Composition

Announcements

Linked Lists

Linked List Structure

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3 , 4 , 5

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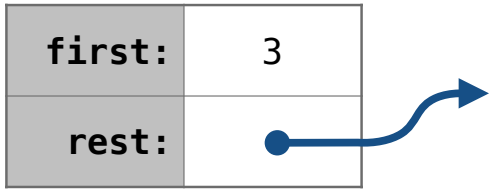
Link instance



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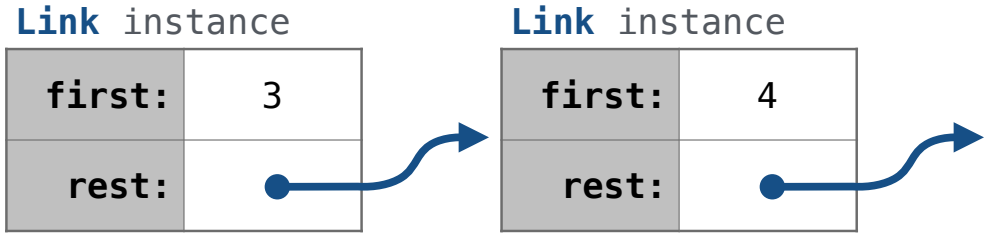

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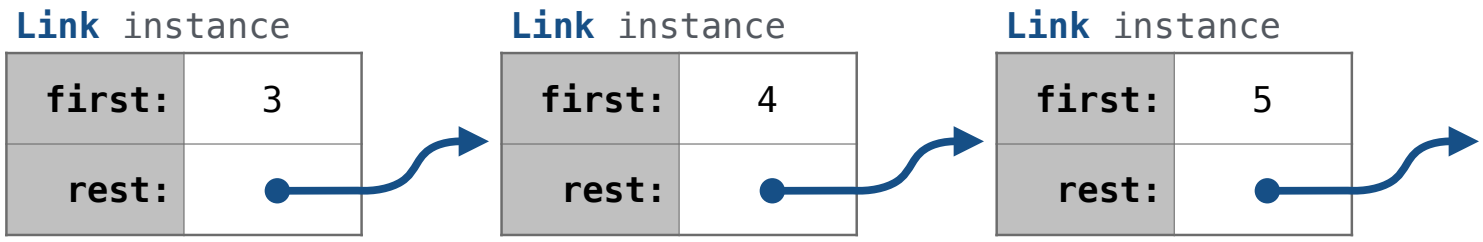

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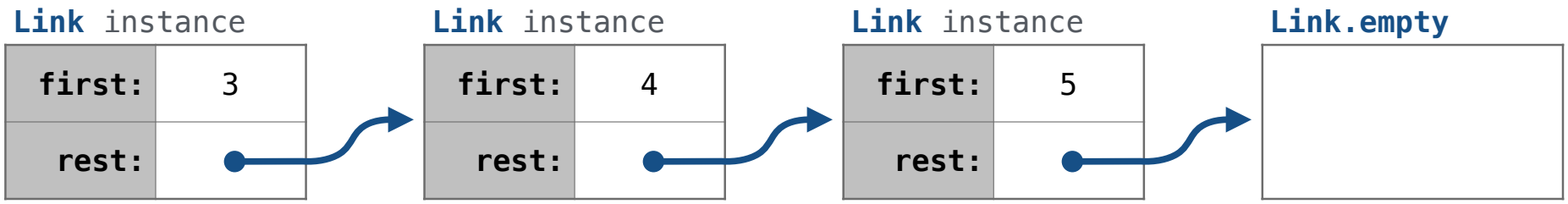

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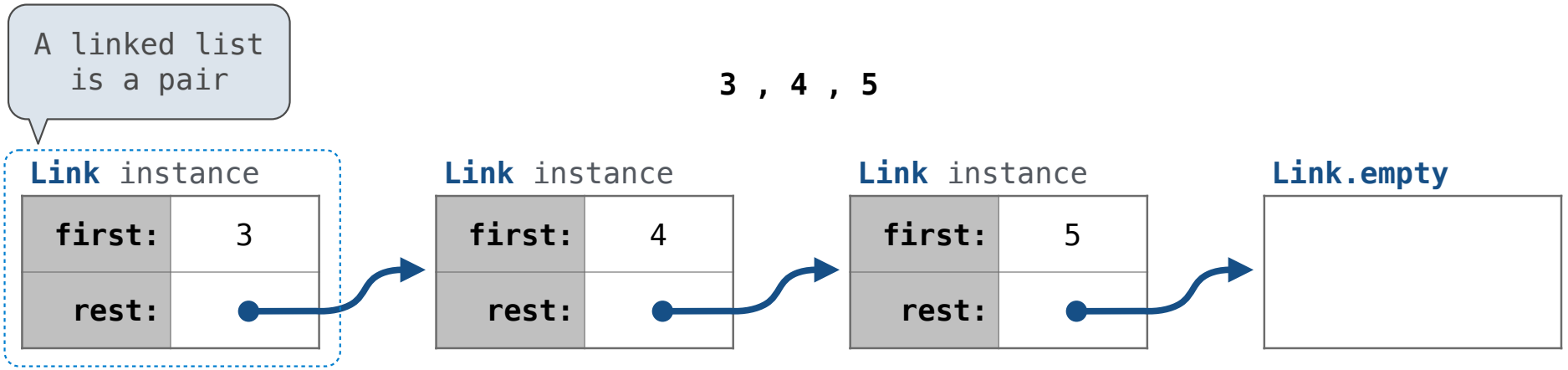

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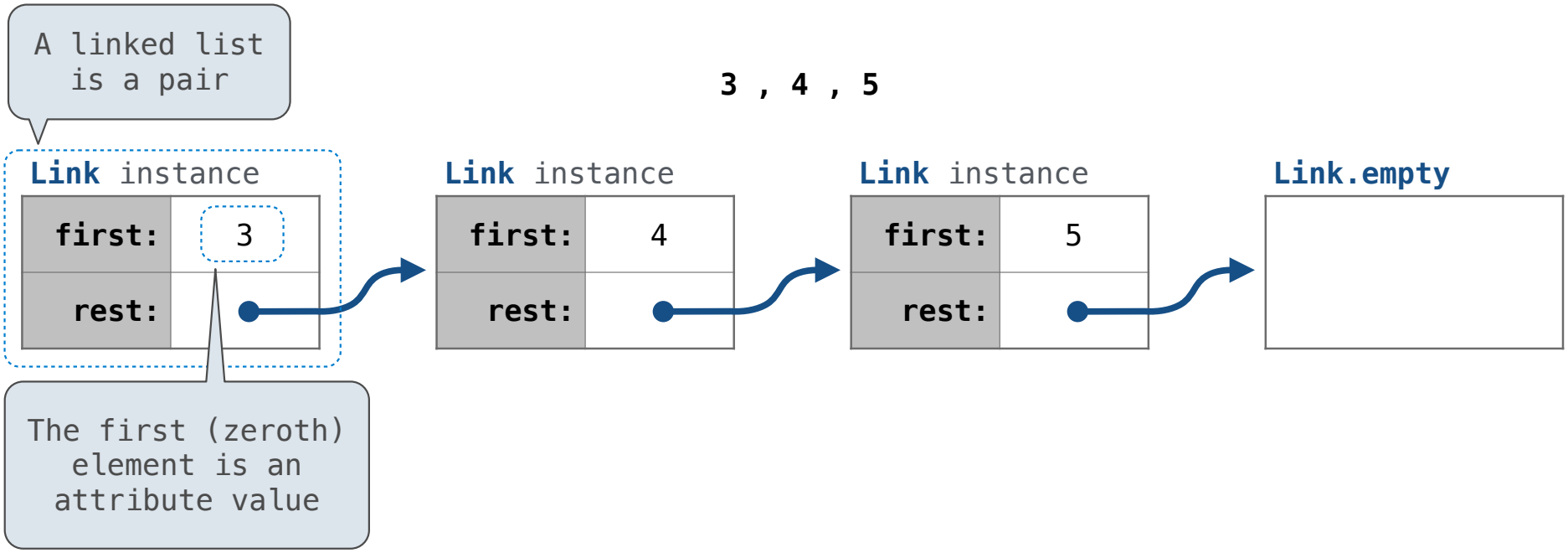

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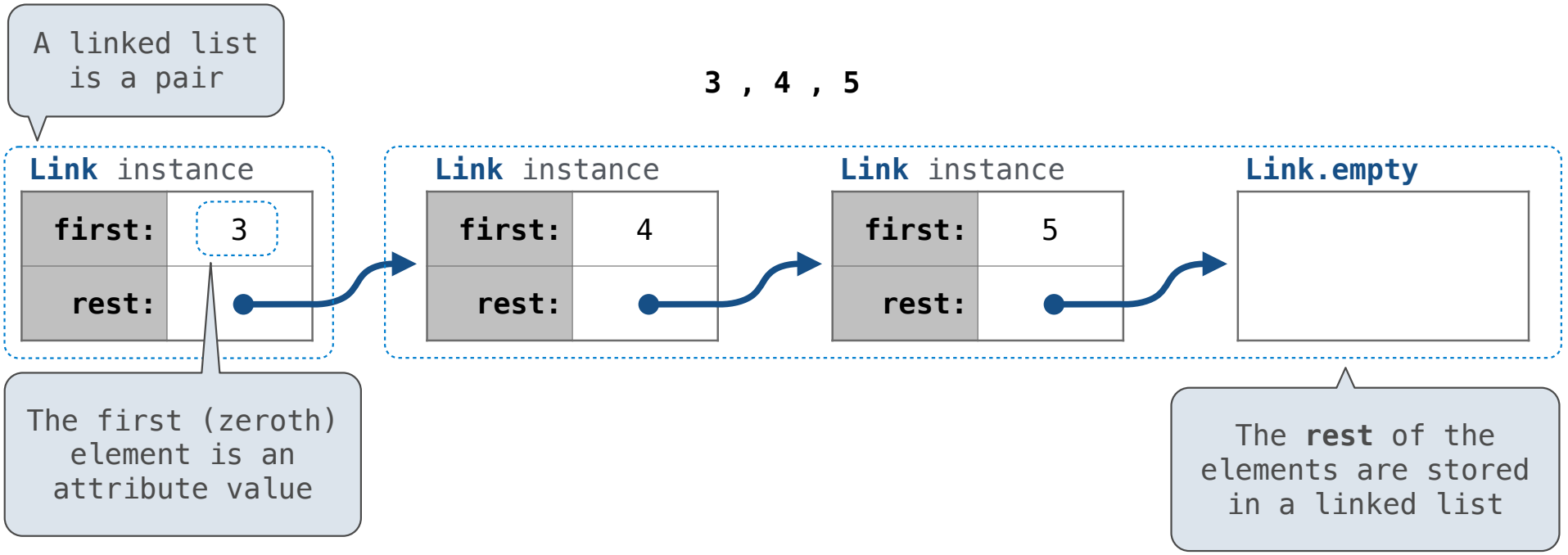

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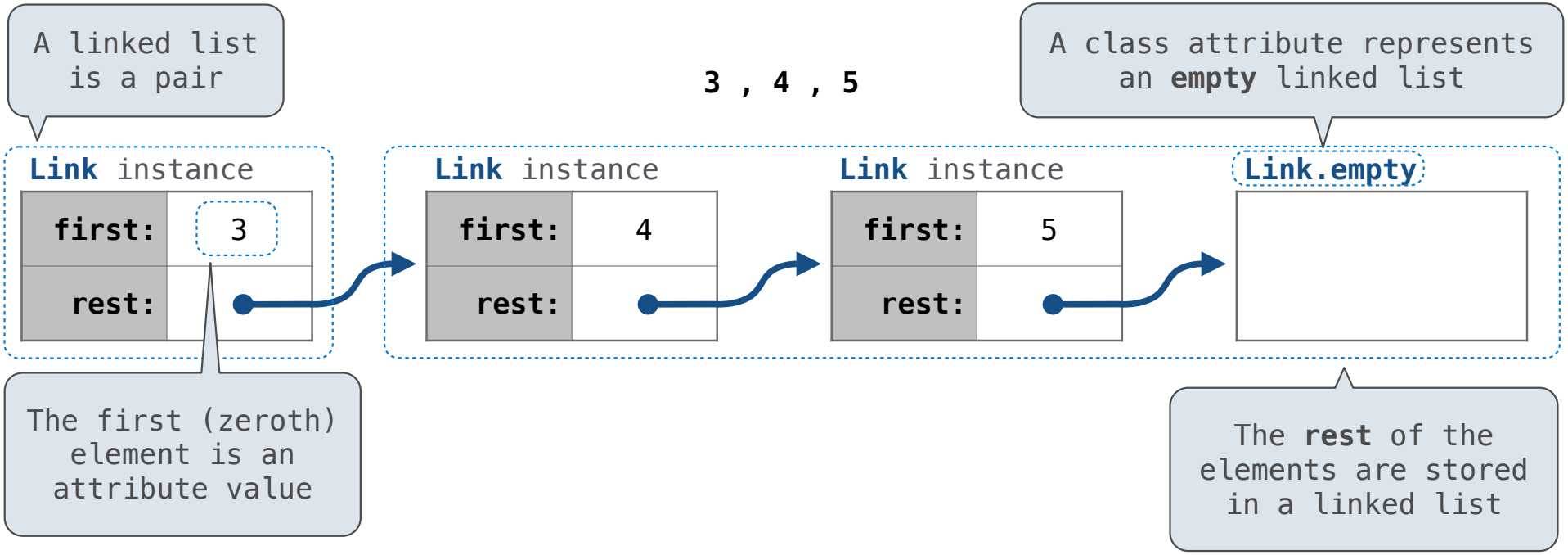

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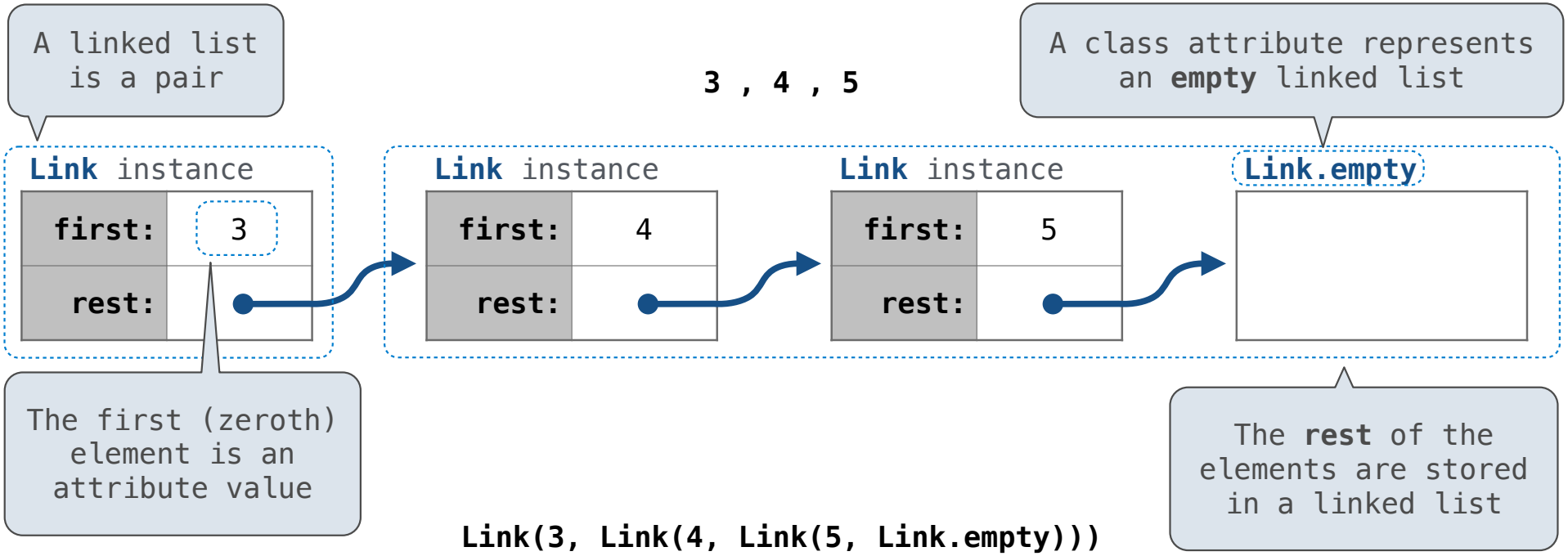

