## Final Examples

## Announcements

## Trees

## Tree-Structured Data

```
def tree(label, branches=[]):
    return [label] + list(branches)

def label(t):
    return t[0]

def branches(t):
    return t[1:]

def is_leaf(t):
    return not branches(t)

class Tree:
    def __init__(self, label, branches=[]):
        self.label = label
        self.branches = list(branches)

    def is_leaf(self):
        return not self.branches
```

A tree can contains other trees:

```
[5, [6, 7], 8, [[9], 10]]

(+ 5 (- 6 7) 8 (* (- 9) 10))

(S
  (NP (JJ Short) (NNS cuts))
  (VP (VBP make)
      (NP (JJ long) (NNS delays)))
  (. .))
```

```
<ul>
  <li>Midterm <b>1</b></li>
  <li>Midterm <b>2</b></li>
</ul>
```

Tree processing often involves recursive calls on subtrees

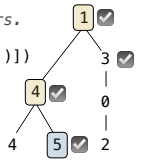
# Tree Processing

## Solving Tree Problems

Implement **bigs**, which takes a Tree instance t containing integer labels. It returns the number of nodes in t whose labels are larger than all labels of their ancestor nodes.

```
def bigs(t):
    """Return the number of nodes in t that are larger than all their ancestors.

    >>> a = Tree(1, [Tree(4, [Tree(4), Tree(5)]), Tree(3, [Tree(0, [Tree(2)])])])
    >>> bigs(a)
    4
```




The root label is always larger than all of its ancestors

```
    if t.is_leaf():
        return ___
    else:
        return ___([___ for b in t.branches])
```

Somehow increment the total count

Somehow track a list of ancestors

```
    if node.label > max(ancestors):
```

Somehow track the largest ancestor

```
    if node.label > max_ancestors:
```



## Solving Tree Problems

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    >>> bigs(a)
    4
    """
    def f(a, x):
        """ A node, max_ancestor
        if a.label > x:
            return 1 + sum([f(b, a.label) for b in a.branches])
        else:
            return sum([f(b, x) for b in a.branches])
    return f(t, t.label - 1 )
```

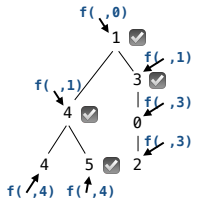
Somehow track the largest ancestor

node.label > max_ancestors

Somehow increment the total count

Root label is always larger than its ancestors

Some initial value for the largest ancestor so far...






# Recursive Accumulation

## Solving Tree Problems

Implement **bigs**, which takes a Tree instance t containing integer labels. It returns the number of nodes in t whose labels are larger than any labels of their ancestor nodes.

```
def bigs(t):
    """Return the number of nodes in t that are larger than all their ancestors."""
    n = [0]
    def f(a, x):
        if a.label > x:
            n[0] += 1
        for b in a.branches:
            f(b, max(a.label, x))
    f(t, t.label - 1)
    return n[0]
```

Somehow track the largest ancestor

node.label > max_ancestors

Somehow increment the total count

Root label is always larger than its ancestors



## Designing Functions

## How to Design Programs

**From Problem Analysis to Data Definitions**
Identify the information that must be represented and how it is represented in the chosen programming language. Formulate data definitions and illustrate them with examples.

**Signature, Purpose Statement, Header**
State what kind of data the desired function consumes and produces. Formulate a concise answer to the question *what* the function computes. Define a stub that lives up to the signature.

**Functional Examples**
Work through examples that illustrate the function's purpose.

**Function Template**
Translate the data definitions into an outline of the function.

**Function Definition**
Fill in the gaps in the function template. Exploit the purpose statement and the examples.

**Testing**
Articulate the examples as tests and ensure that the function passes all. Doing so discovers mistakes. Tests also supplement examples in that they help others read and understand the definition when the need arises—and it will arise for any serious program.

https://htdp.org/2018-01-06/Book/



## Applying the Design Process

## Designing a Function

Implement **smalls**, which takes a Tree instance t containing integer labels. It returns the non-leaf nodes in t whose labels are smaller than any labels of their descendant nodes.

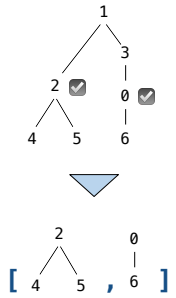
```
def smalls(t):                                  Signature: Tree -> List of Trees
    """Return the non-leaf nodes in t that are smaller than all their descendants.

    >>> a = Tree(1, [Tree(2, [Tree(4), Tree(5)]), Tree(3, [Tree(0, [Tree(6)])])])
    >>> sorted([t.label for t in smalls(a)])
    [0, 2]

    """
    result = []                                 Signature: Tree -> number
    def process(t):                             "Find smallest label in t & maybe add t to result"
        if t.is_leaf():
            return t.label
        else:




            return min(...)
    process(t)
    return result
```



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    [0, 2]

    """
    result = []                                 Signature: Tree -> number
    def process(t):                             "Find smallest label in t & maybe add t to result"
        if t.is_leaf():
            return ____________________
                   t.label
        else:
            smallest = ____________________________________________
                       min([process(b) for b in t.branches])
smallest label in a branch of t
            if ____________________________:
               t.label < smallest
               ____________________
               result.append( t )
            return min(smallest, t.label)
    process(t)
    return result
```

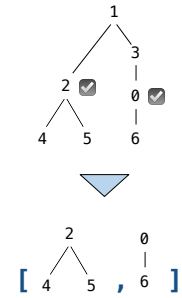




## Interpreters

## Interpreter Analysis

What expressions are passed to scheme_eval when evaluating the following expressions?

```
(define x (+ 1 2))

(define (f y) (+ x y))

(f (if (> 3 2) 4 5))
```