# Tail Calls

# Announcements

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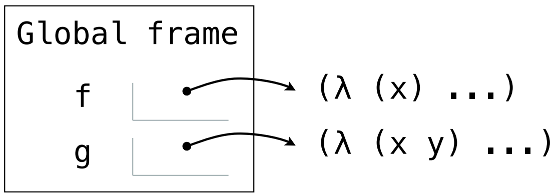


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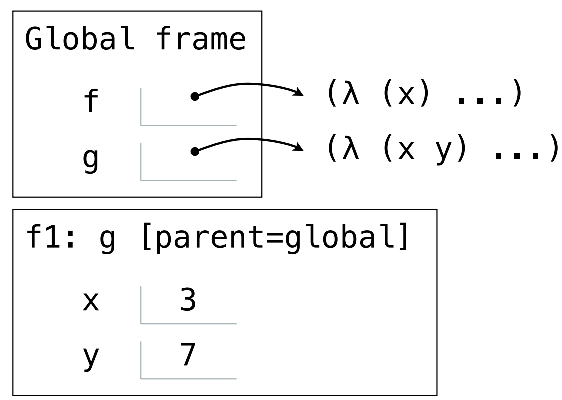


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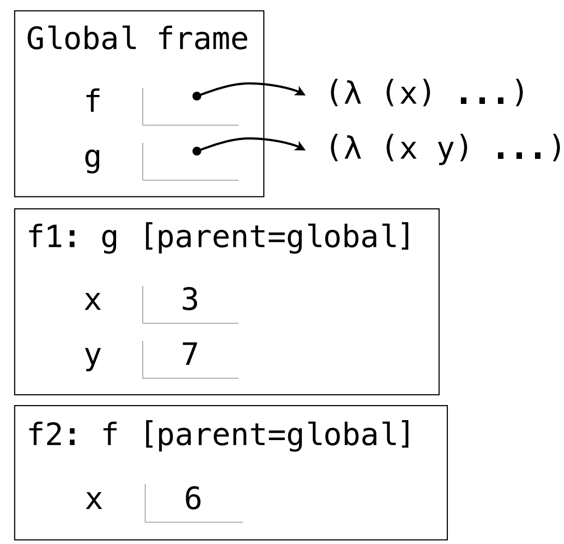

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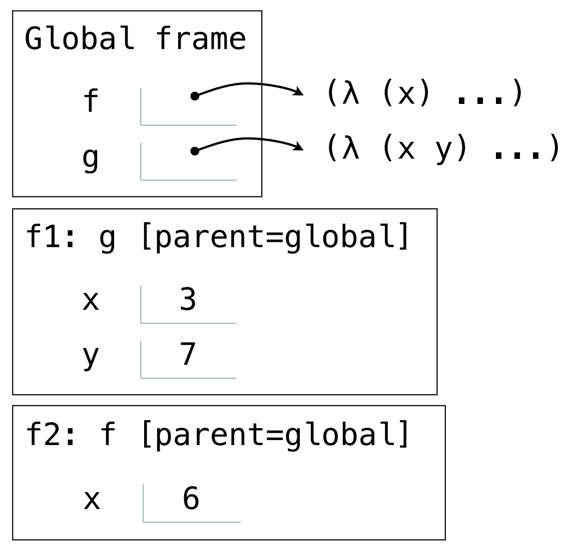

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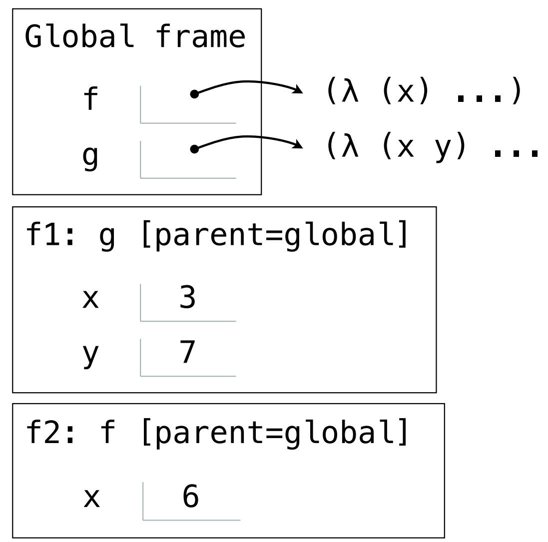


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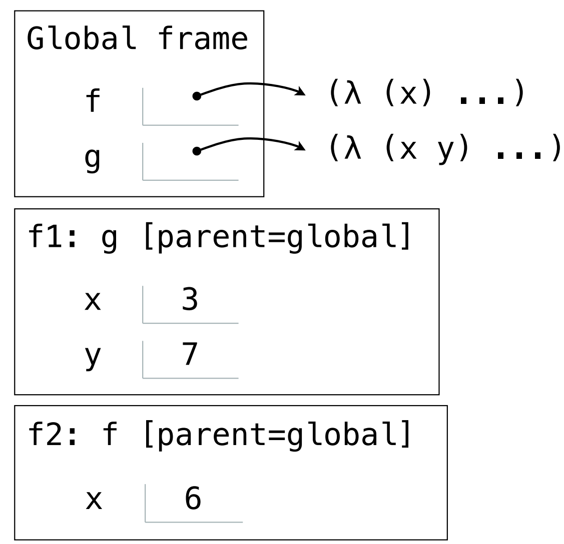

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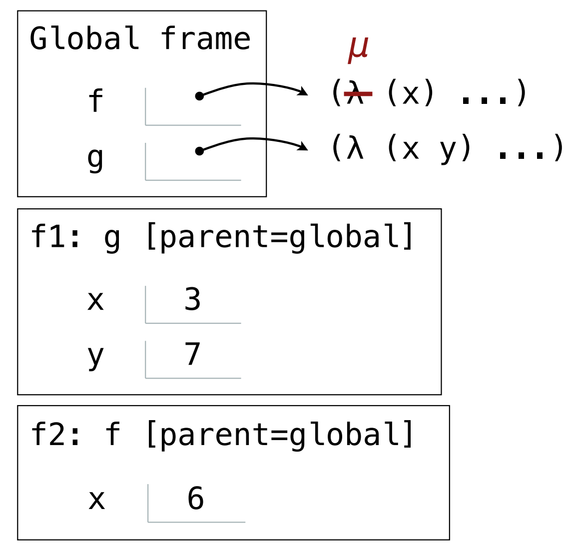


