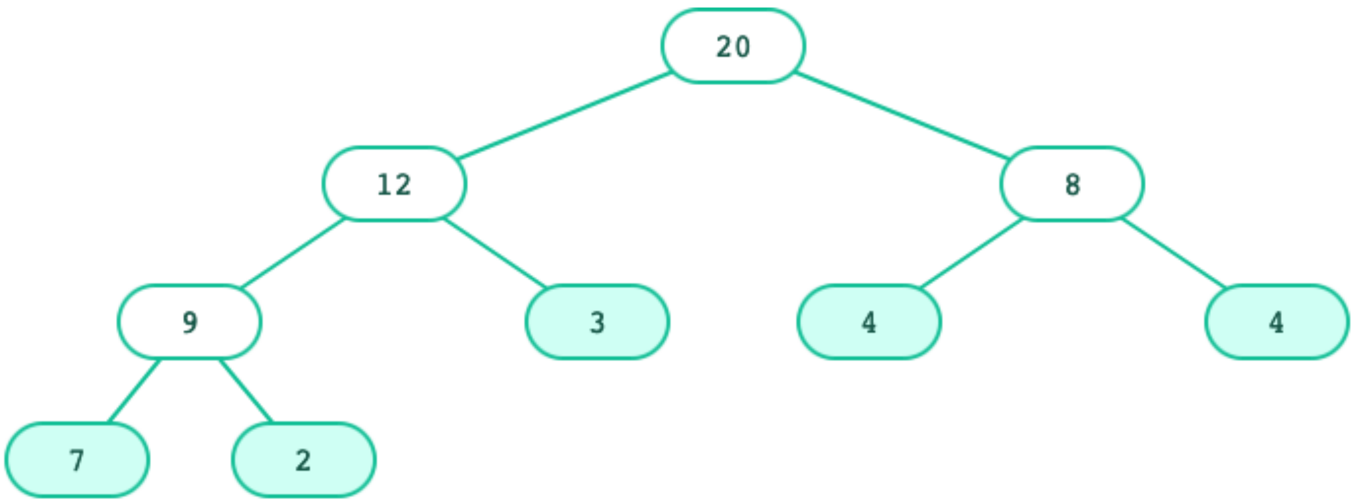
A tree is a recursive structure.

Each tree has:

- A label
- 0 or more children, each a tree

Recursive structure implies recursive algorithm!

Tree processing: Counting leaves

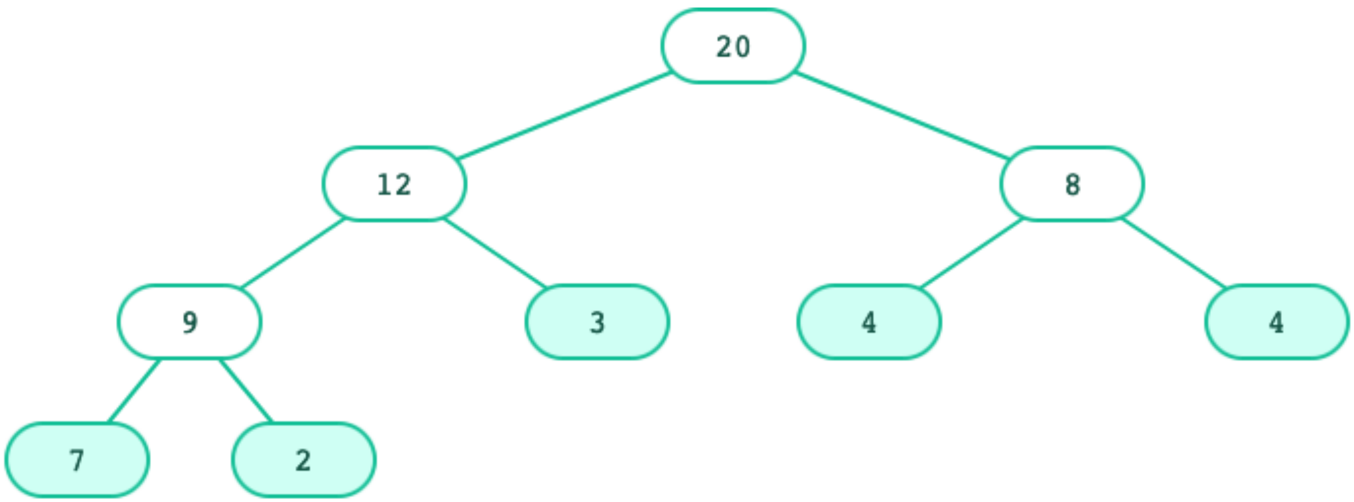



```
def count_leaves(t):
    """Returns the number of leaf nodes in T."""
    if

    else:
```

What's the base case? What's the recursive call?