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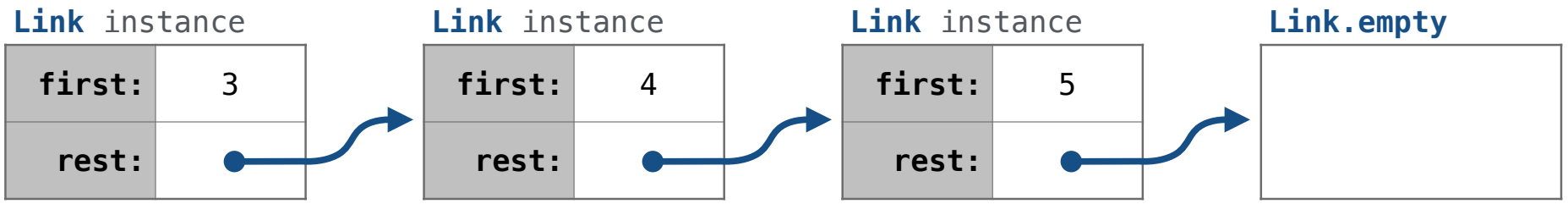


```
Link(3, Link(4, Link(5, Link.empty)))
```

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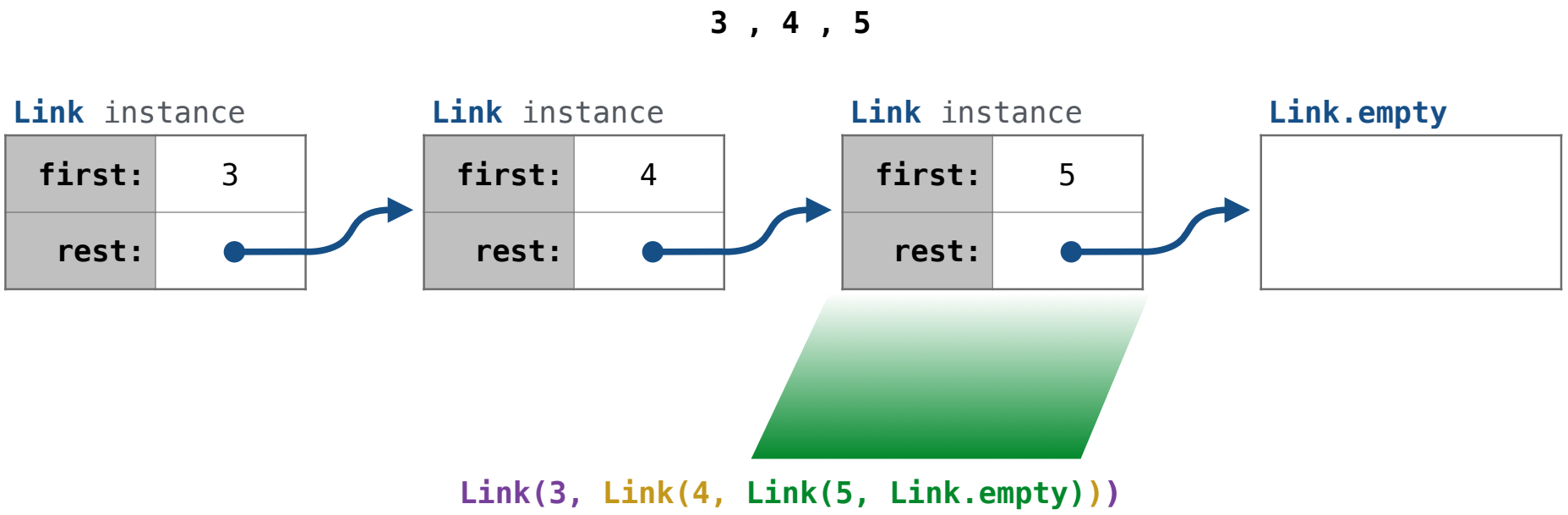

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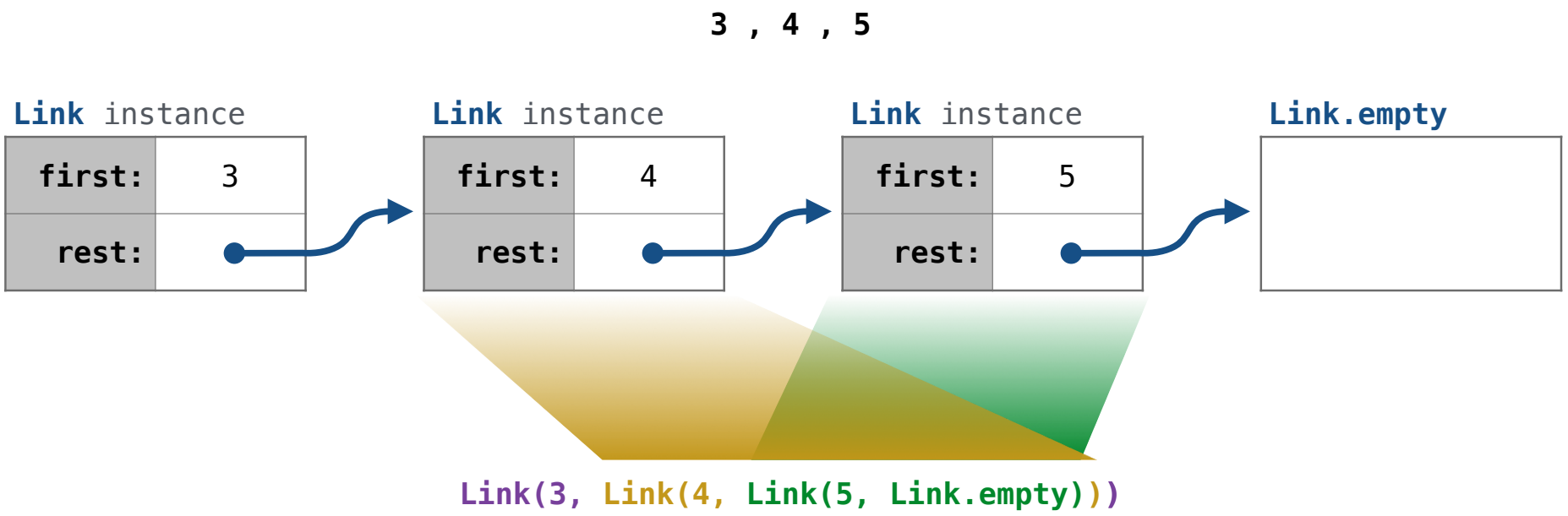

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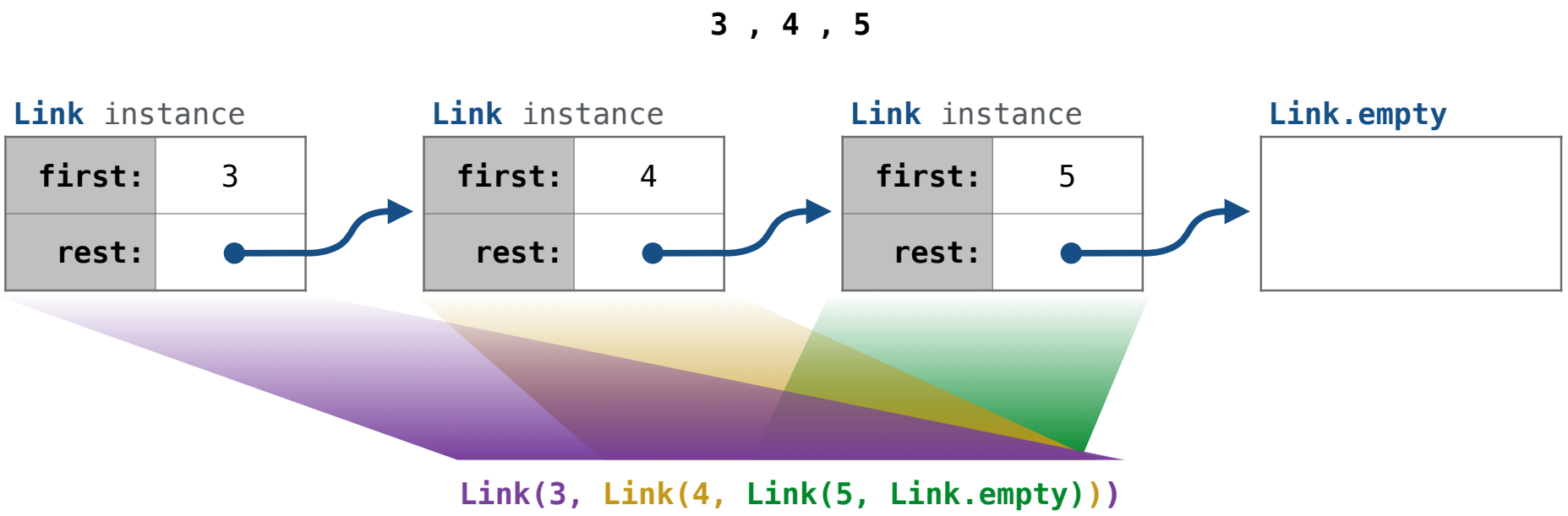

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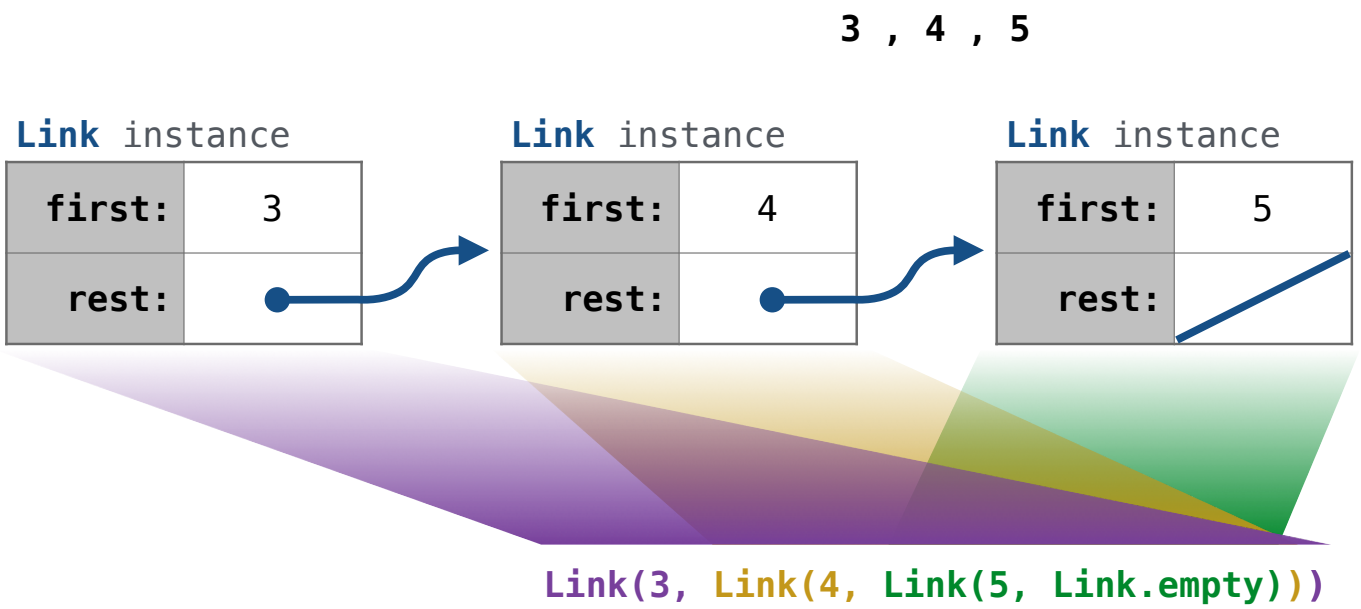

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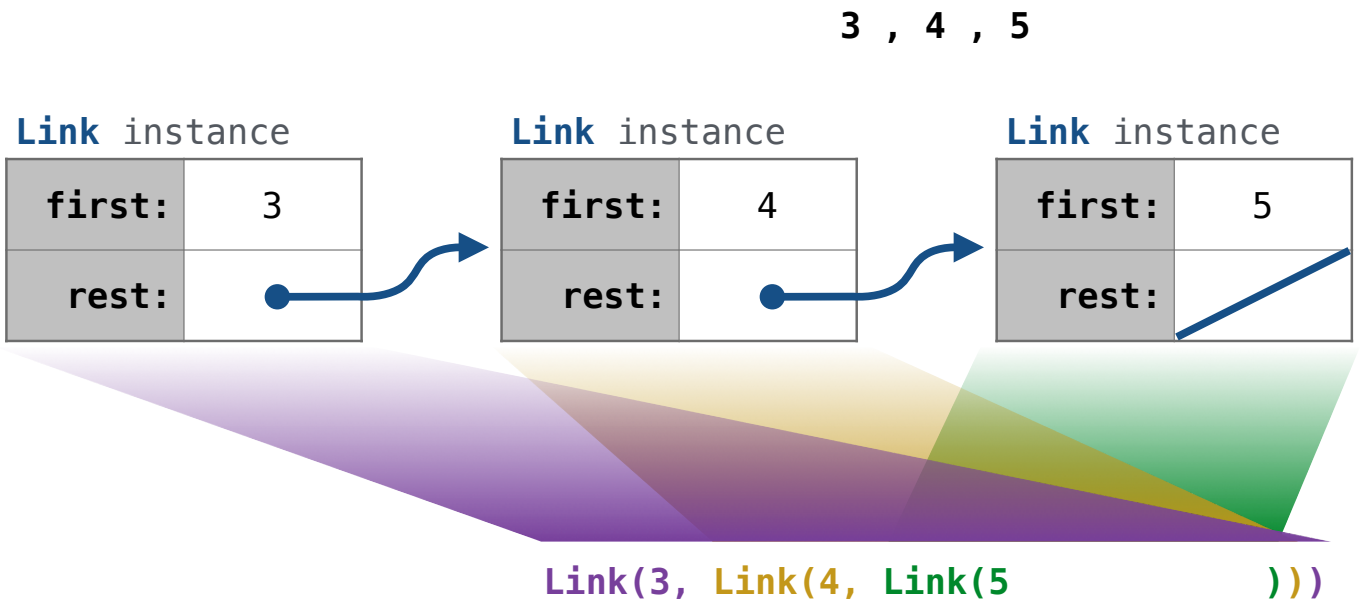

Linked List Class

```
Link(3, Link(4, Link(5            )))
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Linked List Class

Linked list class: attributes are passed to `__init__`

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```
class Link:
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Link(3, Link(4, Link(5            )))
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Linked List Class

Linked list class: attributes are passed to `__init__`

```
class Link:

    def __init__(self, first, rest=empty):
```

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Link(3, Link(4, Link(5            )))
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Linked List Class

Linked list class: attributes are passed to __init__

```
class Link:


    def __init__(self, first, rest=empty):
        assert rest is Link.empty or isinstance(rest, Link)
```

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Link(3, Link(4, Link(5            )))
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Linked List Class

Linked list class: attributes are passed to __init__

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class Link:


    def __init__(self, first, rest=empty):
        assert rest is Link.empty or isinstance(rest, Link)
        self.first = first
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Link(3, Link(4, Link(5            )))
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Linked list class: attributes are passed to `__init__`

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class Link:

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Returns whether rest is a Link

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Link(3, Link(4, Link(5            )))
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class Link:


    def __init__(self, first, rest=empty):
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```

Returns whether rest is a Link

help(isinstance): Return whether an object is an instance of a class or of a subclass thereof.