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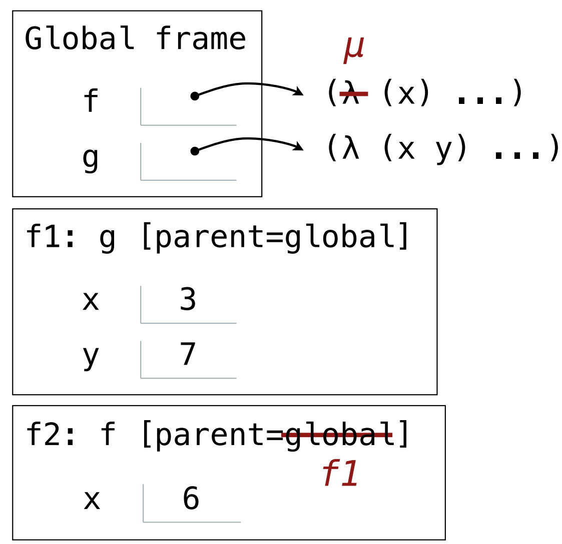


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*13*

Global frame

| | |
|---|---|
| f | → (~~λ~~ $\mu$ (x) ...) |
| g | → (λ (x y) ...) |

f1: g [parent=global]

| | |
|---|---|
| x | 3 |
| y | 7 |

f2: f [parent=~~global~~ *f1*]

| | |
|---|---|
| x | 6 |

# Tail Recursion

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But... no for/while statements! Can we make basic iteration efficient? Yes!

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(Demo)

pythontutor.com/composingprograms.html#code=def%20factorial%28n,%20k%29%3A%0A%20%20%20%20if%20n%20%3D%3D%200%3A%0A%20%20%20%20%20%20%20%20return%20k%0A%20%20%20%20else%3A%0A%20%20%20%20%20%20%20%20return%20factorial%28n-1,%20k*n%29%0A%20%20%20%20%0Afactorial%284,%2010%29&mode=display&origin=composingprograms.js&cumulative=true&py=3&rawInputLstJSON=[]&curInstr=0

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Recursive call is a tail call

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# Tail Recursion Examples

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# Map and Reduce

# Example: Reduce

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(define (reduce procedure s start)
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```
(reduce * '(3 4 5) 2)                                                        120
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# Example: Map with Only a Constant Number of Frames

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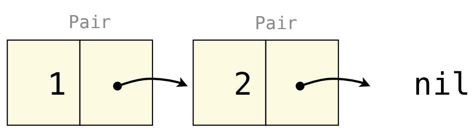

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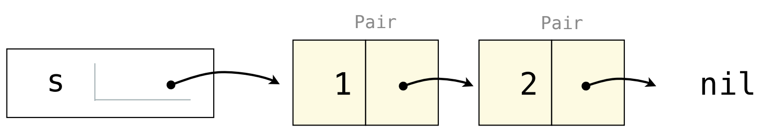

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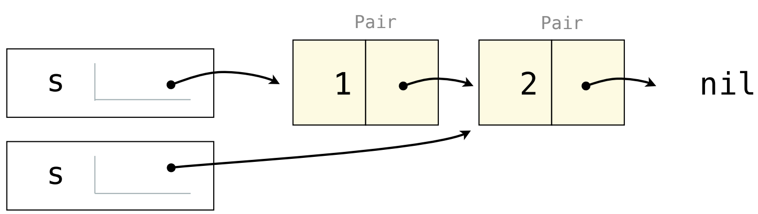

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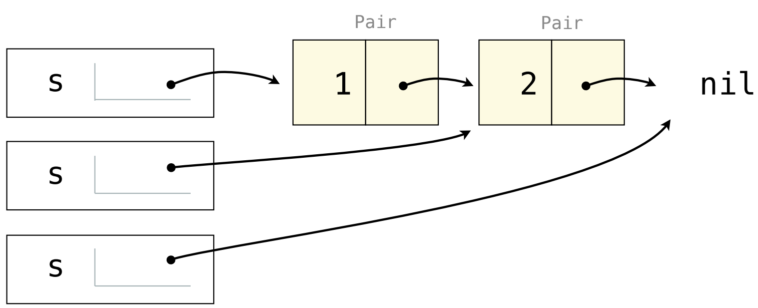

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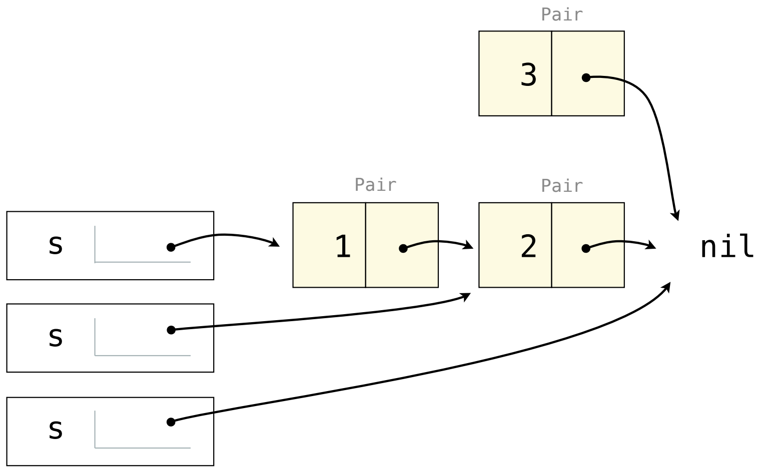