Tree processing: Counting leaves

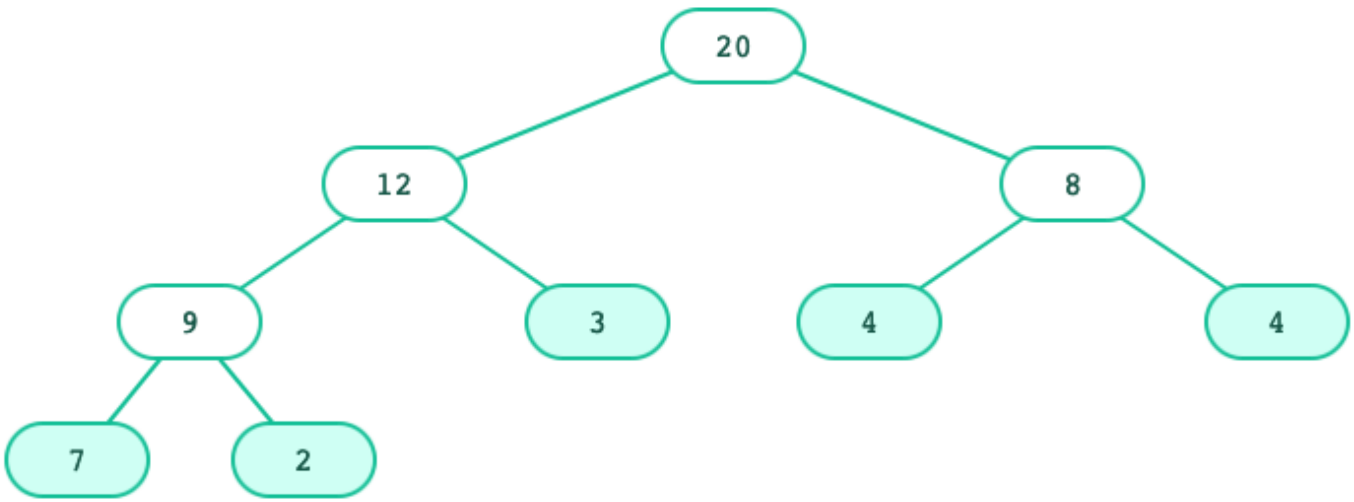



```
def count_leaves(t):
    """Returns the number of leaf nodes in T."""
    if is_leaf(t):

    else:
```

What's the base case? What's the recursive call?

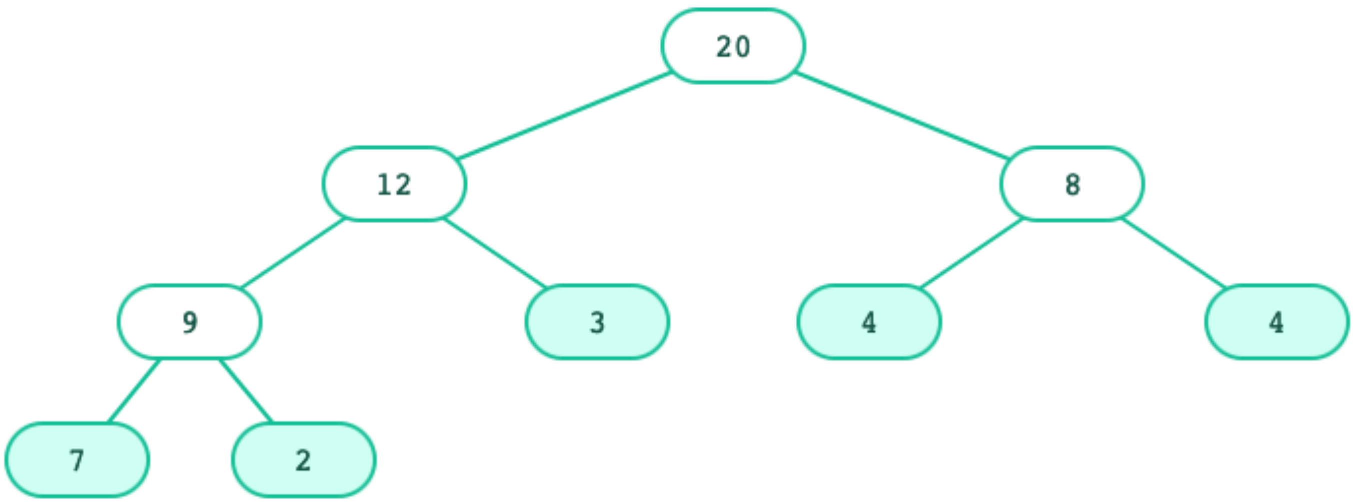
Tree processing: Counting leaves




```
def count_leaves(t):
    """Returns the number of leaf nodes in T."""
    if is_leaf(t):
        return 1
    else:
```