```
Link(3, Link(4, Link(5            )))
```

Linked List Class

Linked list class: attributes are passed to __init__

```
class Link:

    empty = ()

    def __init__(self, first, rest=empty):
        assert rest is Link.empty or isinstance(rest, Link)
        self.first = first
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```

Returns whether rest is a Link

help(isinstance): Return whether an object is an instance of a class or of a subclass thereof.

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Linked List Class

Linked list class: attributes are passed to `__init__`

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class Link:
    empty = ()

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```

Some zero-length sequence

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Link(3, Link(4, Link(5            )))
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Link(3, Link(4, Link(5            )))
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(Demo)

Linked List Processing

Example: Range, Map, and Filter for Linked Lists

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```
square, odd = lambda x: x * x, lambda x: x % 2 == 1
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square, odd = lambda x: x * x, lambda x: x % 2 == 1
list(map(square, filter(odd, range(1, 6))))           # [1, 9, 25]
```

Example: Range, Map, and Filter for Linked Lists

```
square, odd = lambda x: x * x, lambda x: x % 2 == 1
list(map(square, filter(odd, range(1, 6))))           # [1, 9, 25]
map_link(square, filter_link(odd, range_link(1, 6)))  # Link(1, Link(9, Link(25)))
```

Example: Range, Map, and Filter for Linked Lists

```
square, odd = lambda x: x * x, lambda x: x % 2 == 1
list(map(square, filter(odd, range(1, 6))))           # [1, 9, 25]
map_link(square, filter_link(odd, range_link(1, 6)))  # Link(1, Link(9, Link(25)))

def range_link(start, end):
    """Return a Link containing consecutive integers from start to end.
    >>> range_link(3, 6)
    Link(3, Link(4, Link(5)))
    """
```

Example: Range, Map, and Filter for Linked Lists

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square, odd = lambda x: x * x, lambda x: x % 2 == 1
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    """

def map_link(f, s):
    """Return a Link that contains f(x) for each x in Link s.
    >>> map_link(square, range_link(3, 6))
    Link(9, Link(16, Link(25)))
    """
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Example: Range, Map, and Filter for Linked Lists

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square, odd = lambda x: x * x, lambda x: x % 2 == 1
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    """Return a Link that contains f(x) for each x in Link s.
    >>> map_link(square, range_link(3, 6))
    Link(9, Link(16, Link(25)))
    """

def filter_link(f, s):
    """Return a Link that contains only the elements x of Link s for which f(x)
    is a true value.
    >>> filter_link(odd, range_link(3, 6))
    Link(3, Link(5))
    """
```

Linked Lists Mutation

Linked Lists Can Change

Attribute assignment statements can change first and rest attributes of a Link

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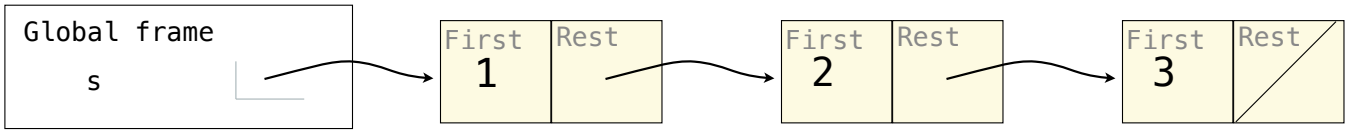

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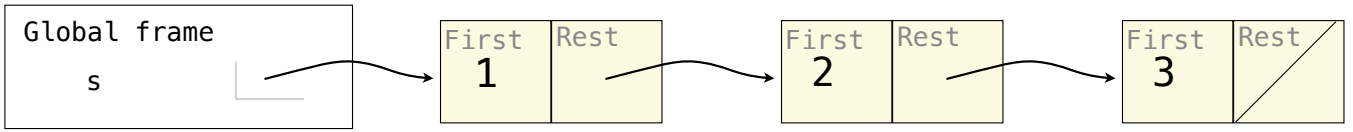

Note: The actual environment diagram is much more complicated.

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>>> s = Link(1, Link(2, Link(3)))
>>> s.first = 5
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Attribute assignment statements can change first and rest attributes of a Link

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>>> s = Link(1, Link(2, Link(3)))
>>> s.first = 5
>>> t = s.rest
```

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The rest of a linked list can contain the linked list as a sub-list

```
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>>> s.first = 5
>>> t = s.rest
>>> t.rest = s
>>> s.first
5
```

Note: The actual environment diagram is much more complicated.

Linked Lists Can Change

Attribute assignment statements can change first and rest attributes of a Link

The rest of a linked list can contain the linked list as a sub-list

```
>>> s = Link(1, Link(2, Link(3)))
>>> s.first = 5
>>> t = s.rest
>>> t.rest = s
>>> s.first
5
>>> s.rest.rest.rest.rest.rest.first
```

Note: The actual environment diagram is much more complicated.

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Attribute assignment statements can change first and rest attributes of a Link

The rest of a linked list can contain the linked list as a sub-list

```
>>> s = Link(1, Link(2, Link(3)))
>>> s.first = 5
>>> t = s.rest
>>> t.rest = s
>>> s.first
5
>>> s.rest.rest.rest.rest.rest.first
2
```

Note: The actual environment diagram is much more complicated.

Linked Lists Can Change

Attribute assignment statements can change first and rest attributes of a Link

The rest of a linked list can contain the linked list as a sub-list

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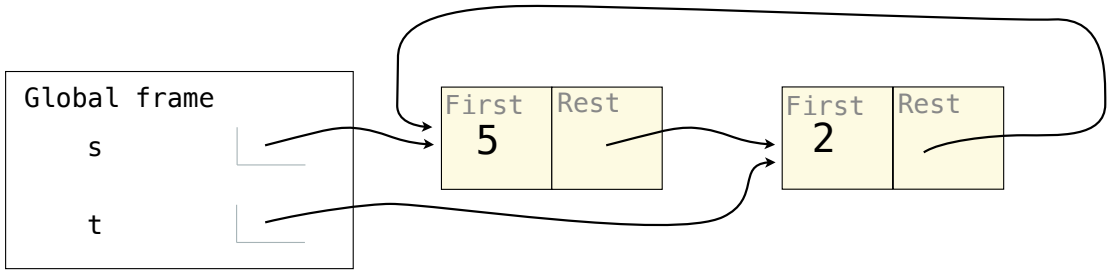
>>> s = Link(1, Link(2, Link(3)))
>>> s.first = 5
>>> t = s.rest
>>> t.rest = s
>>> s.first
5
>>> s.rest.rest.rest.rest.rest.first
2
```



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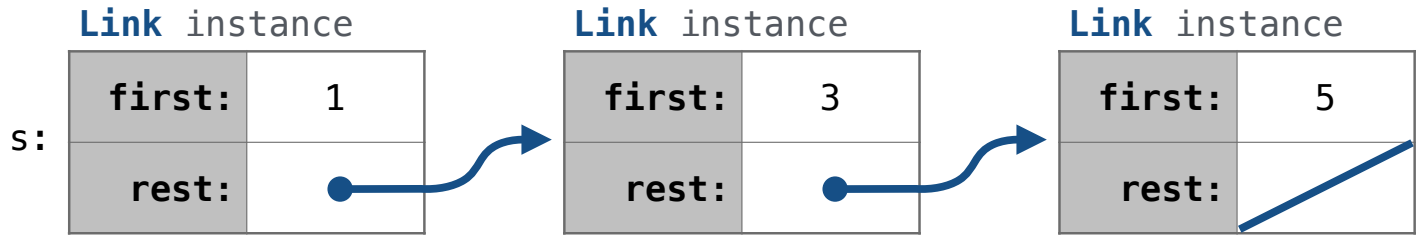
Linked List Mutation Example

Adding to an Ordered List



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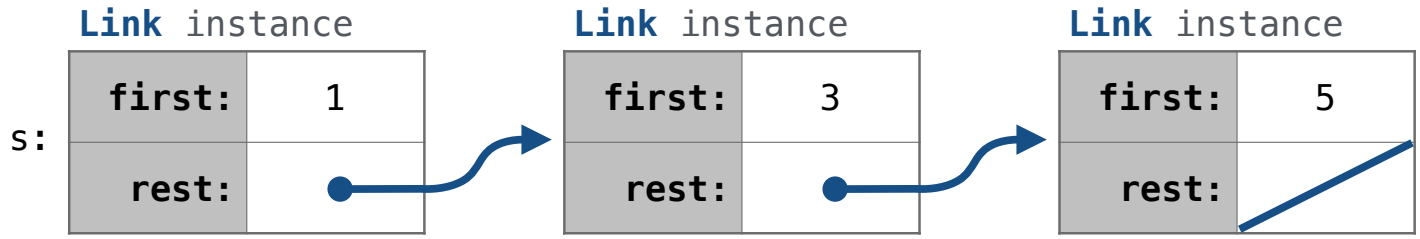
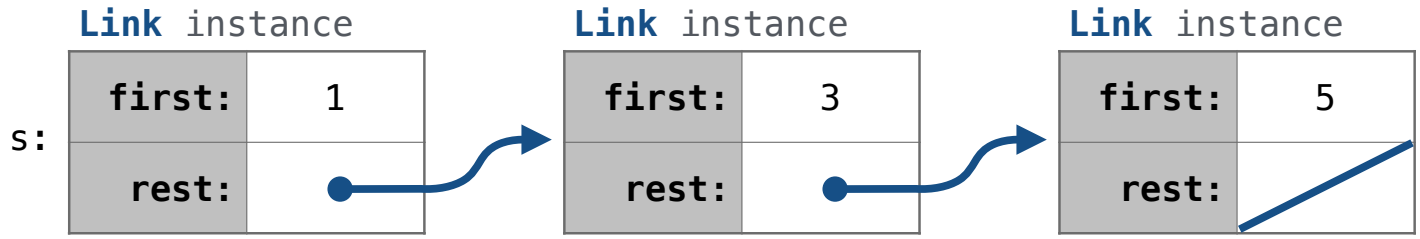


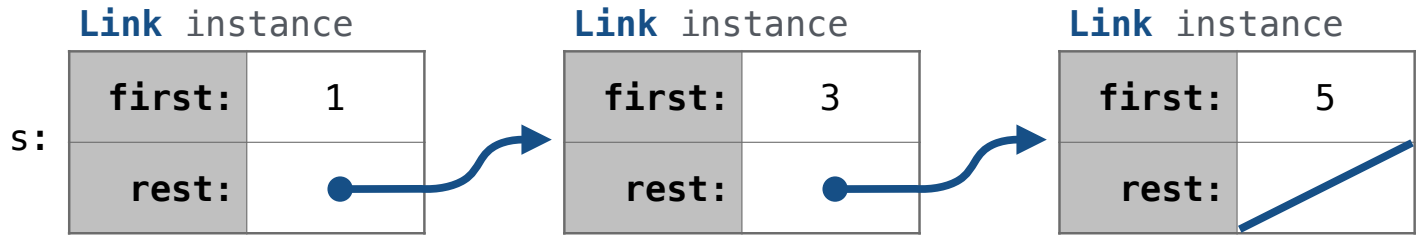

```
def add(s, v):
    """Add v to an ordered list s with no repeats, returning modified s."""
```

Adding to an Ordered List




```
def add(s, v):
    """Add v to an ordered list s with no repeats, returning modified s."""
    (Note: If v is already in s, then don't modify s, but still return it.)
```

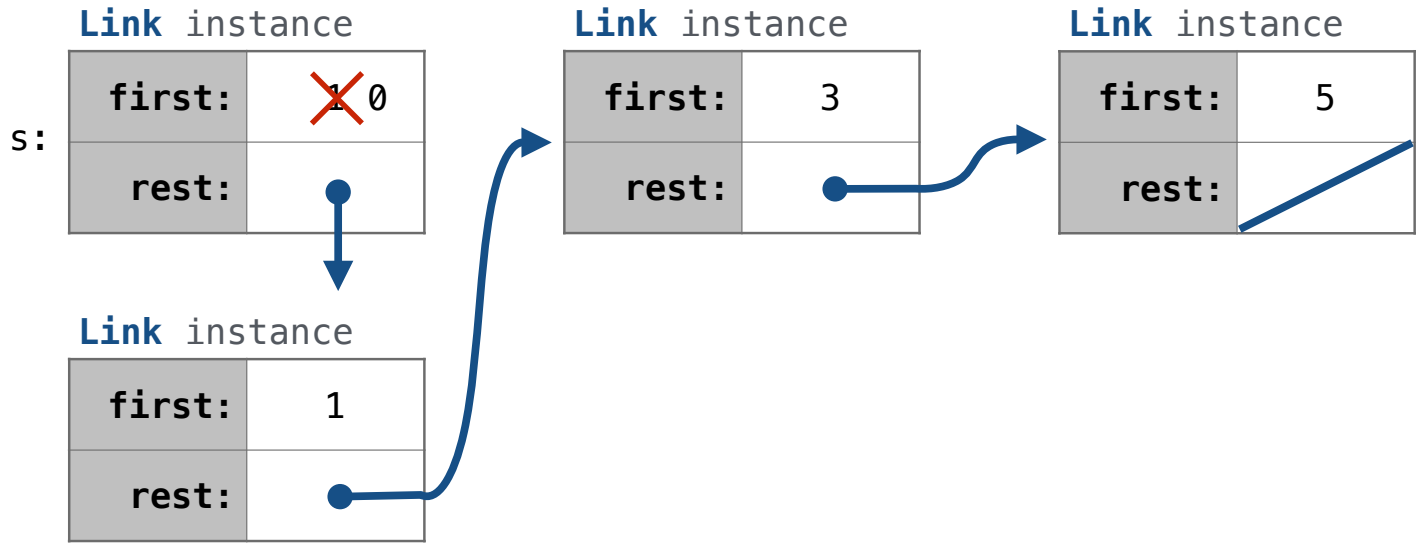
Adding to an Ordered List




```
def add(s, v):
    """Add v to an ordered list s with no repeats, returning modified s."""
    (Note: If v is already in s, then don't modify s, but still return it.)

        add(s, 0)
```

Adding to an Ordered List

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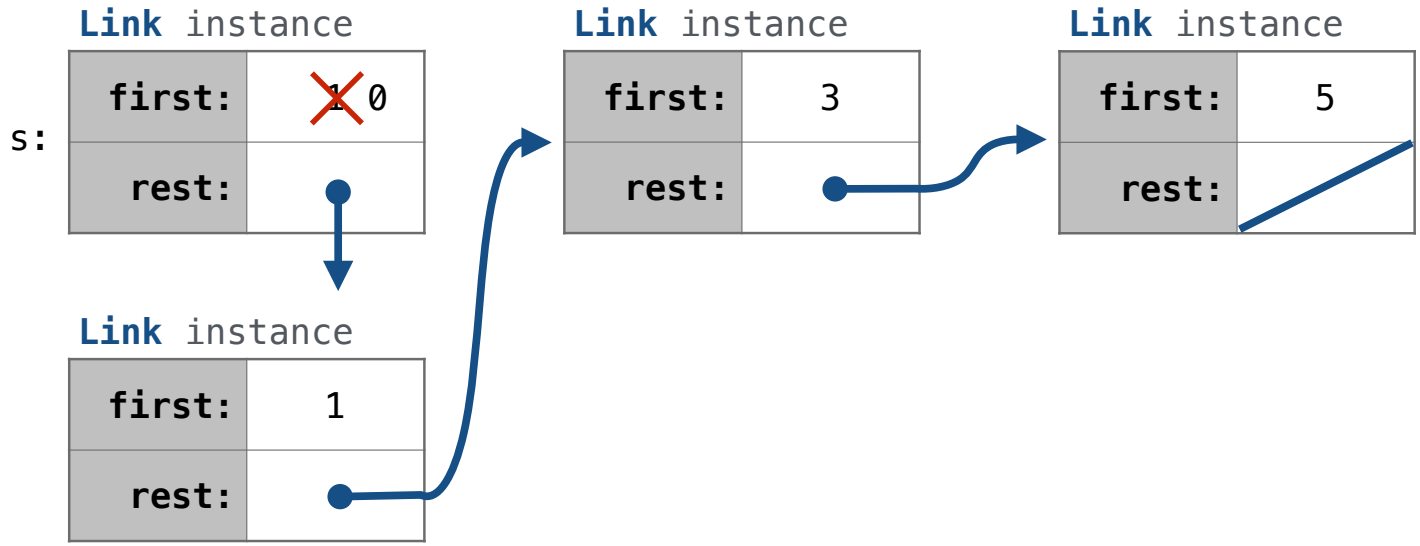
    (Note: If v is already in s, then don't modify s, but still return it.)