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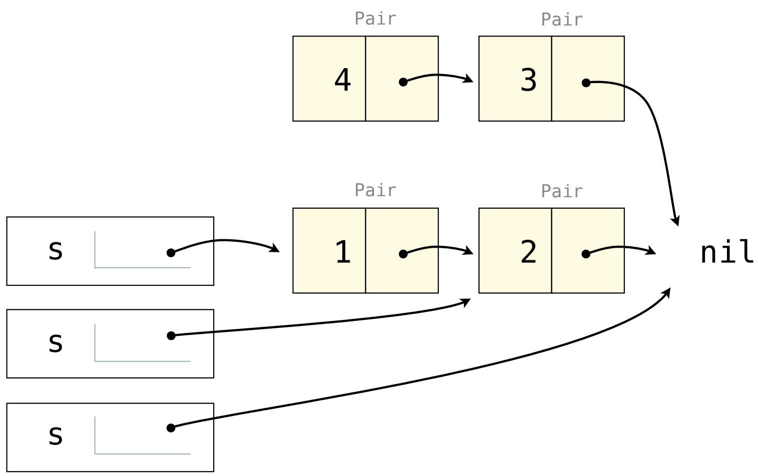

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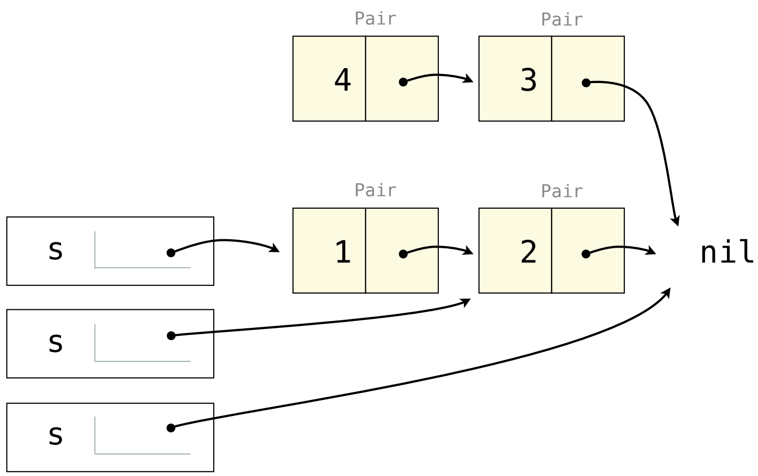

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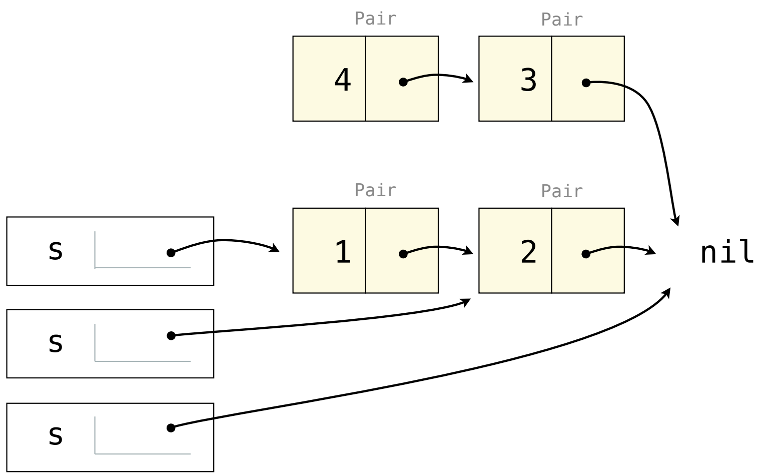

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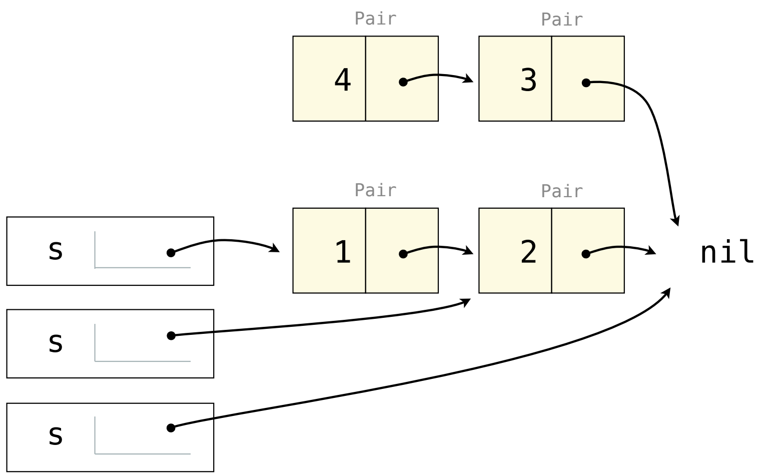

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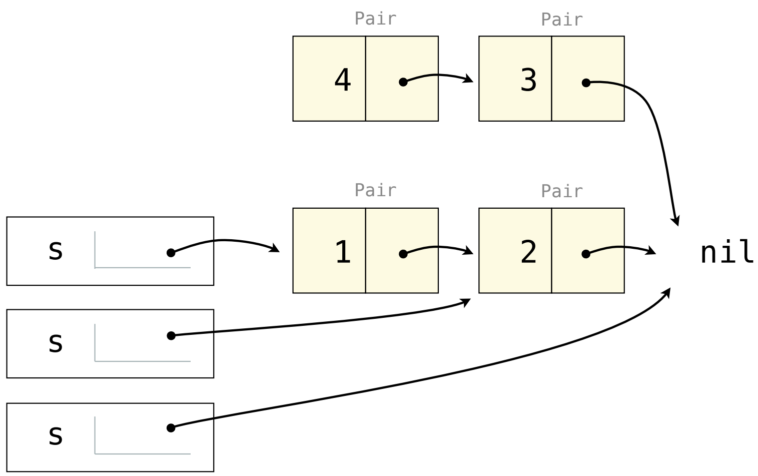