What's the base case? What's the recursive call?

Tree processing: Counting leaves




```python
def count_leaves(t):
    """Returns the number of leaf nodes in T."""
    if is_leaf(t):
        return 1
    else:
        children_leaves = 0
        for c in children(t):
            children_leaves += count_leaves(c)
        return children_leaves
```

What's the base case? What's the recursive call?

Tree processing: Counting leaves

The `sum()` function sums up the items of an iterable.

```
>>> sum([1, 1, 1, 1])
4
```

Tree processing: Counting leaves

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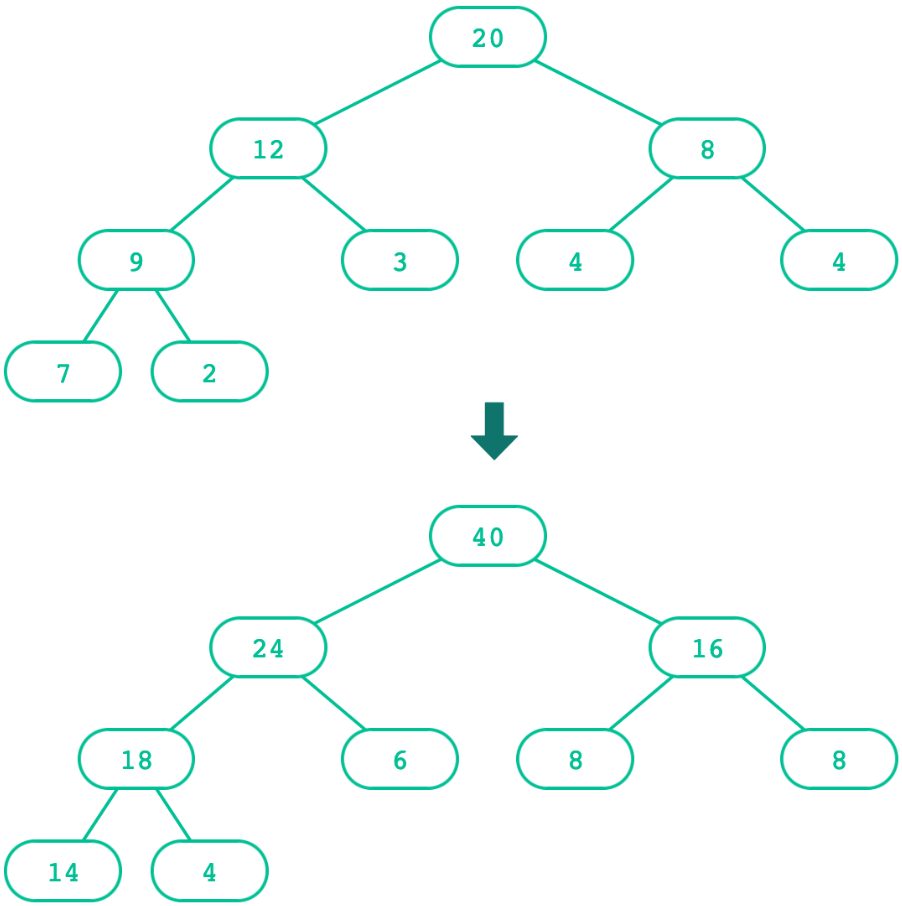
```
>>> sum([1, 1, 1, 1])
4
```

That leads to this shorter function:

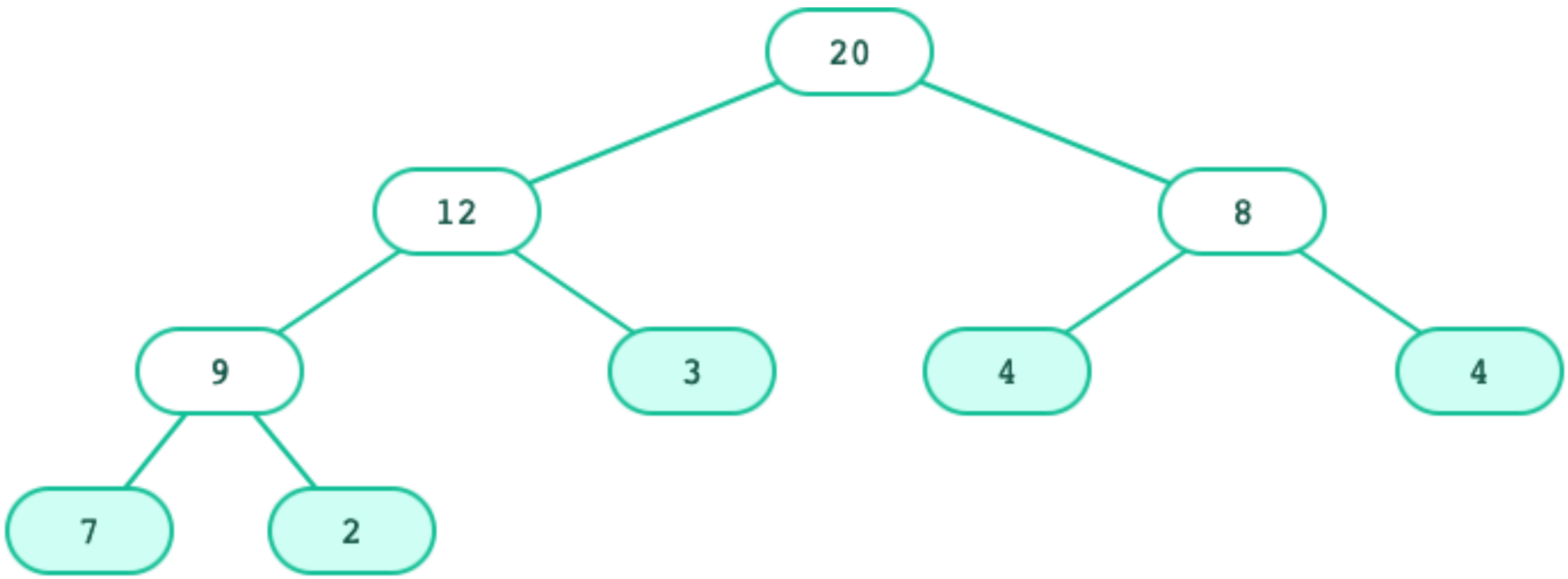
```
def count_leaves(t):
    """Returns the number of leaf nodes in T."""
    if is_leaf(t):
        return 1
    else:
        return sum([count_leaves(c) for c in children(t)])
```

Creating trees

A function that creates a tree from another tree is also often recursive.



Creating trees: Doubling labels

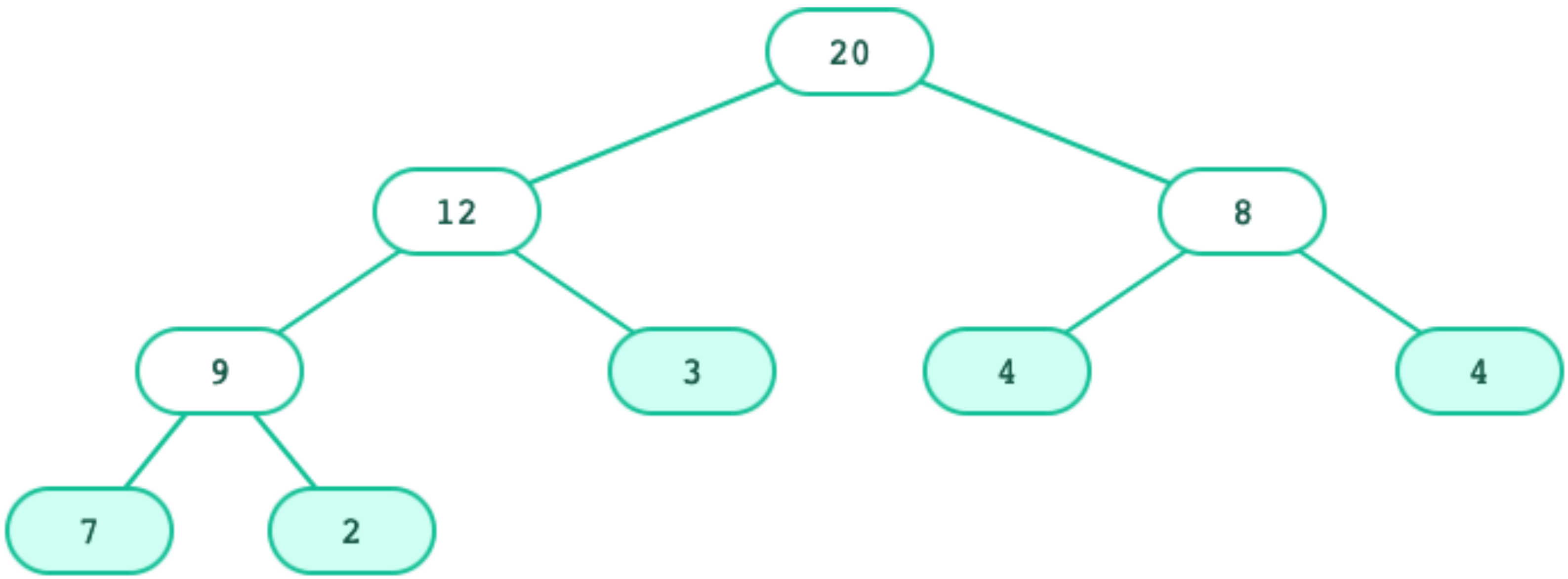



```
def double(t):
    """Returns a tree identical to T, but with all labels doubled."""
    if

    else:
```