        add(s, 0)
```

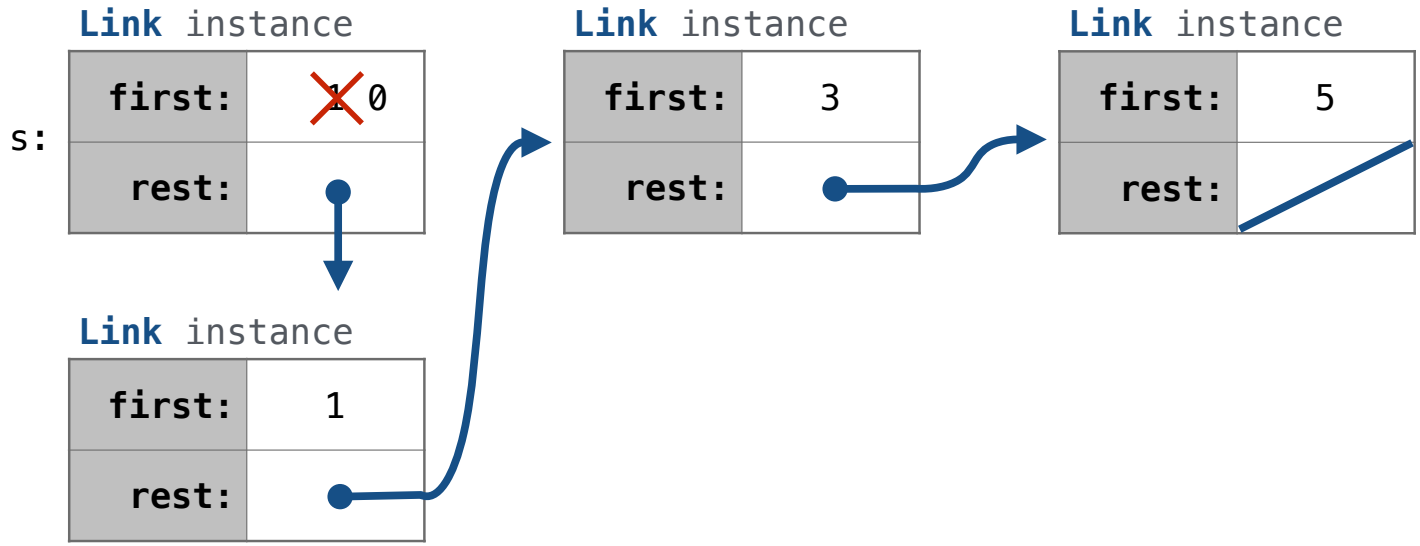
Adding to an Ordered List




```
def add(s, v):
    """Add v to an ordered list s with no repeats, returning modified s."""
    (Note: If v is already in s, then don't modify s, but still return it.)

        add(s, 0)      add(s, 3)
```

Adding to an Ordered List




```
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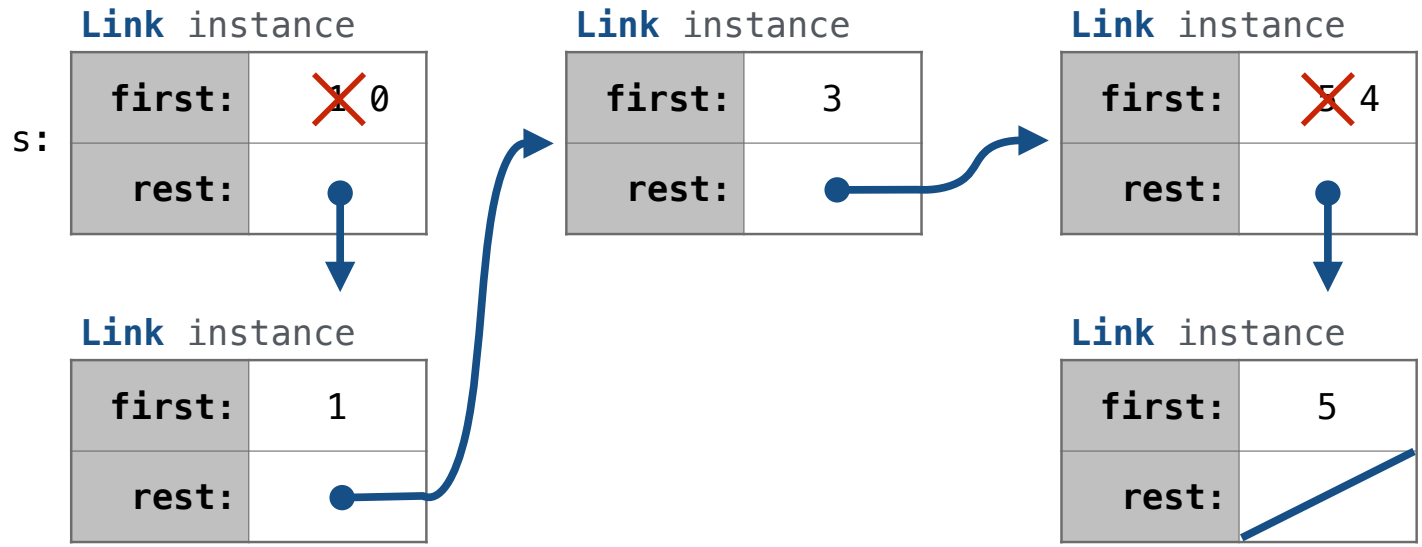
    """Add v to an ordered list s with no repeats, returning modified s."""
    (Note: If v is already in s, then don't modify s, but still return it.)

        add(s, 0)      add(s, 3)      add(s, 4)
```

Adding to an Ordered List

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def add(s, v):
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```

add(s, 0) add(s, 3) add(s, 4)

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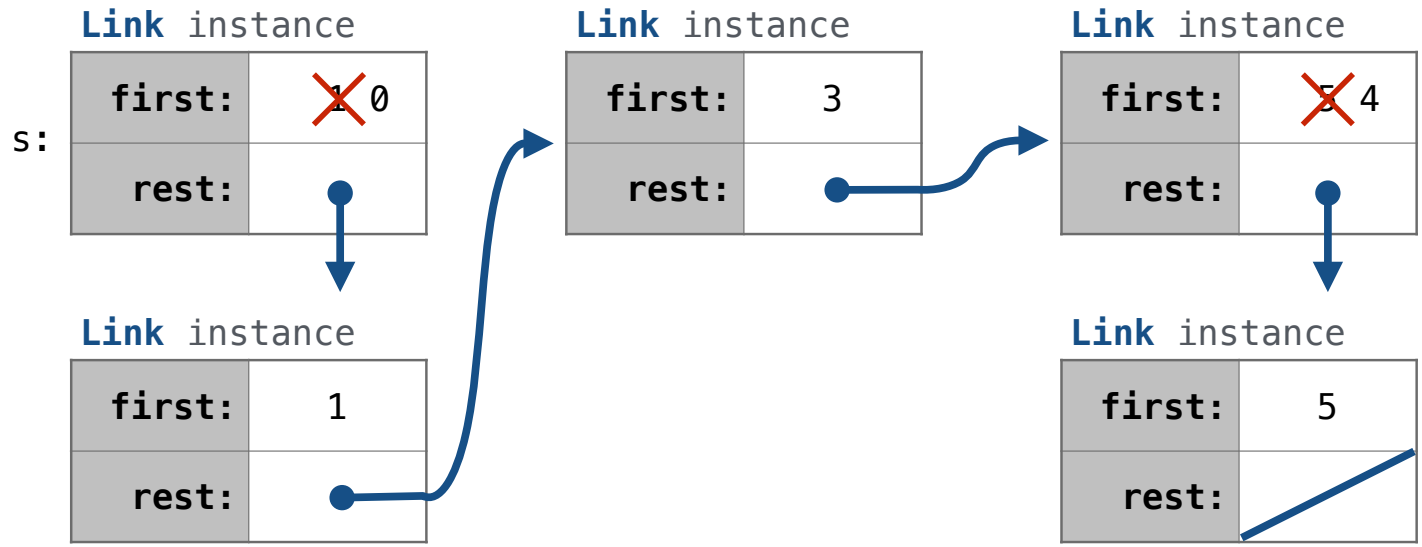
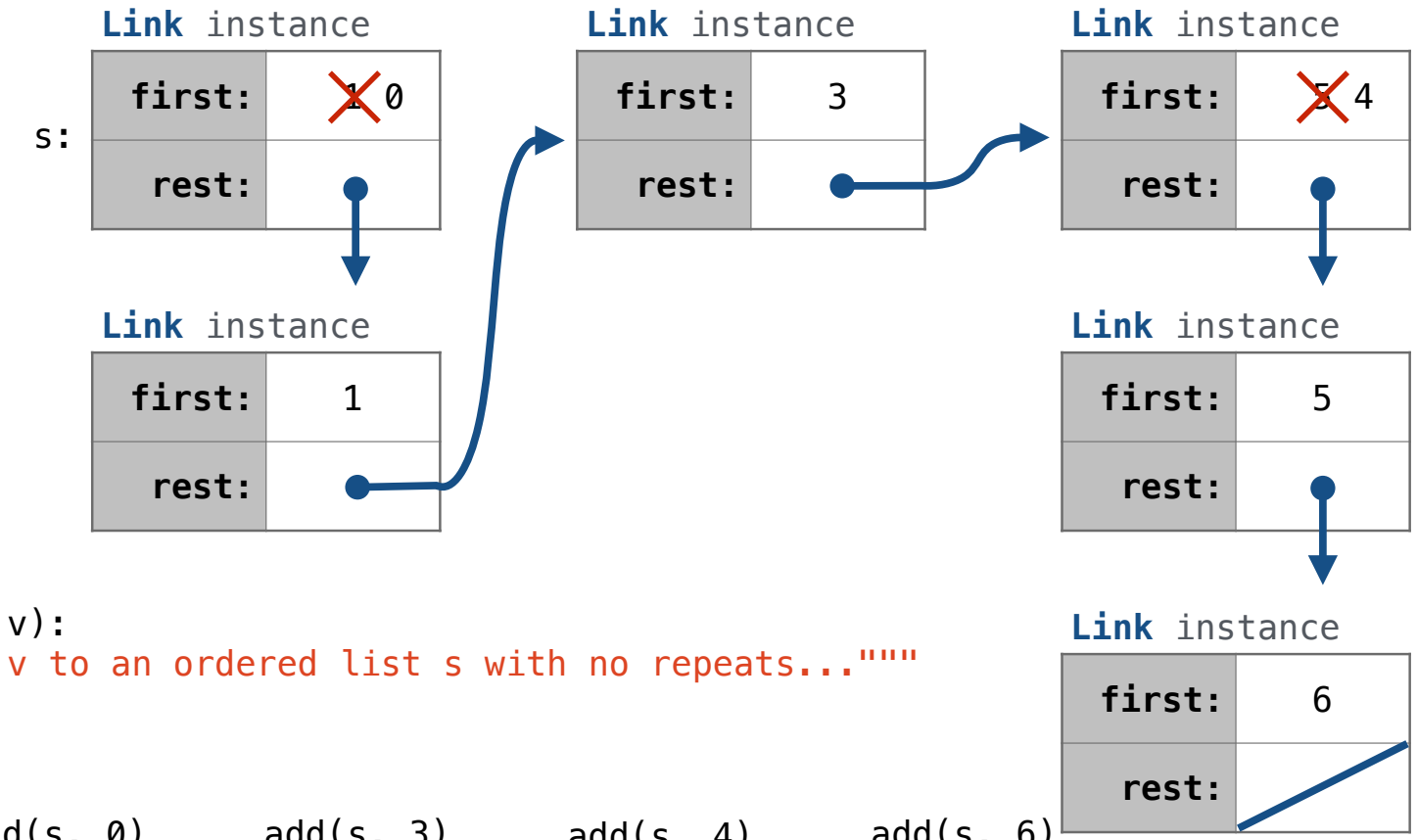



```
def add(s, v):
    """Add v to an ordered list s with no repeats..."""
```

add(s, 0) add(s, 3) add(s, 4) add(s, 6)

Adding to an Ordered List