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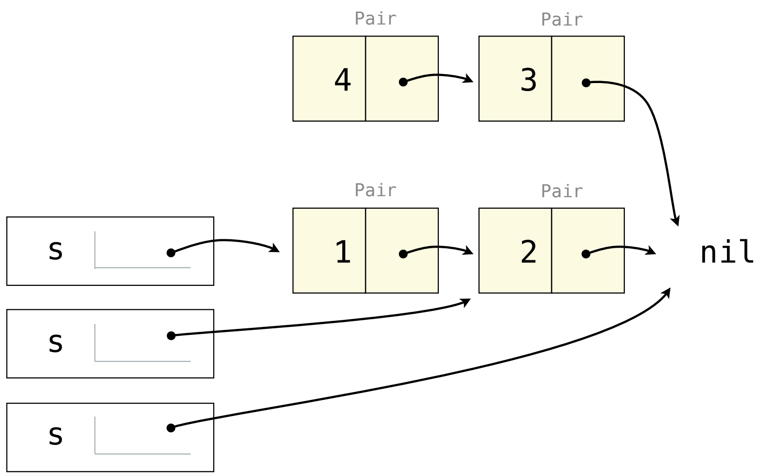

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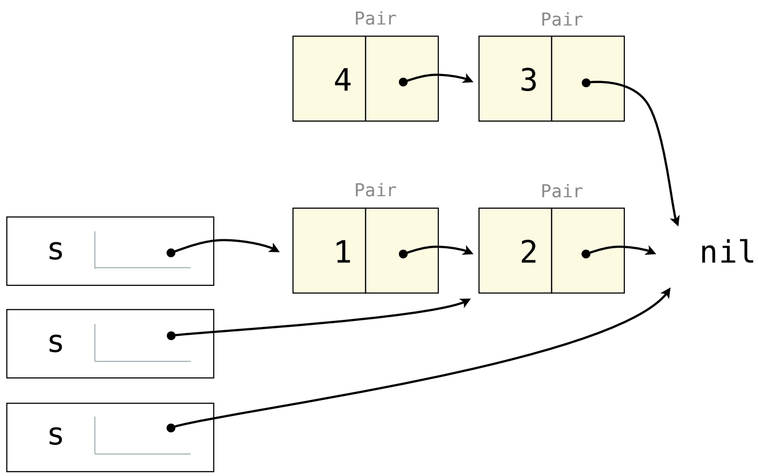

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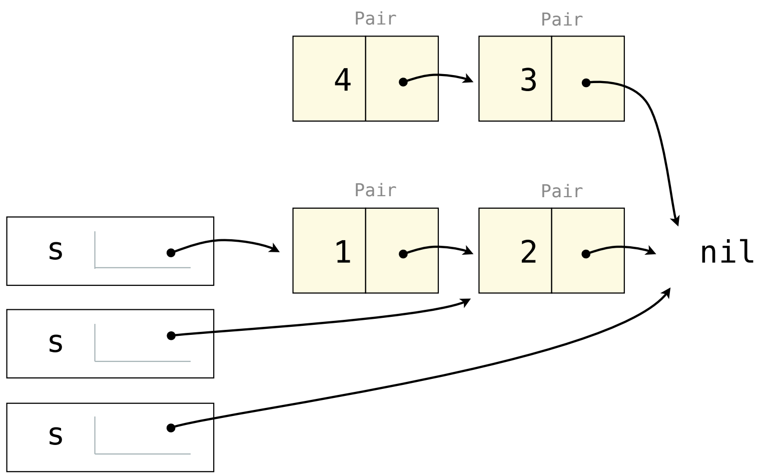

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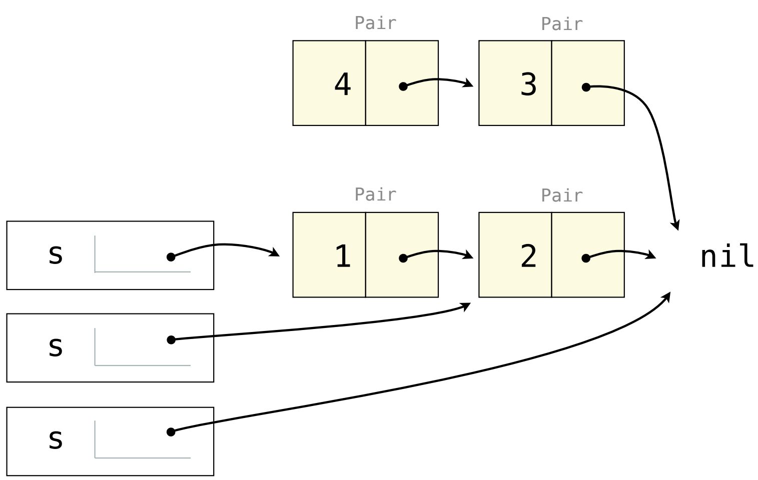


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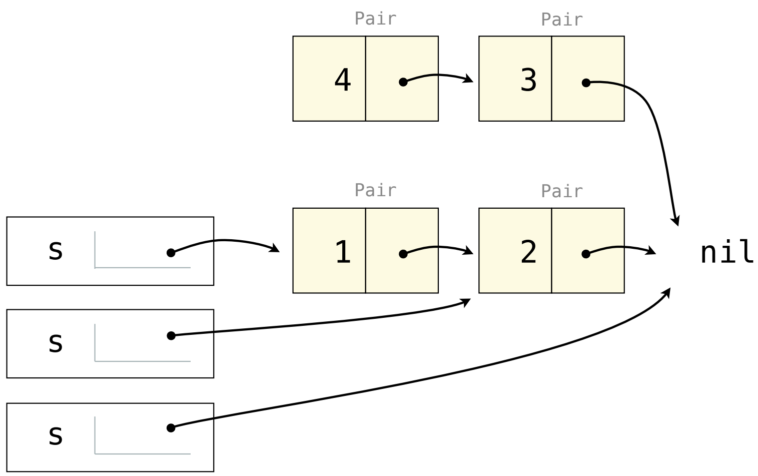