What's the base case? What's the recursive call?

Creating trees: Doubling labels

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def double(t):
    """Returns a tree identical to T, but with all labels doubled."""
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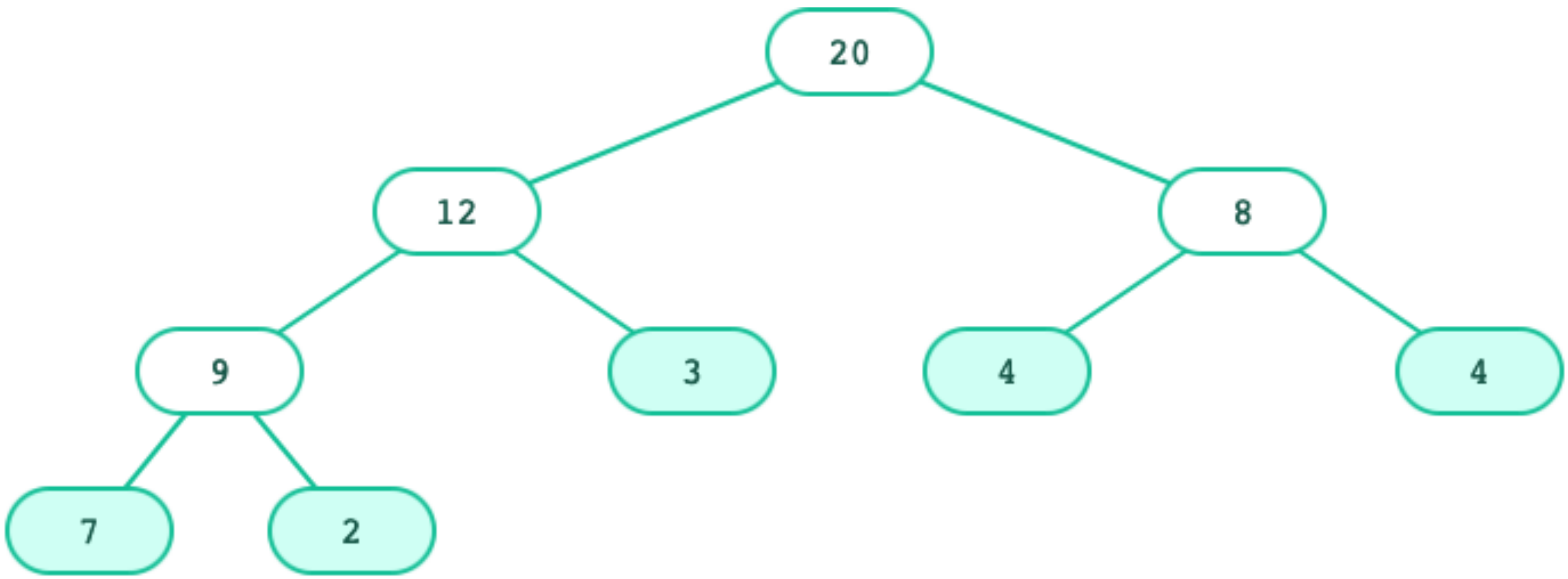
    else:

```

What's the base case? What's the recursive call?

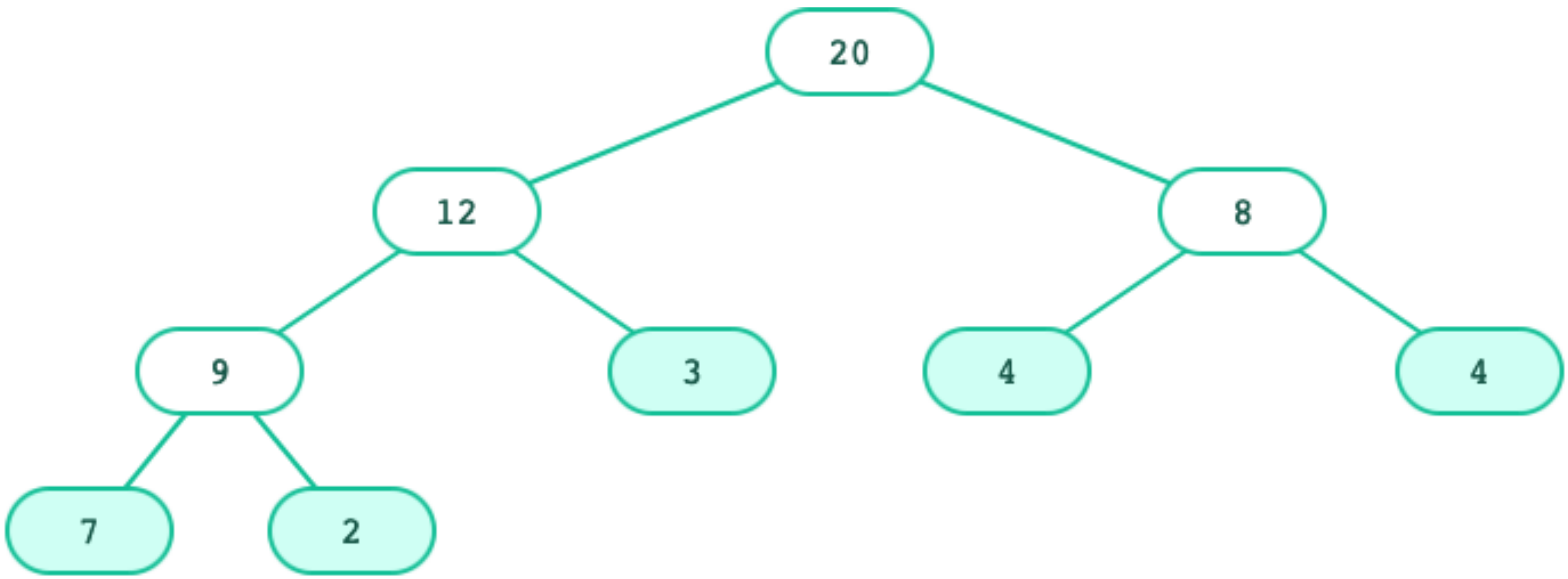
Creating trees: Doubling labels




```
def double(t):
    """Returns a tree identical to T, but with all labels doubled."""
    if is_leaf(t):
        return tree(label(t) * 2)
    else:
```

What's the base case? What's the recursive call?

Creating trees: Doubling labels




```
def double(t):
    """Returns a tree identical to T, but with all labels doubled."""
    if is_leaf(t):
        return tree(label(t) * 2)
    else:
        return tree(label(t) * 2,
            [double(c) for c in children(t)])
```

What's the base case? What's the recursive call?

Creating trees: Doubling labels

Longer...

```
def double(t):
        """Returns a tree identical to T, but with all labels doubled."""
    if is_leaf(t):
        return tree(label(t) * 2)
    else:
        doubled_children = []
        for c in children(t):
            doubled_children.append(double(c))
        return tree(label(t) * 2, doubled_children)
```

Shorter!

```
def double(t):
    """Returns the number of leaf nodes in T."""
    return tree(label(t) * 2,
            [double(c) for c in children(t)])
```

Challenge: List of leaves

Try this on your own:

```
def list_of_leaves(t):
    """Return a list containing the leaf labels of T.