```
def add(s, v):
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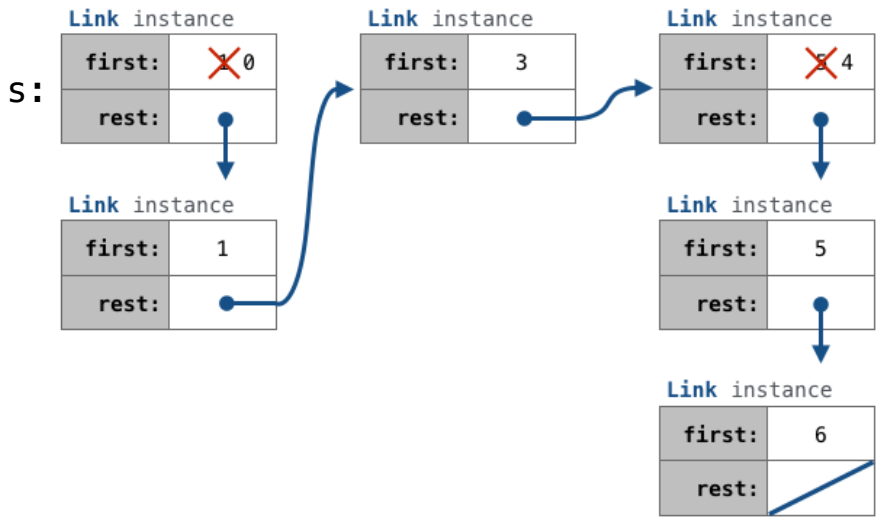
```

add(s, 0) add(s, 3) add(s, 4) add(s, 6)

Adding to a Set Represented as an Ordered List



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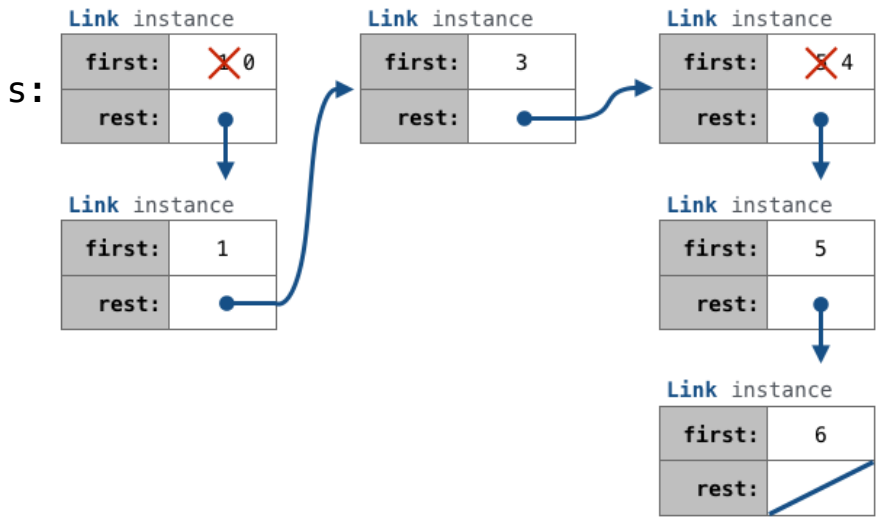

Adding to a Set Represented as an Ordered List

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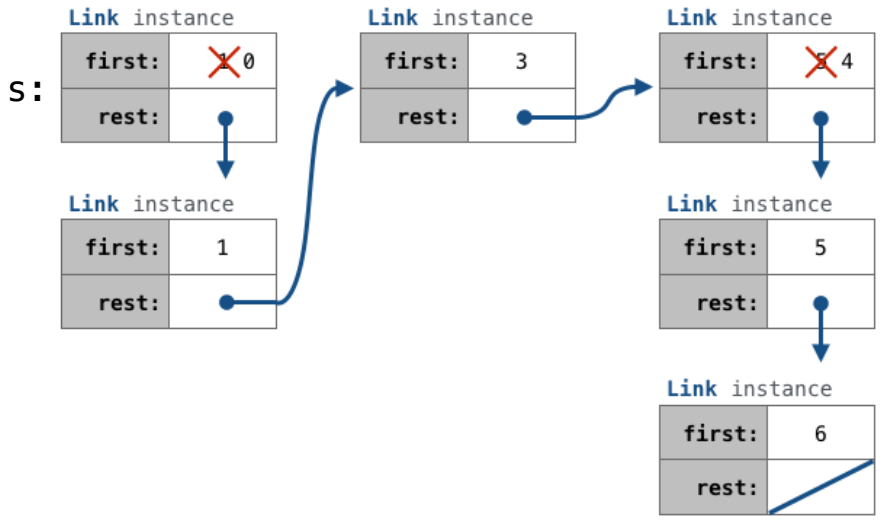

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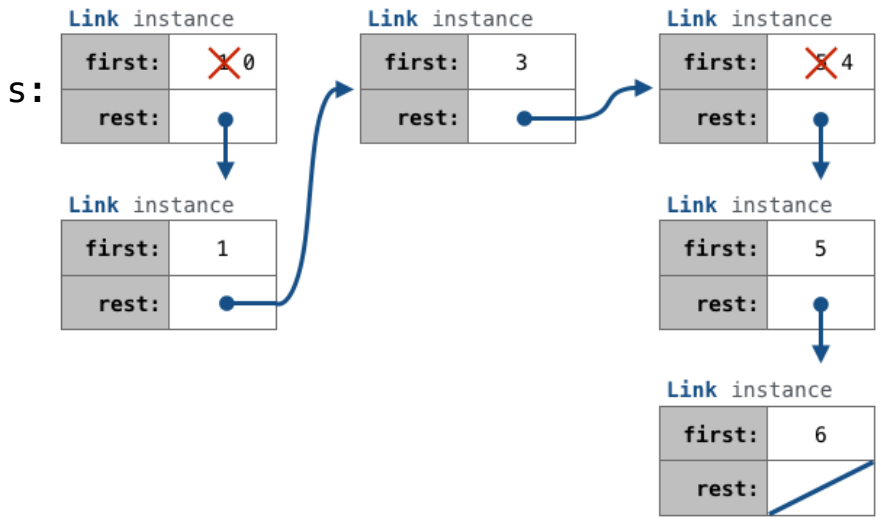

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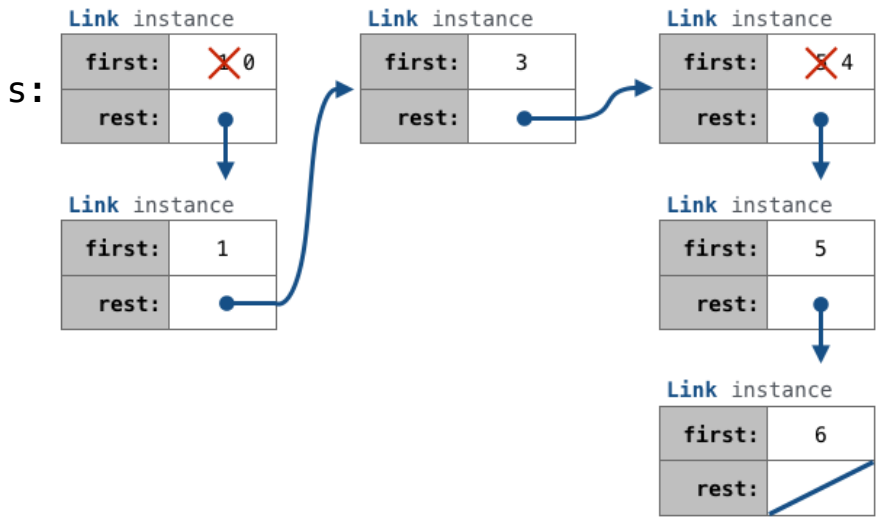

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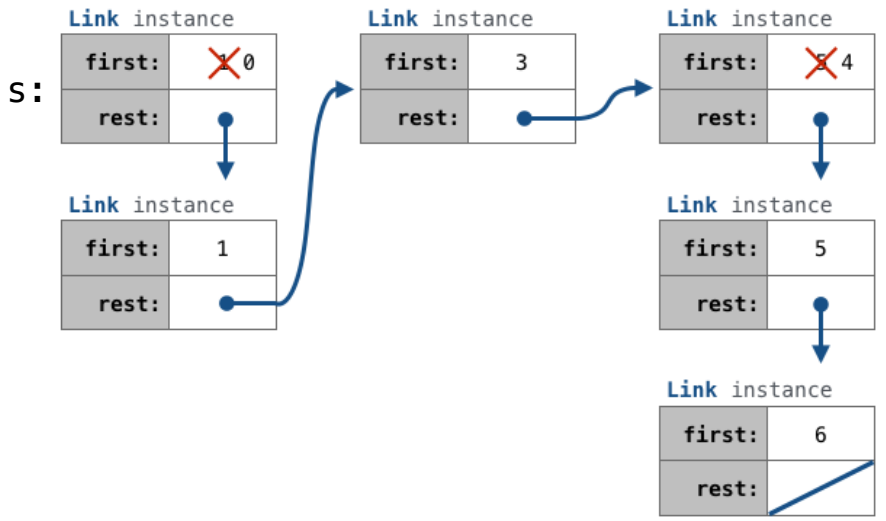

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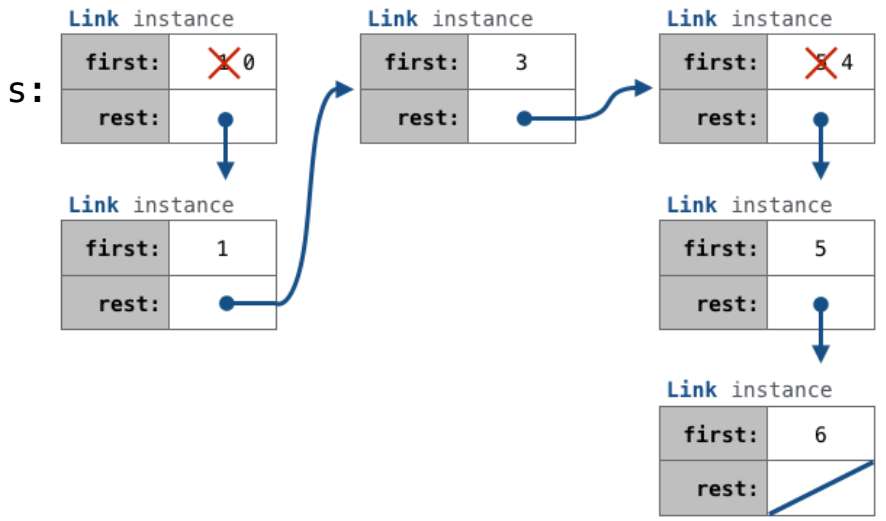

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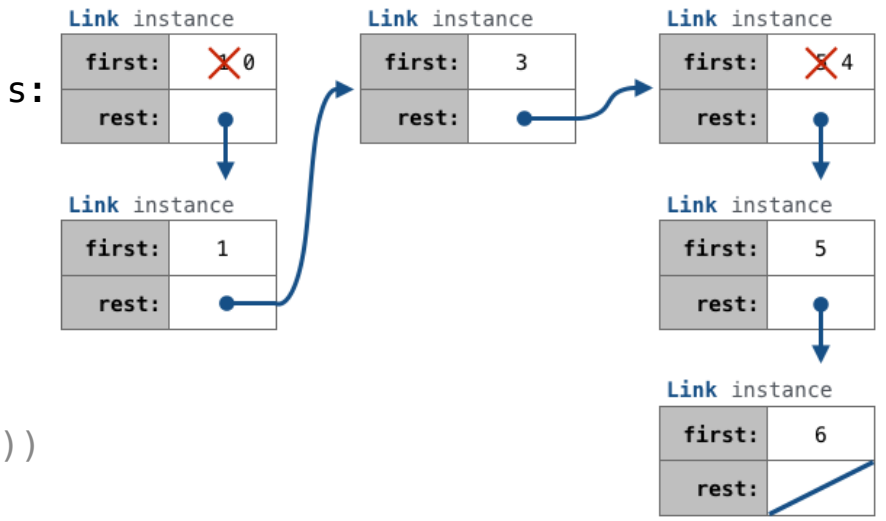

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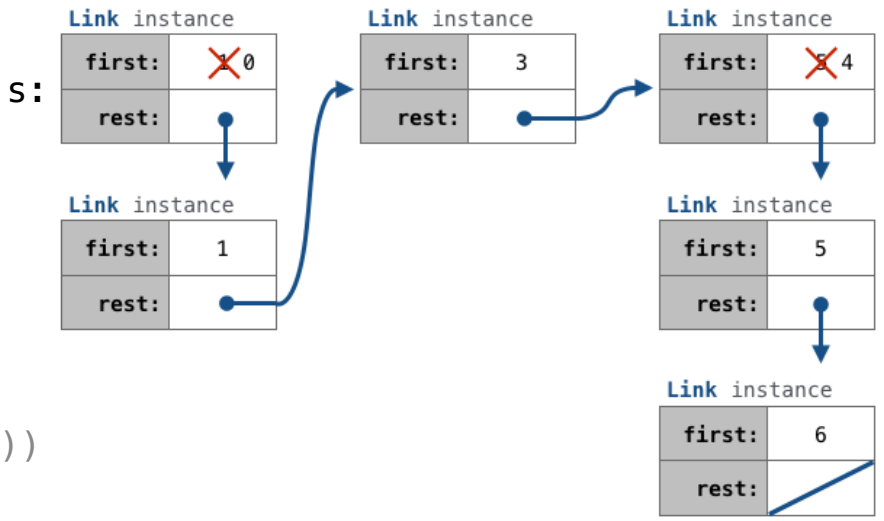

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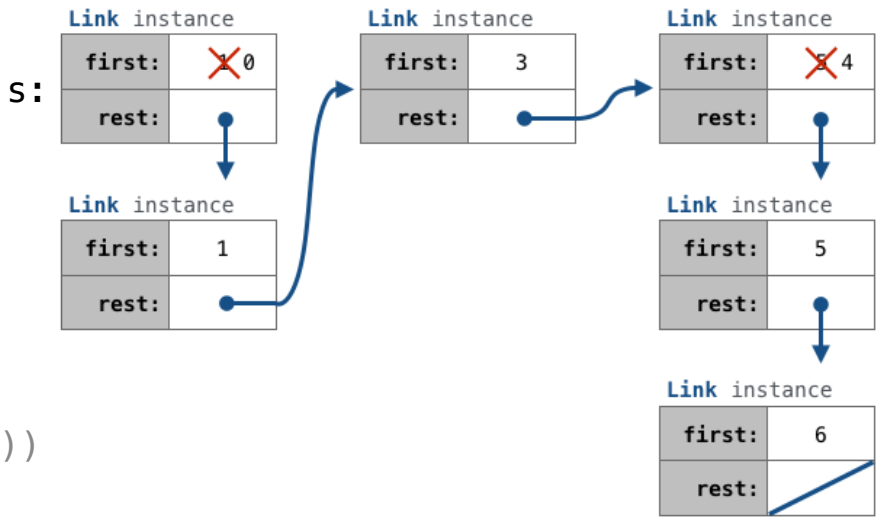

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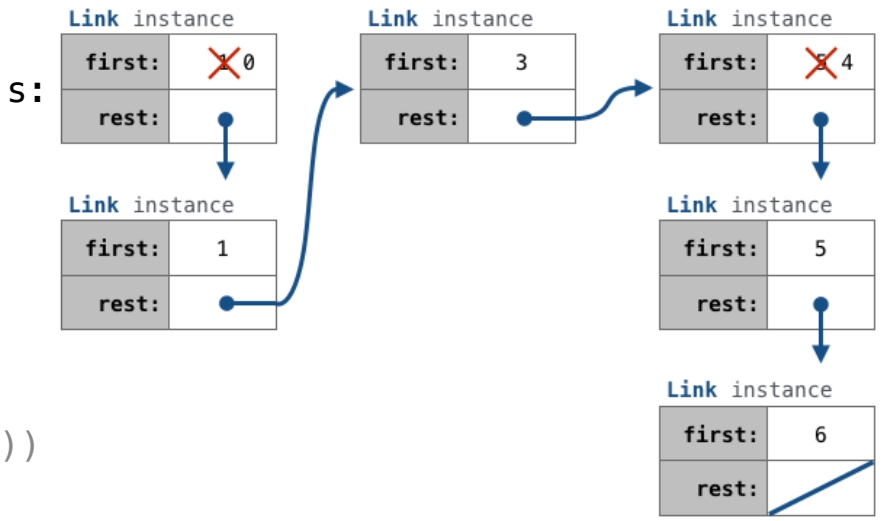

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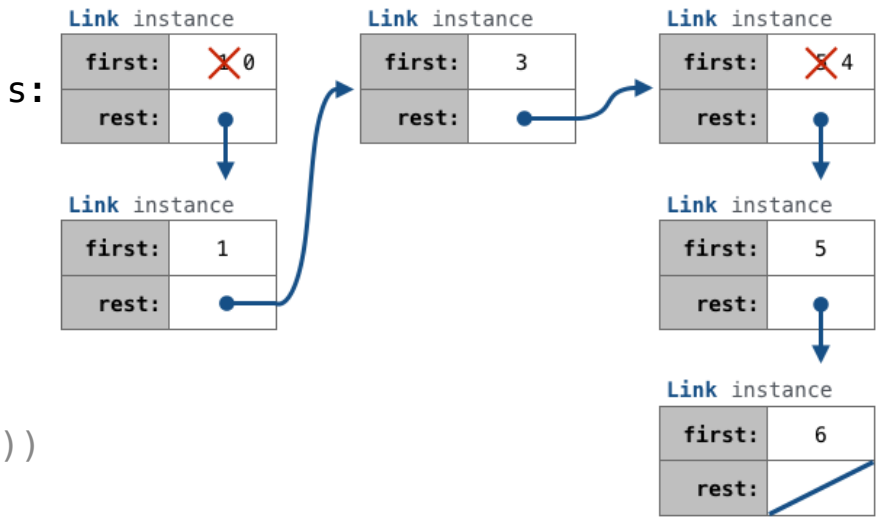

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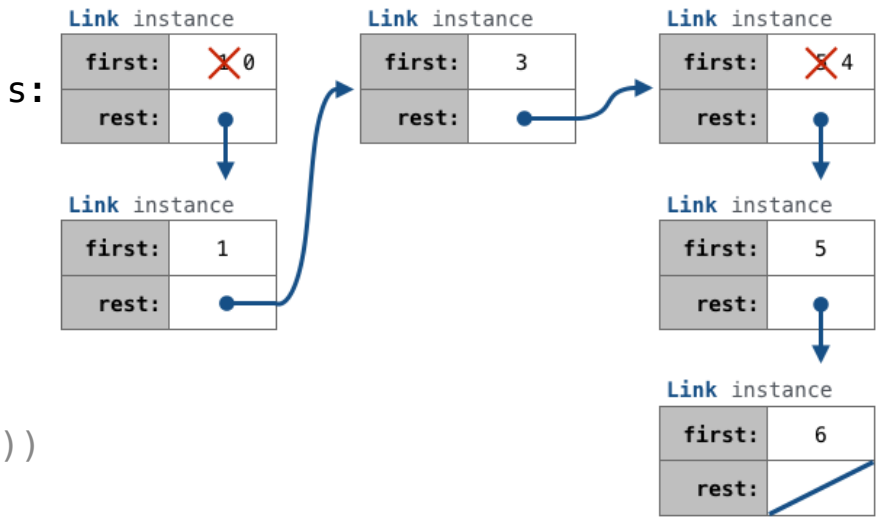
```

v



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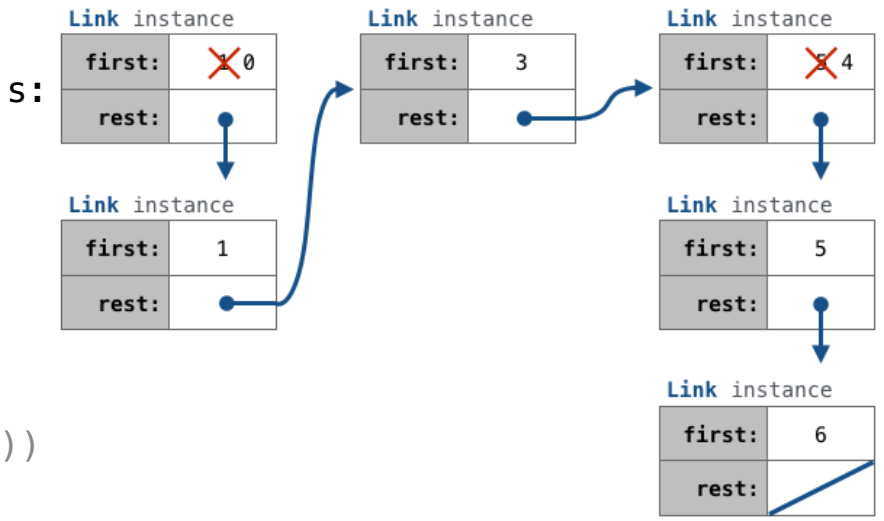

v

Link(s.first, s.rest)