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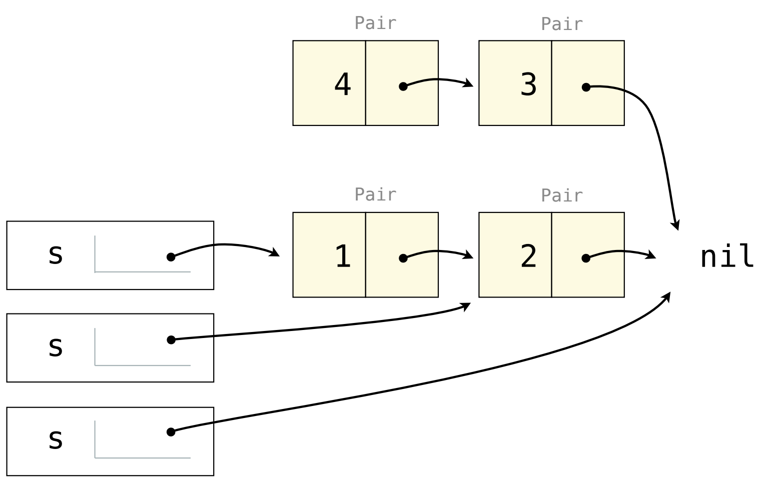


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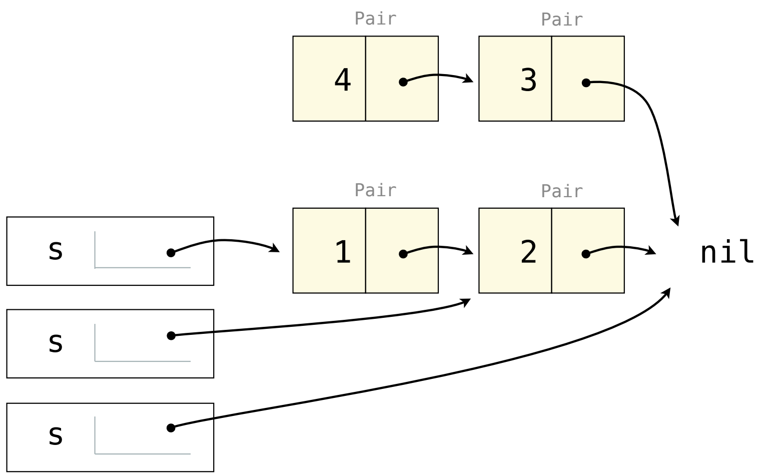


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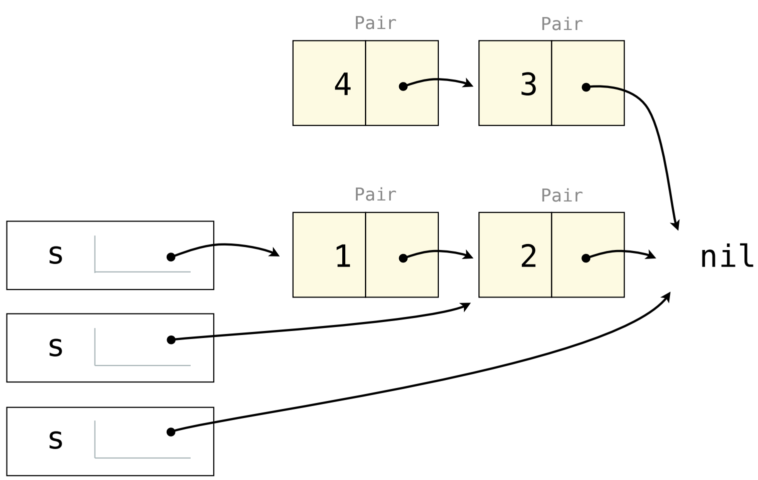


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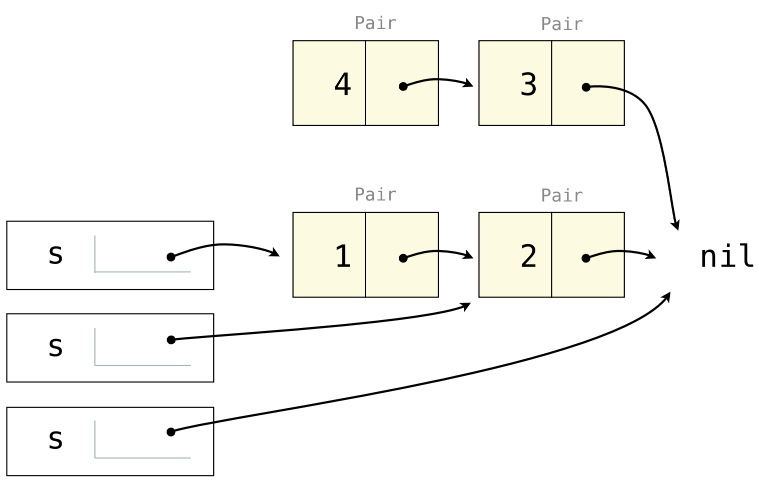


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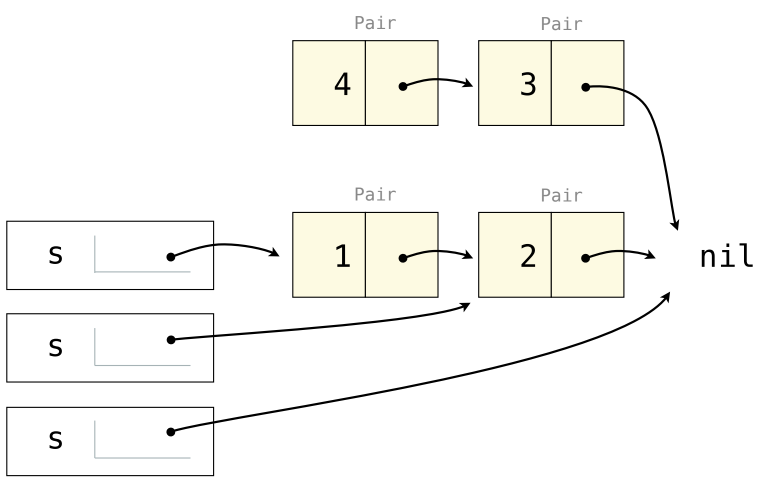