    >>> leaves(t) # Using the t from the slides
    [7, 2, 3, 4, 4]
    """
    if ______:
        return ______
    else:
        ______
        return ______
```

Hint: If you sum a list of lists, you get a list containing the elements of those lists. The sum function takes a second argument, the starting value of the sum.

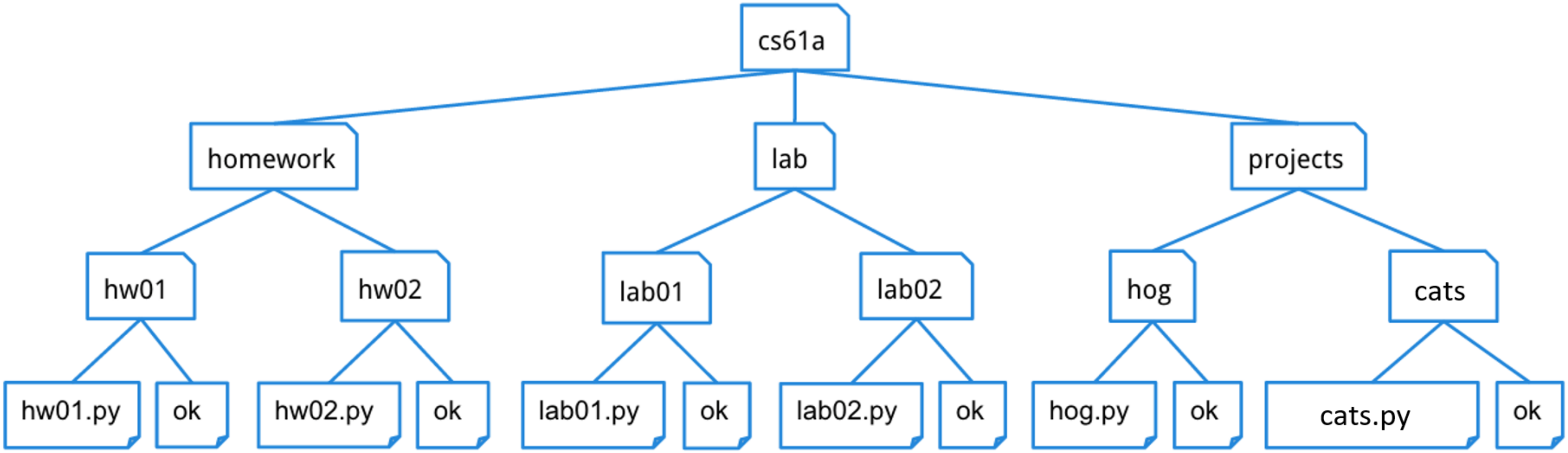
Tree: Layers of abstraction

Primitive Representation	`1 2 3 True False` `(..,..) [..,..] {...}`