Adding to a Set Represented as an Ordered List

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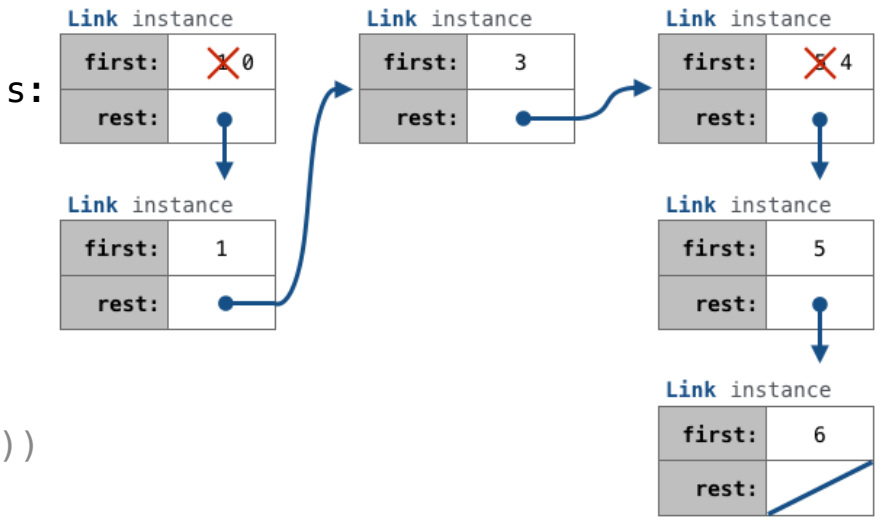
    Link(0, Link(1, Link(3, Link(4, Link(5, Link(6))))
    """
    assert s is not List.empty
    if s.first > v:
        s.first, s.rest = _____v_____ , _____Link(s.first, s.rest)_____
    elif s.first < v and empty(s.rest):
        s.rest = _____Link(v)_____
    elif s.first < v:
        _____
    return s
```



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    if s.first > v:
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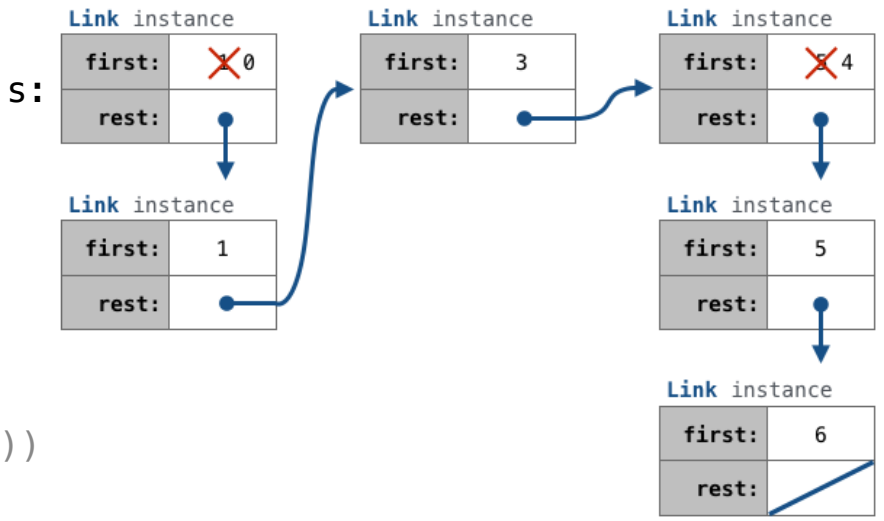
    elif s.first < v and empty(s.rest):
        s.rest = _____
    elif s.first < v:
        _____
    return s
```



v

Link(s.first, s.rest)

Link(v)

add(s.rest, v)