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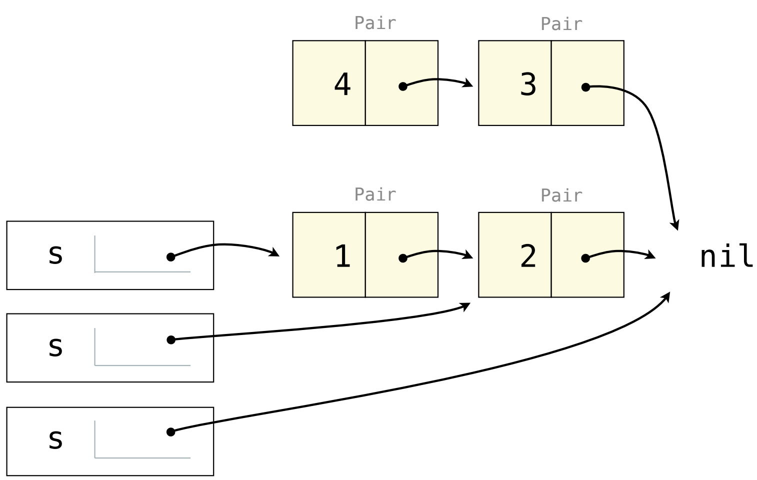


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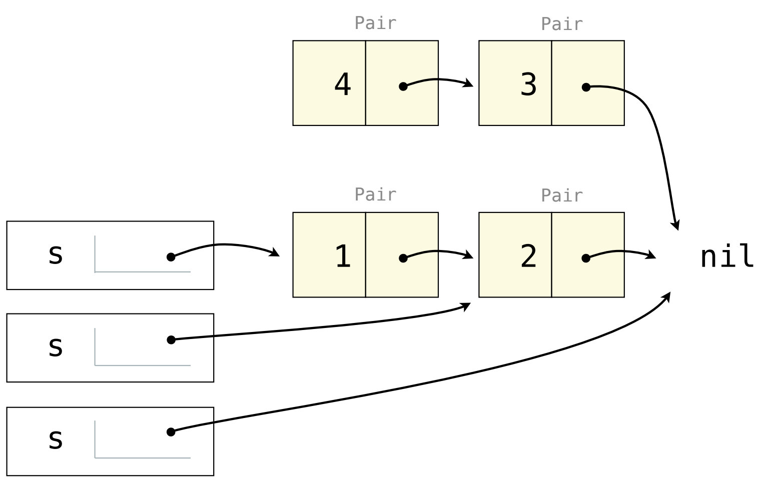


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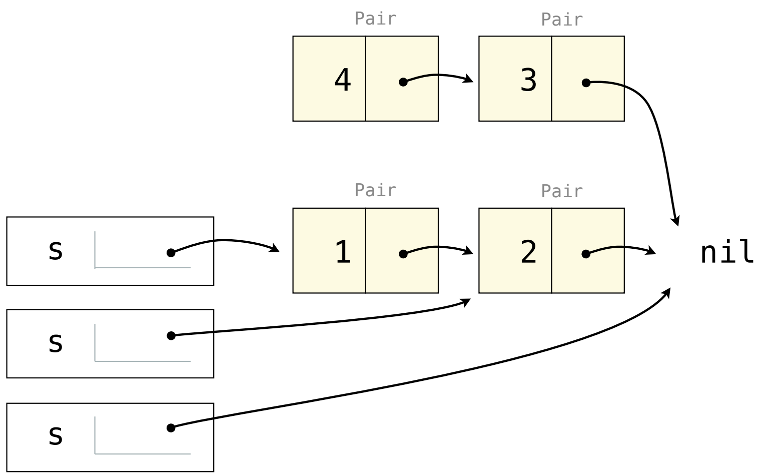


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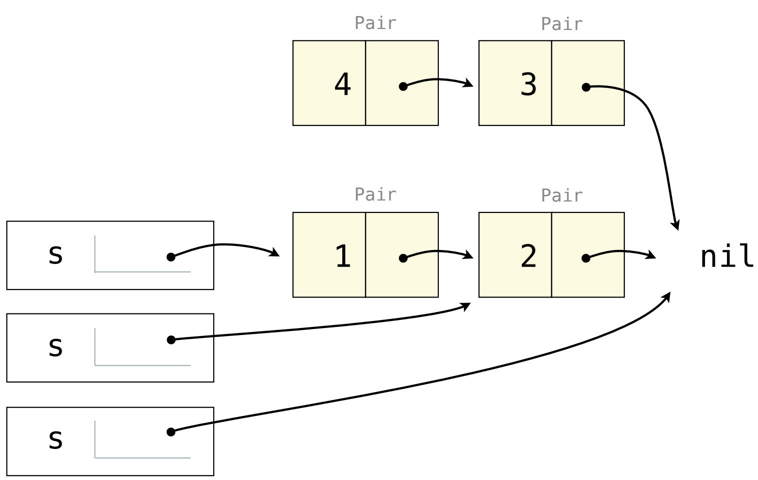


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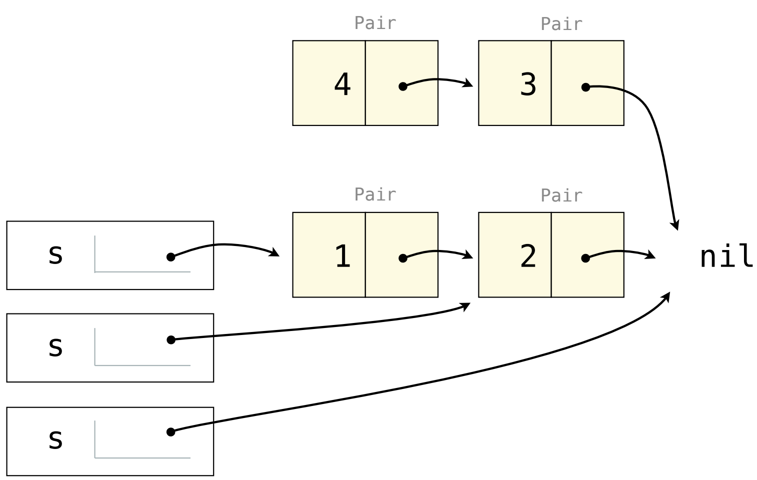