Data abstraction	`tree() children() label()` `is_leaf()`
User program	`double(t)` `count_leaves(t)`

Each layer only uses the layer above it.

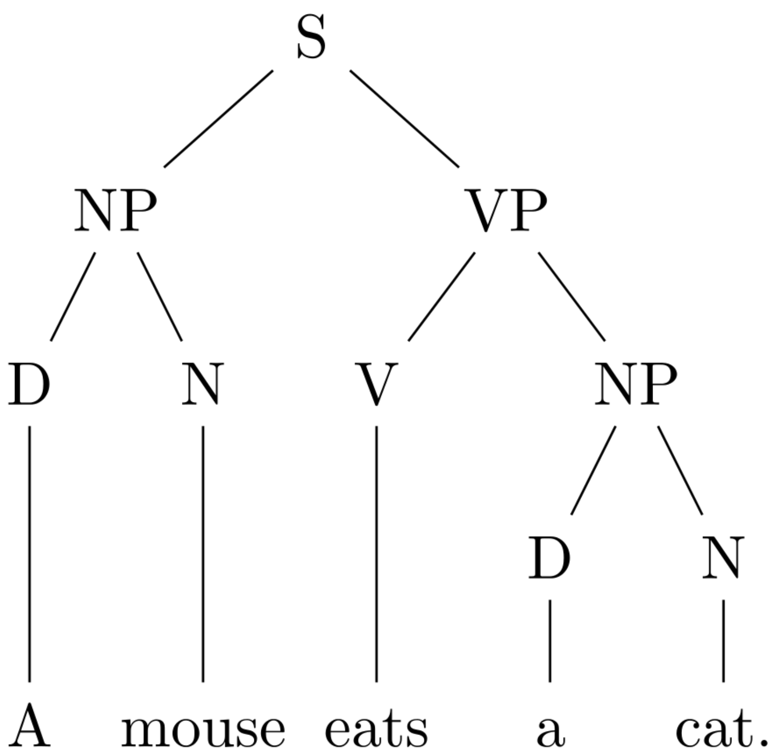
Trees, trees, everywhere!

Directory structures



Parse trees

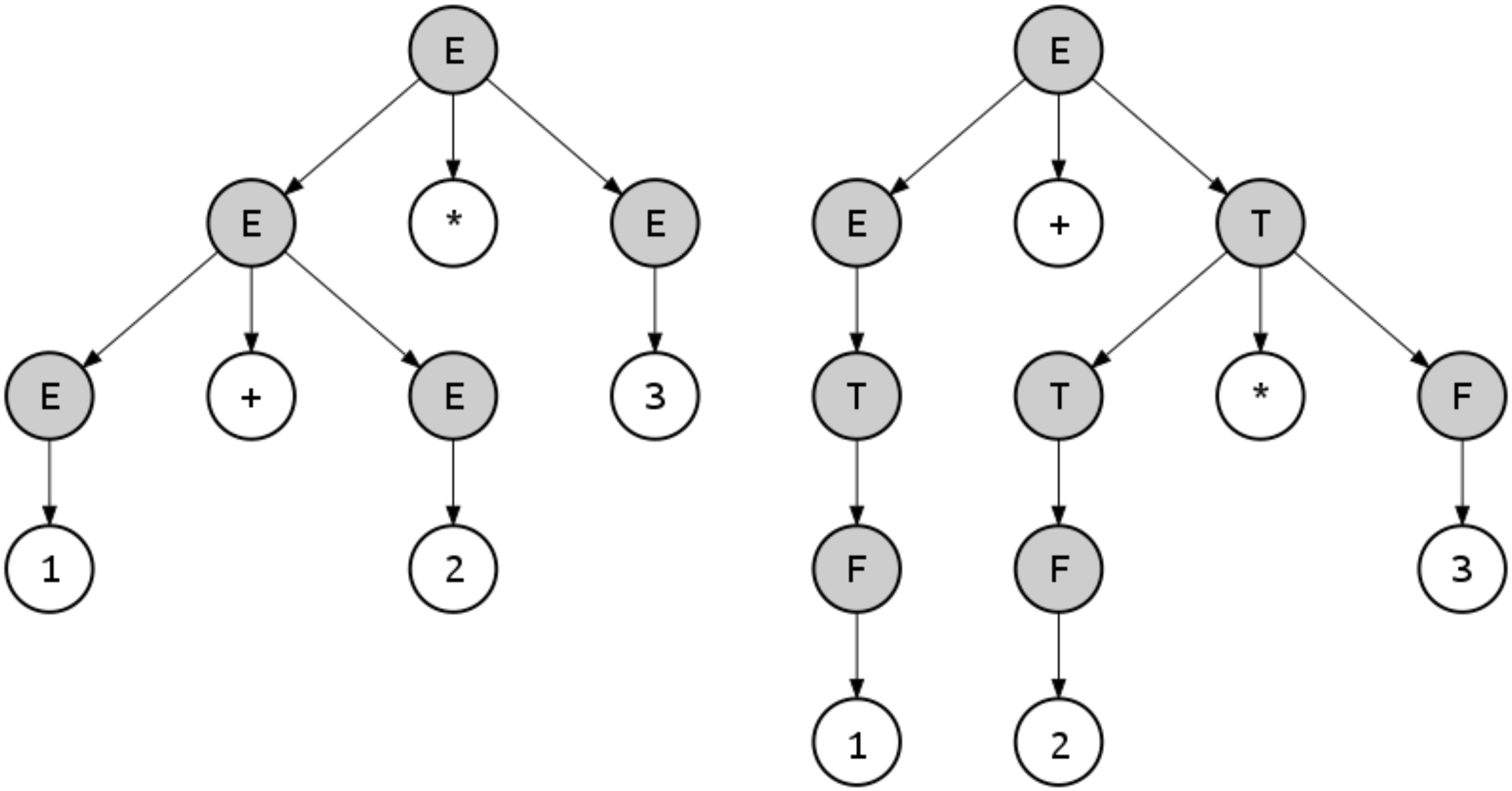
For natural languages...



Key: S = Sentence, NP = Noun phrase, D = Determiner, N = Noun, V = Verb, VP = Verb Phrase

Parse trees

For programming languages, too…



Key: E = expression