Adding to a Set Represented as an Ordered List

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        s.rest = _____Link(v)_____
    elif s.first < v:
        _____add(s.rest, v)_____
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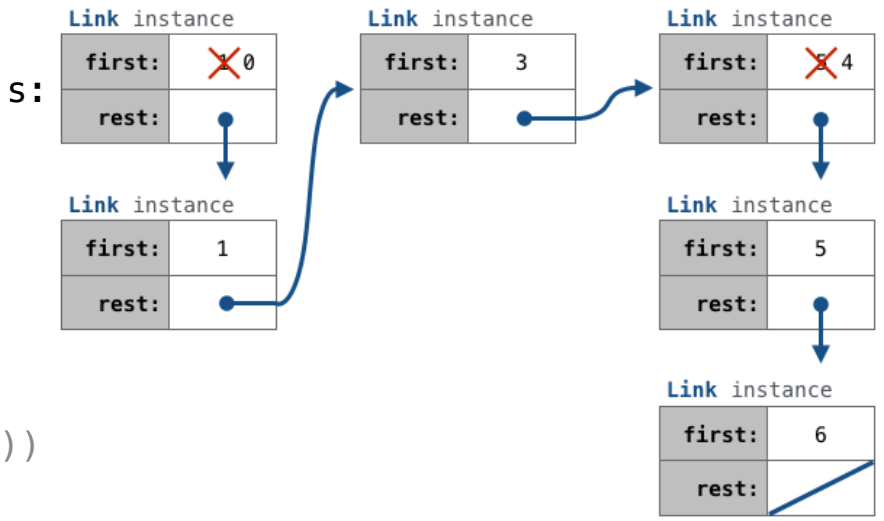
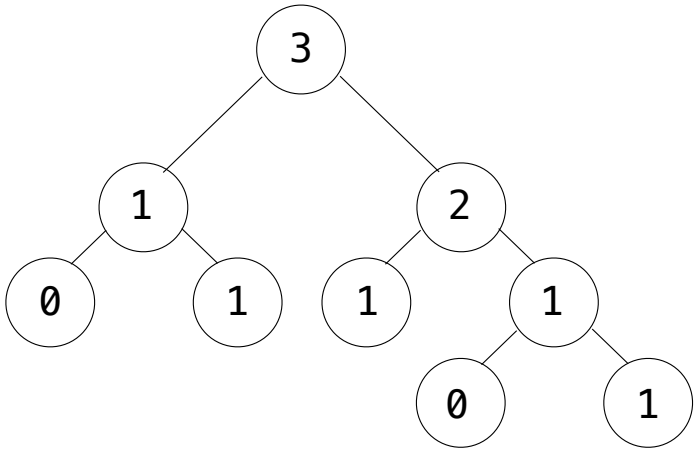
```



Tree Class

Tree Abstraction (Review)



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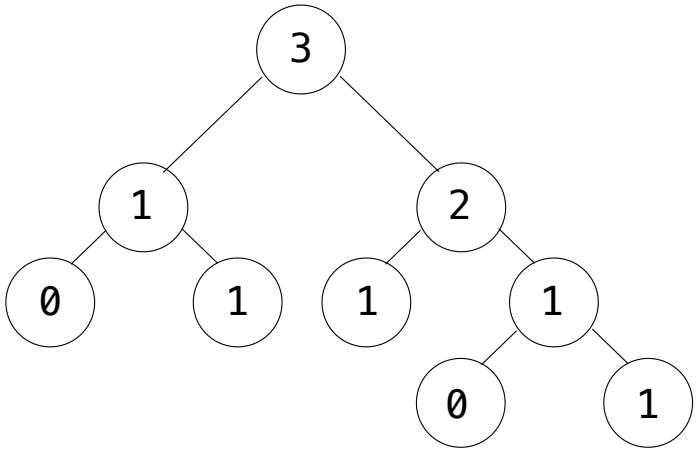
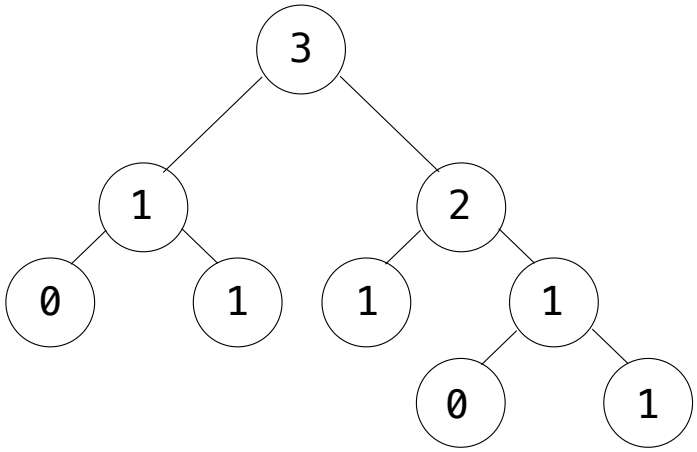


Recursive description (wooden trees):

Relative description (family trees):

Tree Abstraction (Review)



Recursive description (wooden trees):

A **tree** has a **root label** and a list of **branches**

Relative description (family trees):

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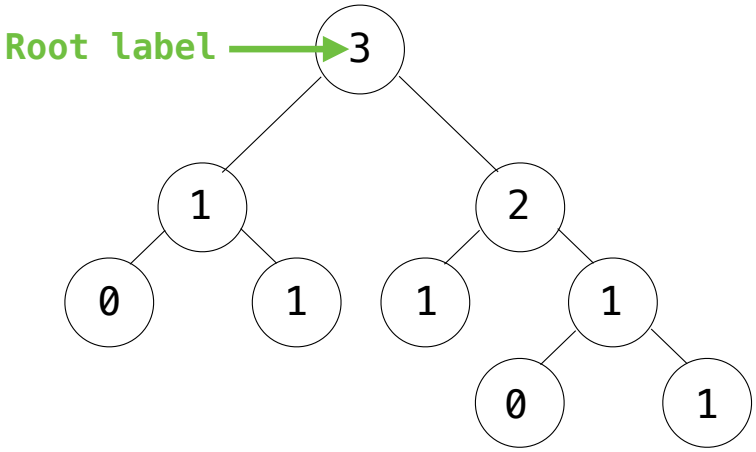

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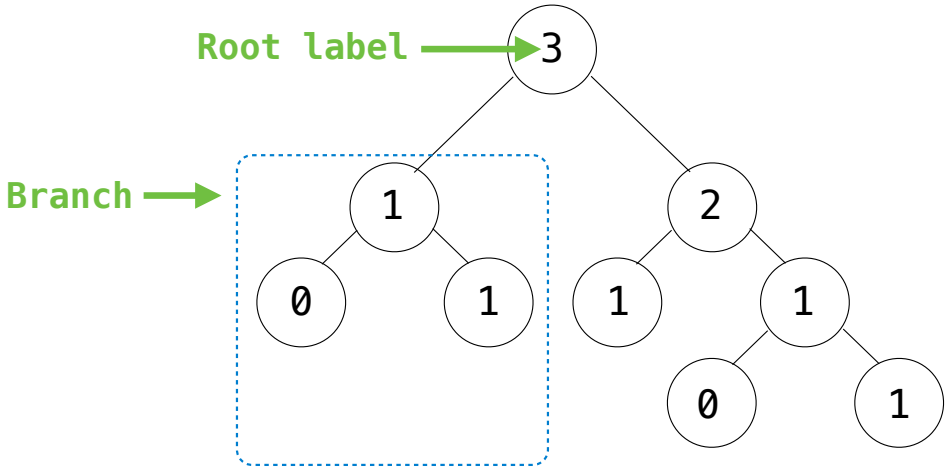

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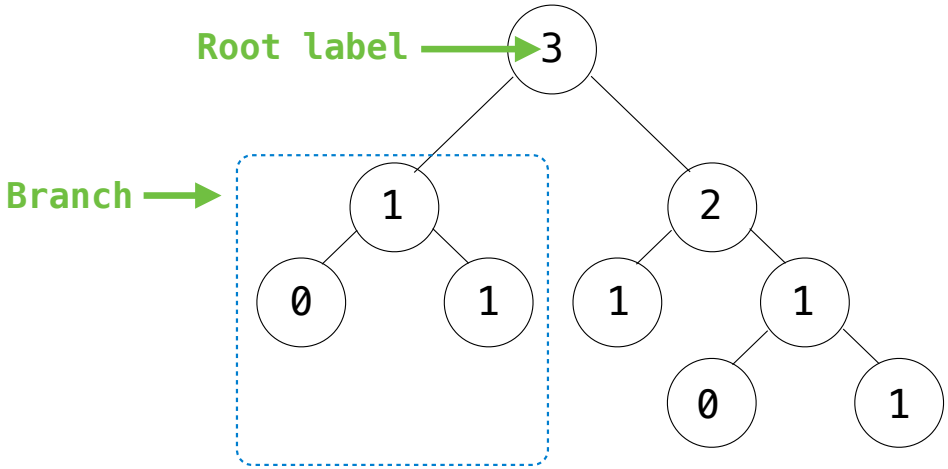

Recursive description (wooden trees):

A **tree** has a **root label** and a list of **branches**

Each **branch** is a **tree**

Relative description (family trees):

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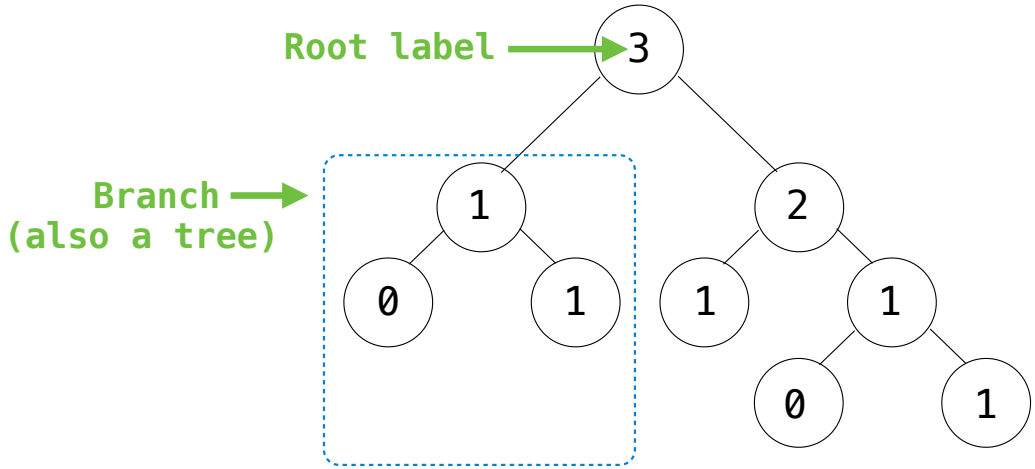

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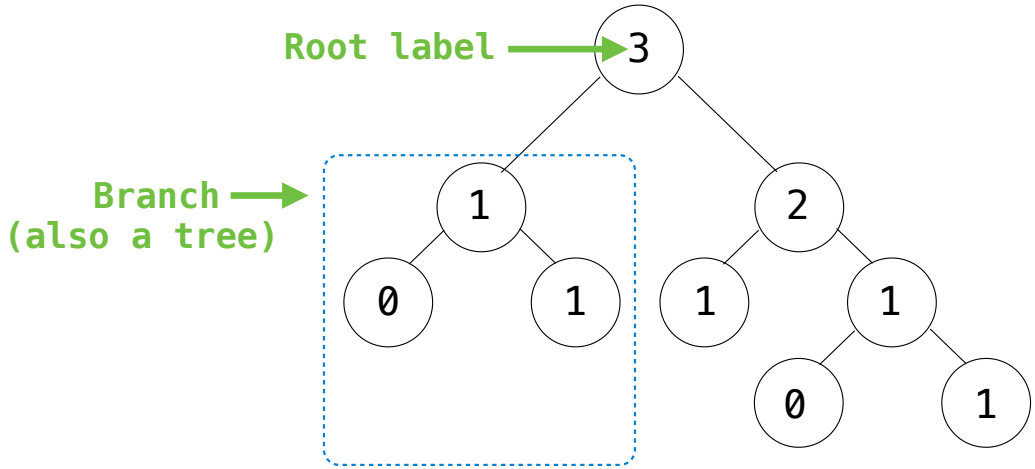

Recursive description (wooden trees):

A **tree** has a **root label** and a list of **branches**

Each **branch** is a **tree**

A **tree** with zero **branches** is called a **leaf**

Relative description (family trees):

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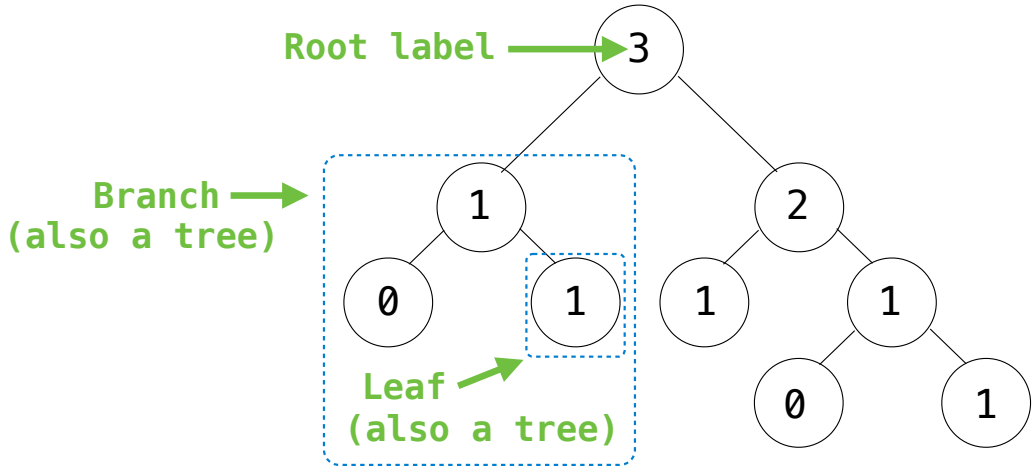

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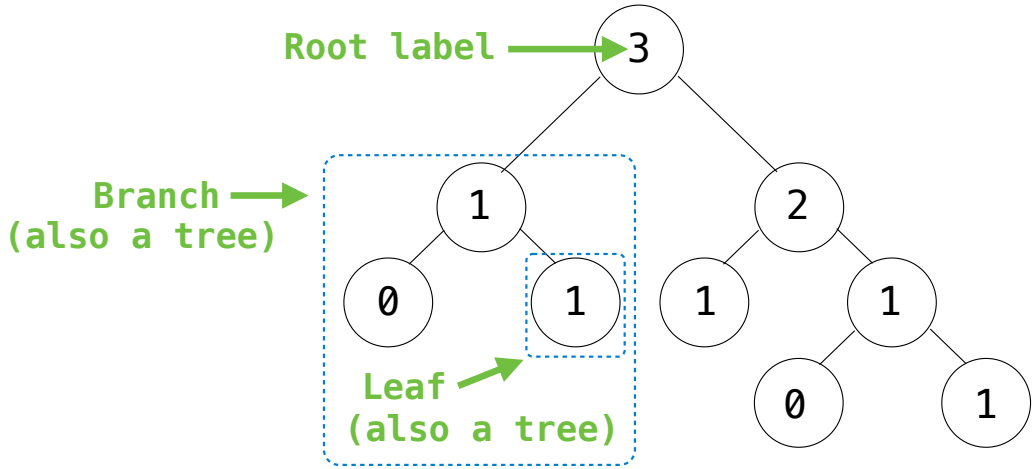

Recursive description (wooden trees):

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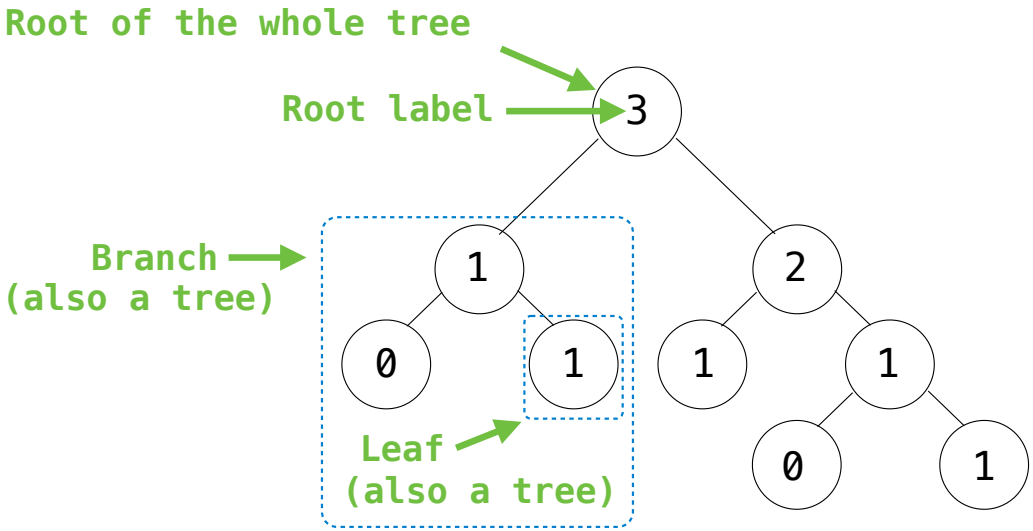
Each **branch** is a **tree**

A **tree** with zero **branches** is called a **leaf**

A **tree** starts at the **root**

Relative description (family trees):

Tree Abstraction (Review)



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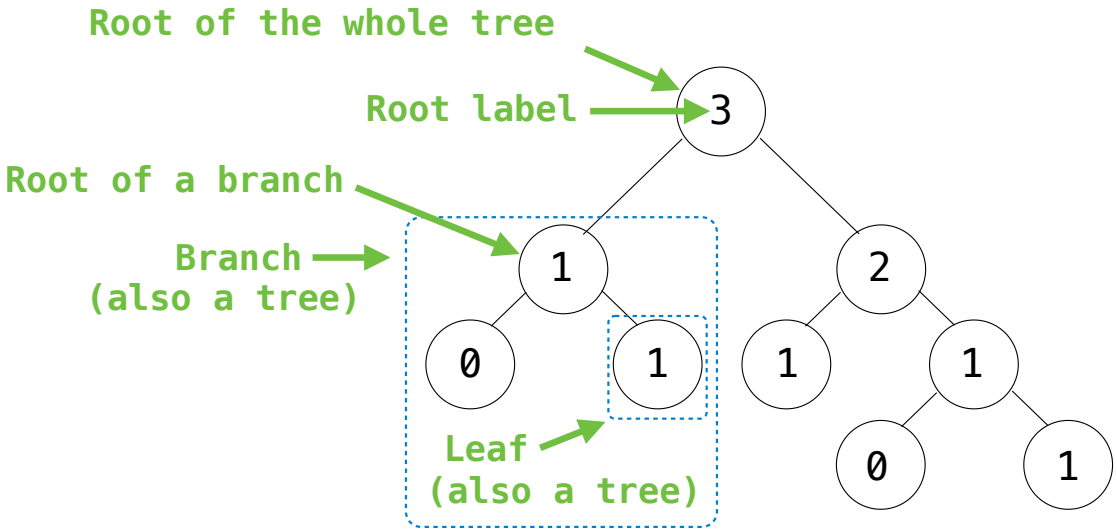

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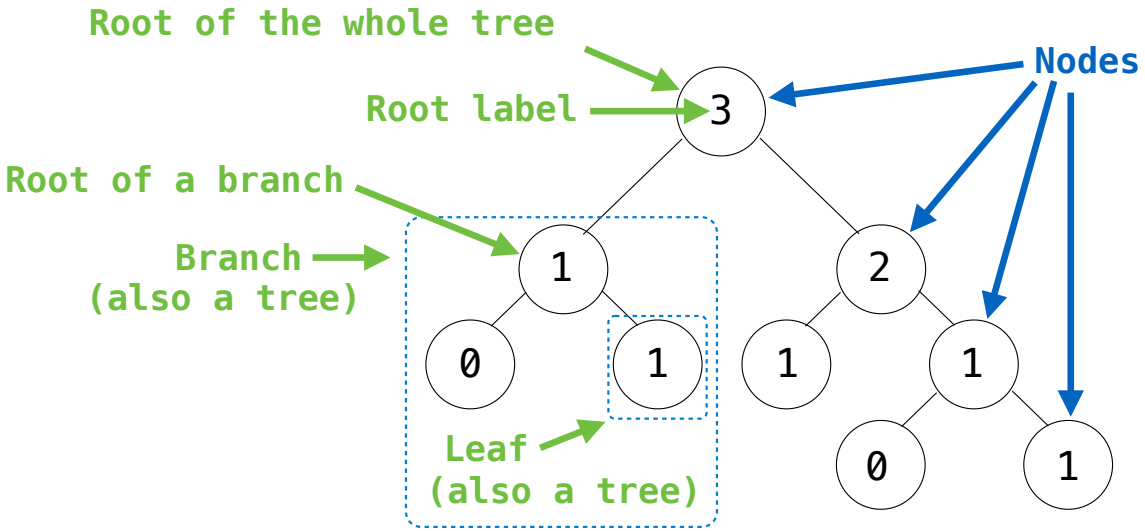

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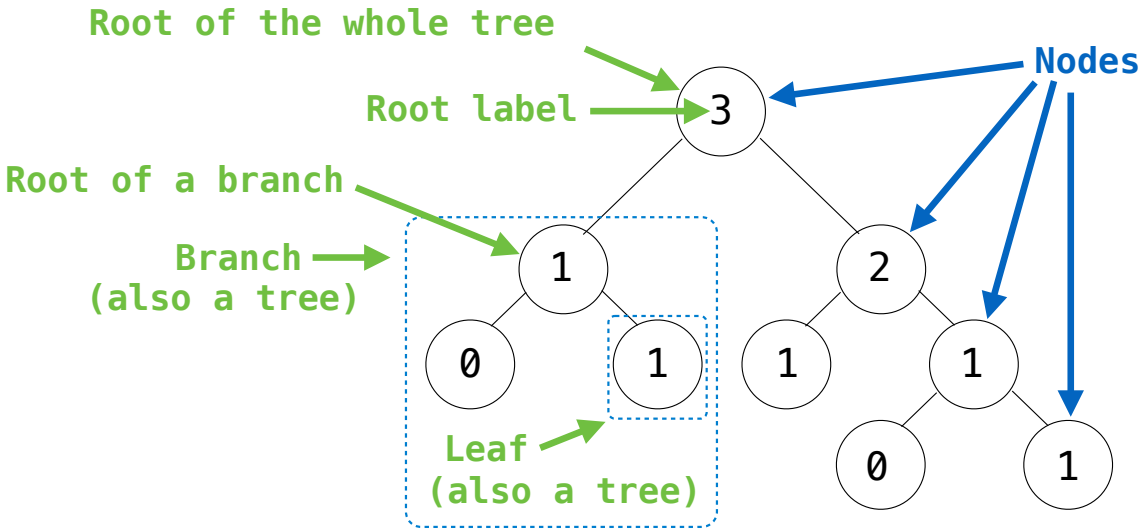
Relative description (family trees):

Each location in a tree is called a **node**

Tree Abstraction (Review)



Recursive description (wooden trees):

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Each **branch** is a **tree**

A **tree** with zero **branches** is called a **leaf**

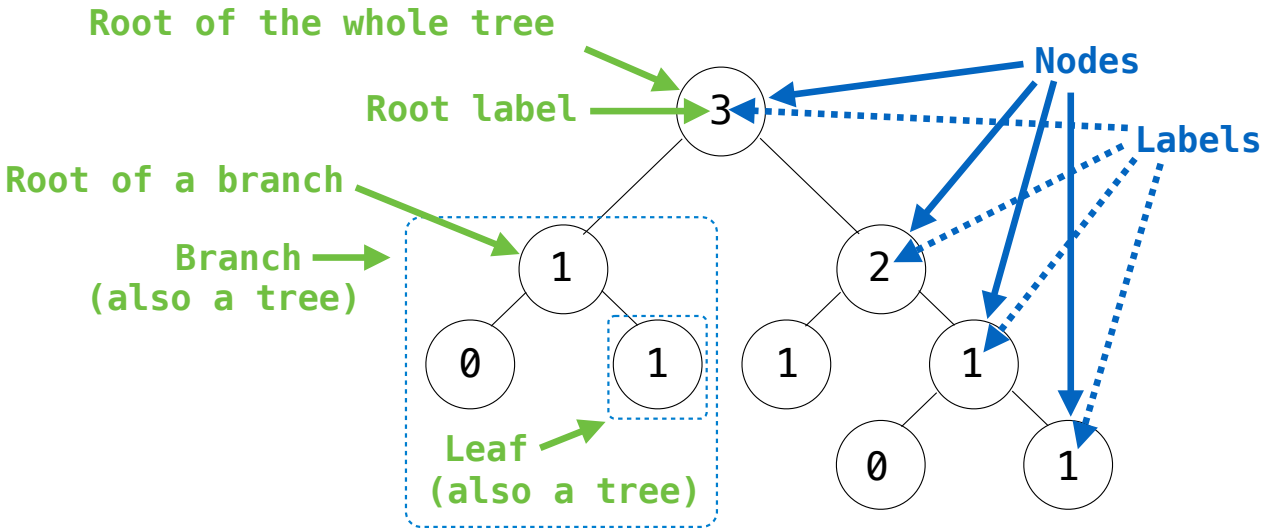
A **tree** starts at the **root**

Relative description (family trees):

Each location in a tree is called a **node**

Each **node** has a **label** that can be any value

Tree Abstraction (Review)



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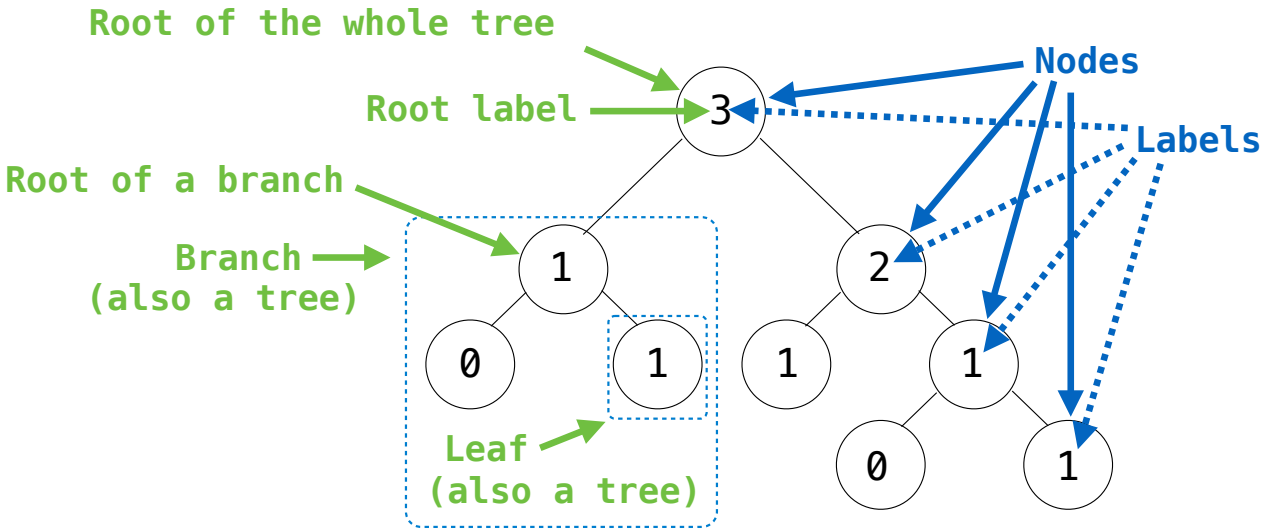
Tree Abstraction (Review)



Recursive description (wooden trees):

A **tree** has a **root label** and a list of **branches**

Each **branch** is a **tree**

A **tree** with zero **branches** is called a **leaf**

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Relative description (family trees):

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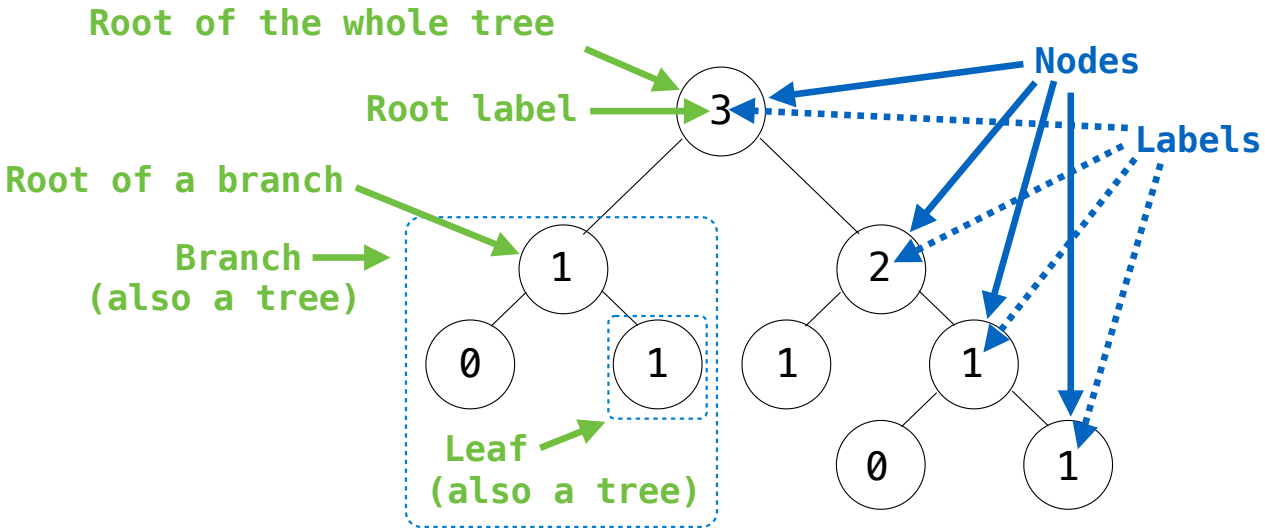

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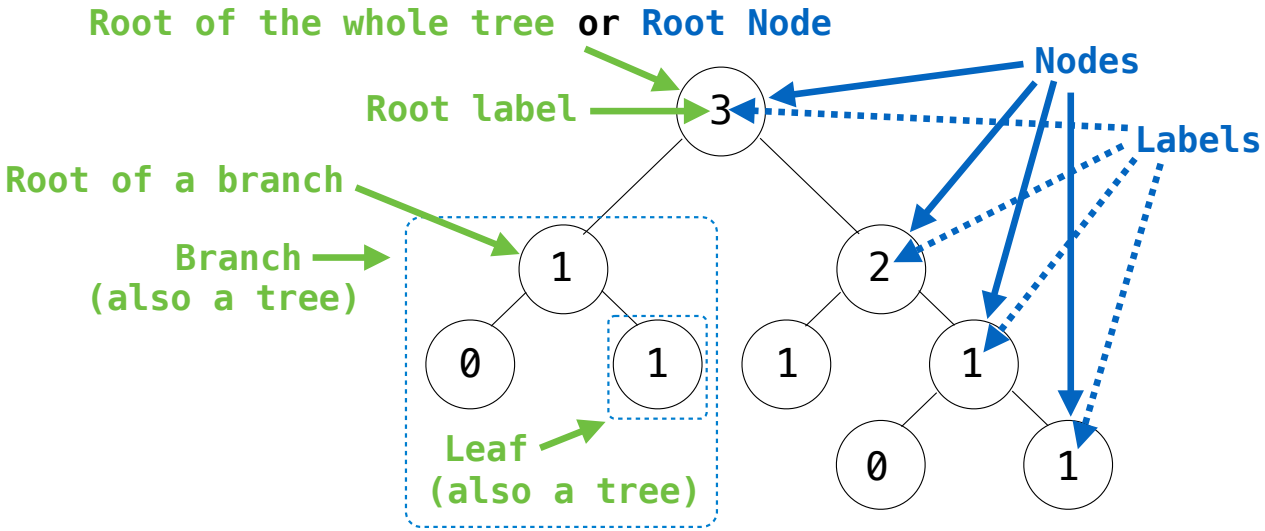

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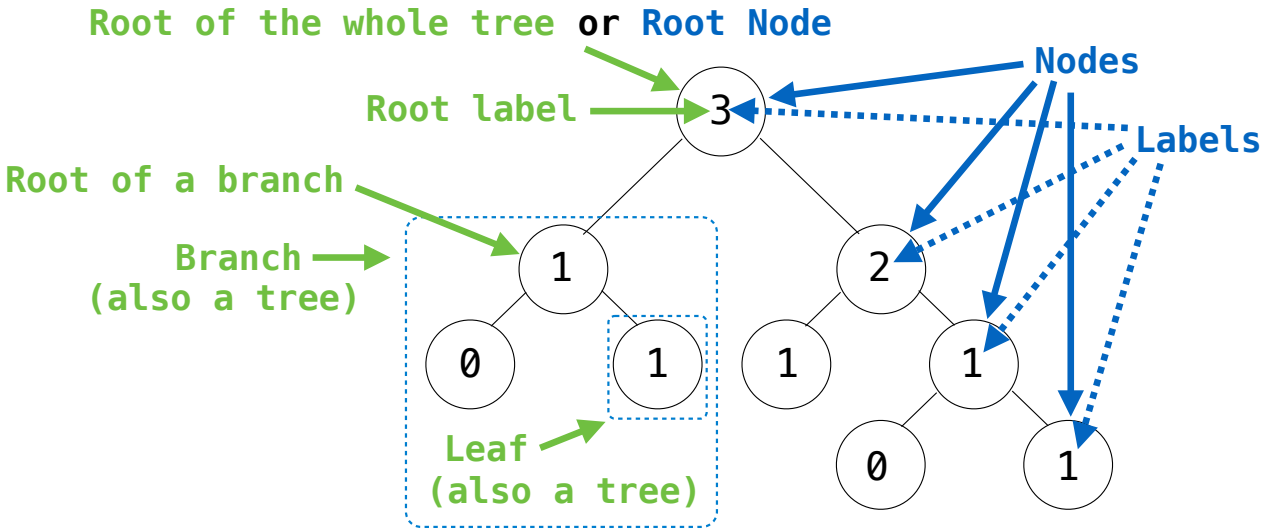

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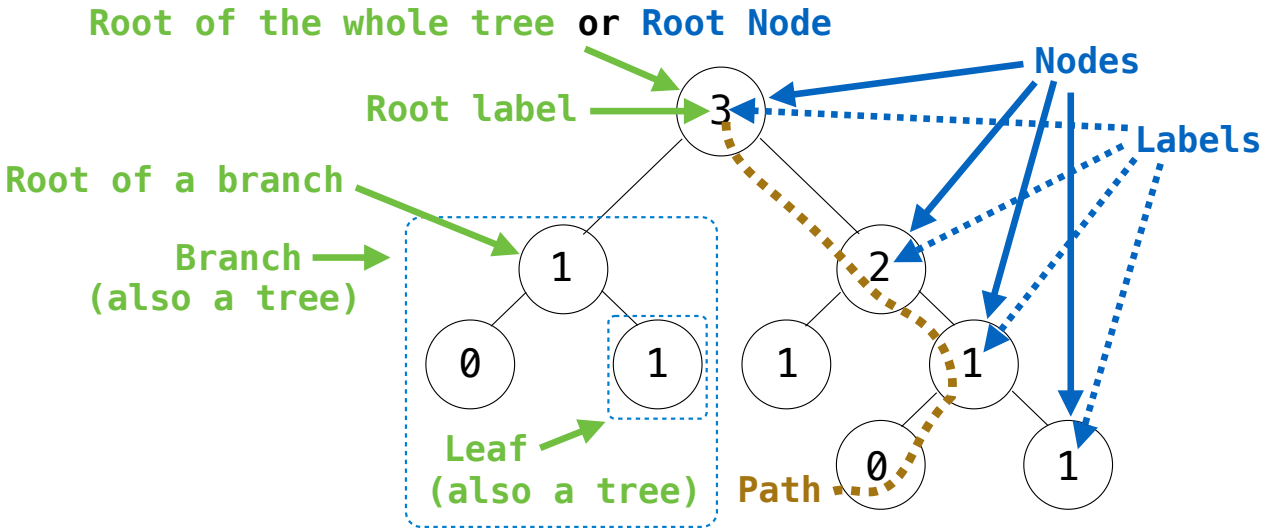

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def tree(label, branches=[]):
    for branch in branches:
        assert is_tree(branch)
    return [label] + list(branches)
def label(tree):
    return tree[0]
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    if n == 0 or n == 1:
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(Demo)

Tree Mutation

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Removing subtrees from a tree is called *pruning*

Prune branches before recursive processing

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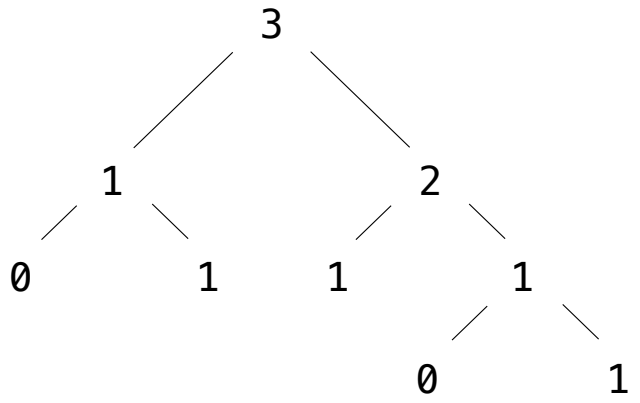

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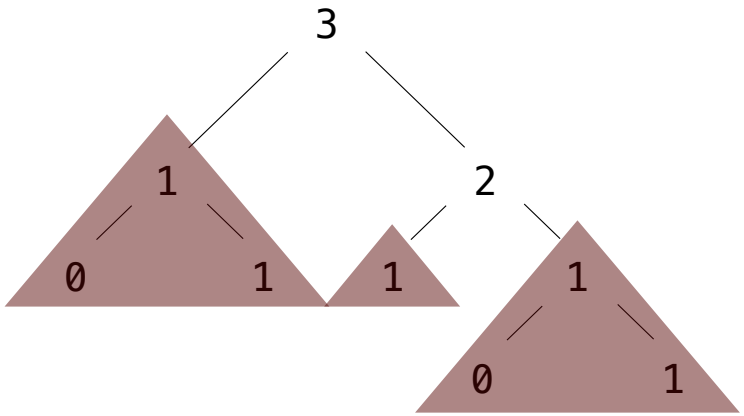

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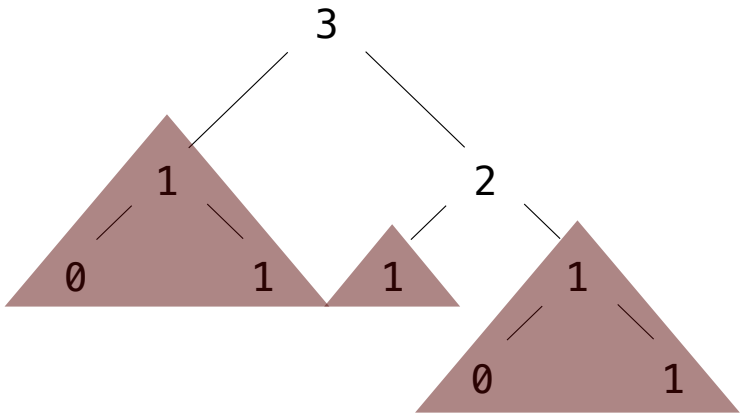