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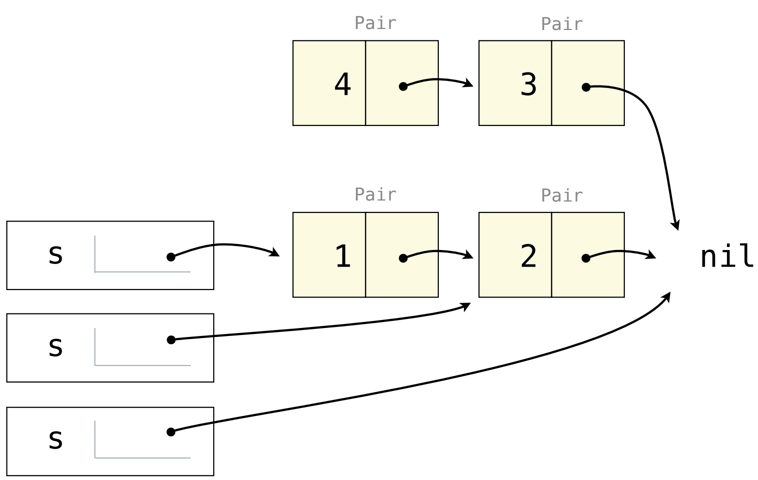


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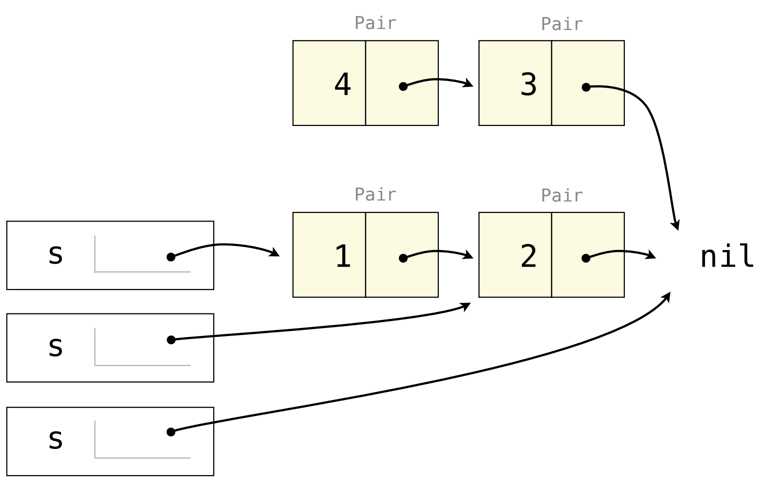


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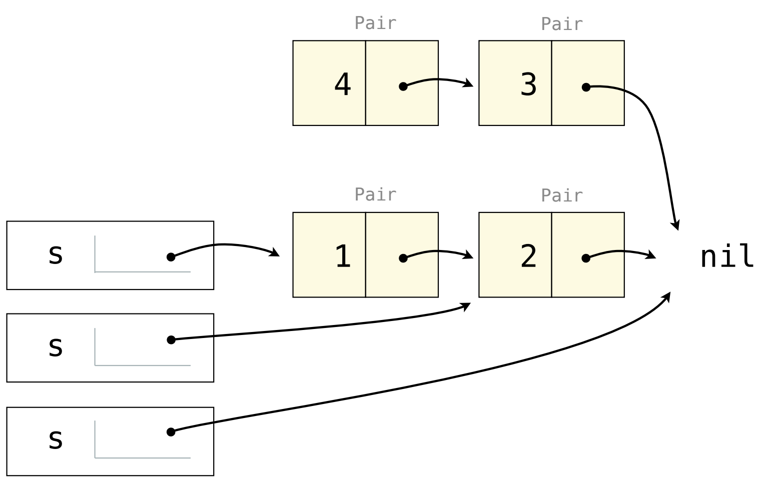


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# General Computing Machines

# An Analogy: Programs Define Machines

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Programs specify the logic of a computational device
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```
Programs specify the logic of a computational device
```

```
factorial
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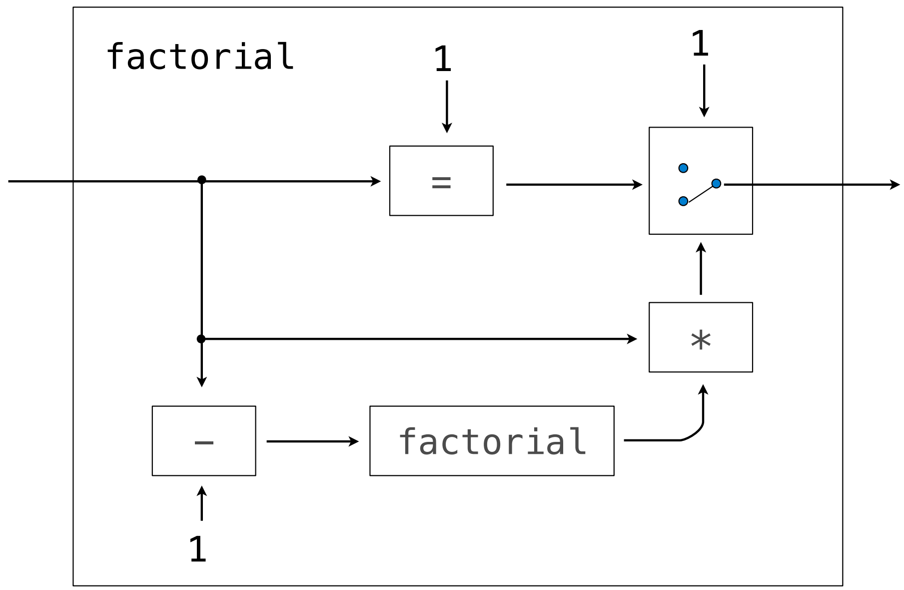

