Tail Calls

Announcements

Dynamic Scope

Dynamic Scope

The way in which names are looked up in Scheme and Python is called lexical scope (or static scope) [You can see what names are in scope by inspecting the definition]

Lexical scope: The parent of a frame is the environment in which a procedure was *defined*

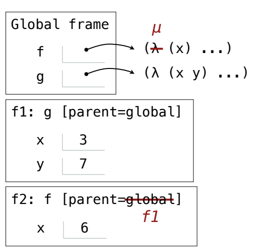
Dynamic scope: The parent of a frame is the environment in which a procedure was *called*

Special form to create dynamically scoped procedures (**mu** special form only exists in Project 4 Scheme)

(define f (~~lambda~~ *mu* (x) (+ x y)))

(define g (lambda (x y) (f (+ x x))))

(g 3 7)



Lexical scope: The parent for f's frame is the global frame

Error: unknown identifier: y

Dynamic scope: The parent for f's frame is g's frame

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Tail Recursion

Functional Programming

All functions are pure functions

No re-assignment and no mutable data types

Name-value bindings are permanent

Advantages of functional programming:

- The value of an expression is independent of the order in which sub-expressions are evaluated
- Sub-expressions can safely be evaluated in parallel or only on demand (lazily)
- **Referential transparency:** The value of an expression does not change when we substitute one of its subexpression with the value of that subexpression

But... no for/while statements! Can we make basic iteration efficient? Yes!

Recursion and Iteration in Python

In Python, recursive calls always create new active frames

factorial(n, k) computes: n! * k

```
def factorial(n, k):
    if n == 0:
        return k
    else:
        return factorial(n-1, k*n)
```

```
def factorial(n, k):
    while n > 0:
        n, k = n-1, k*n
    return k
```

	Time	Space
Recursive	Linear	Linear
Iterative	Linear	Constant

Tail Recursion

From the Revised[7] Report on the Algorithmic Language Scheme:

"Implementations of Scheme are required to be properly tail-recursive. This allows the execution of an iterative computation in constant space, even if the iterative computation is described by a syntactically recursive procedure."

```
(define (factorial n k)
  (if (zero? n) k
    (factorial (- n 1)
               (* k n))))
```

How? Eliminate the middleman!

Should use resources like

```
def factorial(n, k):
    while n > 0:
        n, k = n-1, k*n
    return k
```

Time	Space
Linear	Constant

(Demo)

Tail Calls

Tail Calls

A procedure call that has not yet returned is active. Some procedure calls are tail calls. A Scheme interpreter should support an unbounded number of active tail calls using only a constant amount of space.

A tail call is a call expression in a tail context:

- The last body sub-expression in a **lambda** expression (or procedure definition)
- Sub-expressions 2 & 3 in a tail context **if** expression
- All non-predicate sub-expressions in a tail context **cond**
- The last sub-expression in a tail context **and, or, begin,** or **let**

```
(define (factorial n k)
  (if (= n 0) k
    (factorial (- n 1)
               (* k n)) ) )
```

```
(define factorial (lambda (n k)
  (if (= n 0) k
    (factorial (- n 1)
               (* k n)) ) ))
```


Example: Length of a List

```
(define (length s)
  (if (null? s) 0
    (+ 1 (length (cdr s)) ) ) )
```

Not a tail context

A call expression is not a tail call if more computation is still required in the calling procedure

Linear recursive procedures can often be re-written to use tail calls

```
(define (length-tail s)
  (define (length-iter s n)
    (if (null? s) n
      (length-iter (cdr s) (+ 1 n)) ) )
  (length-iter s 0) )
```

Recursive call is a tail call

Eval with Tail Call Optimization

The return value of the tail call is the return value of the current procedure call

Therefore, tail calls shouldn't increase the environment size

(Demo)

Tail Recursion Examples

Which Procedures are Tail Recursive?

Which of the following procedures run in constant space?

```
;; Compute the length of s.
(define (length s)
  (+ 1 (if (null? s)
           -1
           (length (cdr s))) ) )

;; Return the nth Fibonacci number.
(define (fib n)
  (define (fib-iter current k)
    (if (= k n)
        current
        (fib-iter (+ current
                     (fib (- k 1)))
                  (+ k 1)) ) )
  (if (= 1 n) 0 (fib-iter 1 2)))

;; Return whether s contains v.
(define (contains s v)
  (if (null? s)
      false
      (if (= v (car s))
          true
          (contains (cdr s) v))))

;; Return whether s has any repeated elements.
(define (has-repeat s)
  (if (null? s)
      false
      (if (contains? (cdr s) (car s))
          true
          (has-repeat (cdr s))) ) )
```


Map and Reduce

Example: Reduce

```
(define (reduce procedure s start)
  (if (null? s) start
    (reduce procedure
            (cdr s)
            (procedure start (car s)) ) ) )
```

Recursive call is a tail call

Space depends on what procedure requires

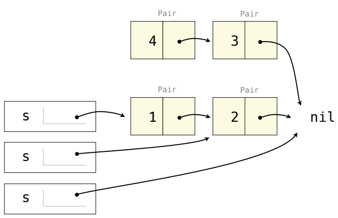
```
(reduce * '(3 4 5) 2)                              120

(reduce (lambda (x y) (cons y x)) '(3 4 5) '(2))   (5 4 3 2)
```

Example: Map with Only a Constant Number of Frames

```
(define (map procedure s)
  (if (null? s)
      nil
      (cons (procedure (car s))
            (map procedure (cdr s)))) )

(map (lambda (x) (- 5 x)) (list 1 2))
```



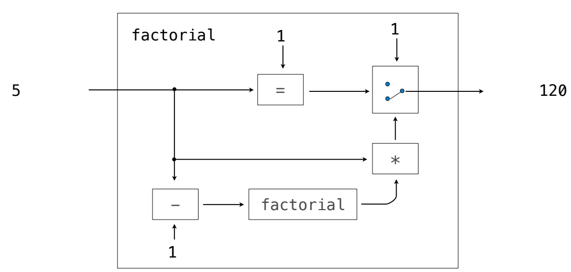

```
(define (map procedure s)
  (define (map-reverse s m)
    (if (null? s)
        m
        (map-reverse (cdr s)
                     (cons (procedure (car s))
                           m))
                                            ) )
  (reverse (map-reverse s nil)))

(define (reverse s)
  (define (reverse-iter s r)
    (if (null? s)
        r
        (reverse-iter (cdr s)
                      (cons (car s) r)) ) )
  (reverse-iter s nil))
```


General Computing Machines

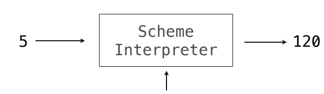
An Analogy: Programs Define Machines

Programs specify the logic of a computational device



Interpreters are General Computing Machine

An interpreter can be parameterized to simulate any machine




```
(define (factorial n)
  (if (zero? n) 1 (* n (factorial (- n 1)))))
```

Our Scheme interpreter is a universal machine

A bridge between the data objects that are manipulated by our programming language and the programming language itself

Internally, it is just a set of evaluation rules