```
def prune(t, n):
    """Prune all sub-trees whose label is n."""
    t.branches = [_____________ for b in t.branches if _____________________]
    for b in t.branches:
        prune(_____________________________, _____________________________)
```

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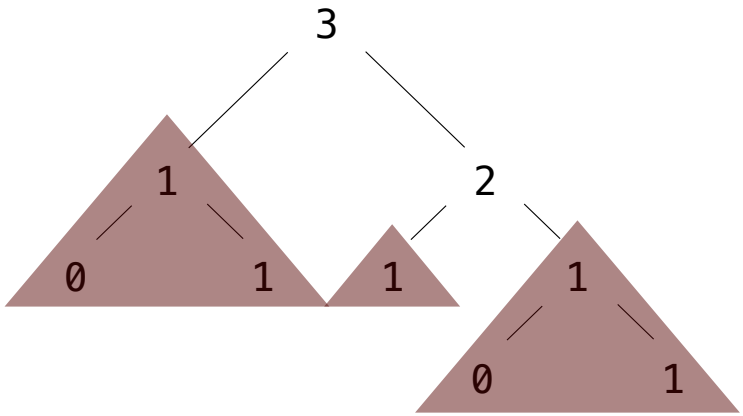


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def prune(t, n):
    """Prune all sub-trees whose label is n."""
    t.branches = [_____b_____ for b in t.branches if ____b.label != n____]
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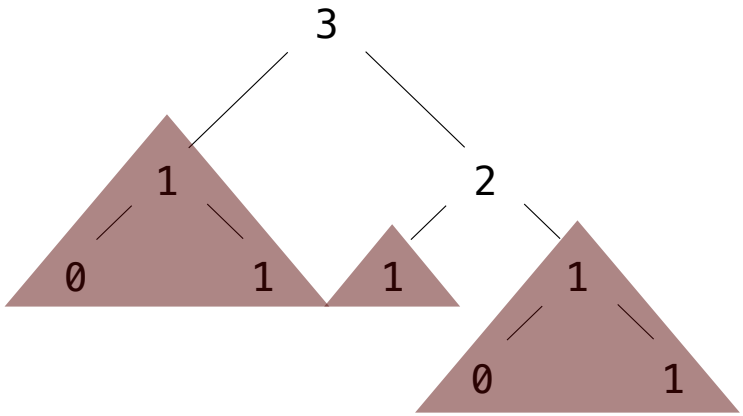


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