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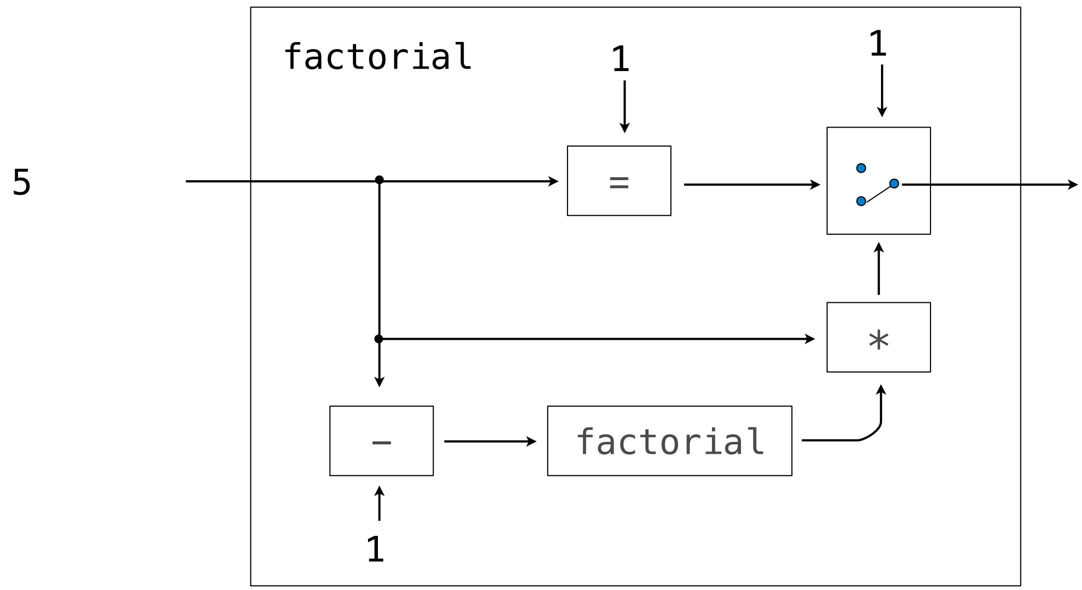

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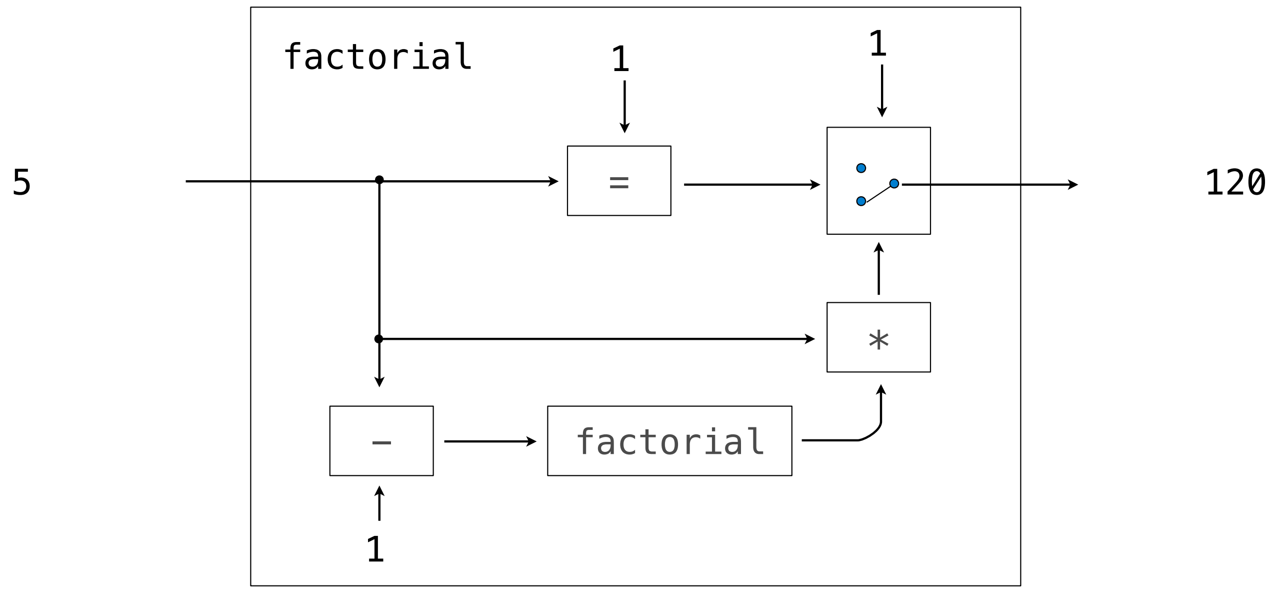

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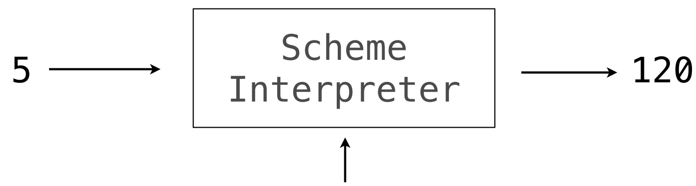


```
(define (factorial n)
  (if (zero? n) 1 (* n (factorial (- n 1)))))
```

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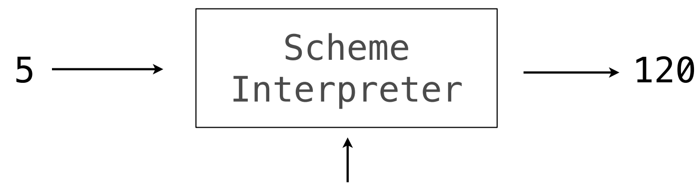


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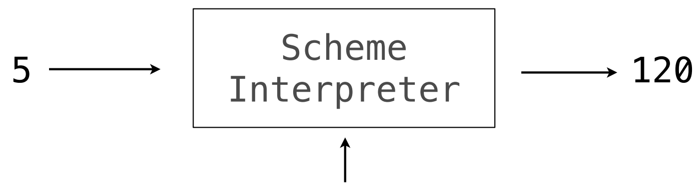
Our Scheme interpreter is a universal machine

# Interpreters are General Computing Machine

An interpreter can be parameterized to simulate any machine

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A bridge between the data objects that are manipulated by our programming language and the programming language itself

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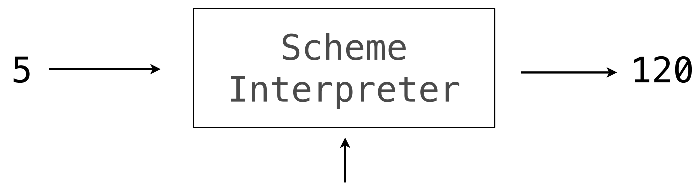


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Internally, it is just a set of evaluation rules