Tree Recursion

Announcements

Order of Recursive Calls

The Cascade Function

(Demo)

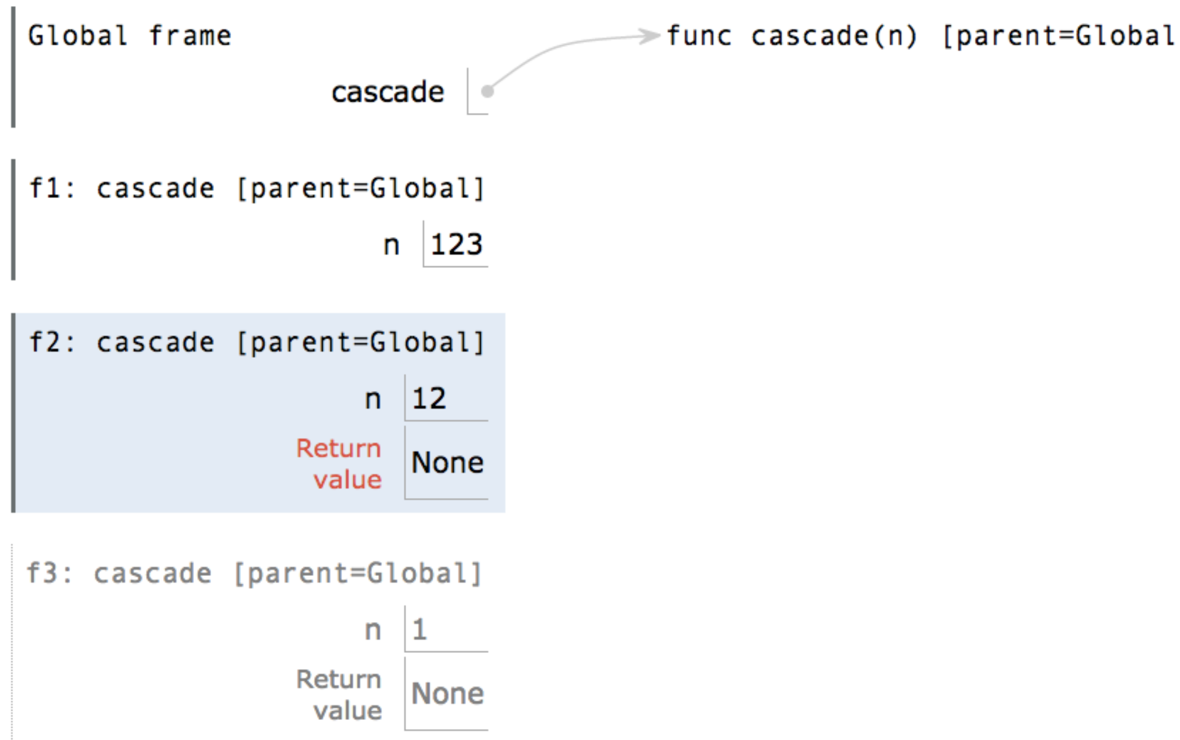
http://pythontutor.com/composingprograms.html#code=def%20cascade%28n%29%3A%20%20%20%20%0A%20%20%20%20if%20n%20%3C%2010%3A%20%20%20%20%20%20%20%20%0A%20%20%20%20%20%20%20%20print%28n%29%20%20%20%20%0A%20%20%20%20else%3A%20%20%20%20%20%20%20%20%0A%20%20%20%20%20%20%20%20print%28n%29%20%20%20%20%20%20%20%20%0A%20%20%20%20%20%20%20%20cascade%28n//
10%29%20%20%20%20%20%20%20%20%0A%20%20%20%20%20%20%20%20print%28n%29%20%20%20%20%20%20%20%20%0A%20%20%20%20%20%20%20%20%0Acascade%28123%29&cumulative=true&curInstr=0&mode=display&origin=composingprograms.js&py=3&rawInputLstJSON=%5B%5D

The Cascade Function

(Demo)

```python
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
        print(n)

cascade(123)
```



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The Cascade Function

(Demo)

```
def cascade(n):
    if n < 10:
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    else:
        print(n)
        cascade(n//10)
        print(n)

cascade(123)
```

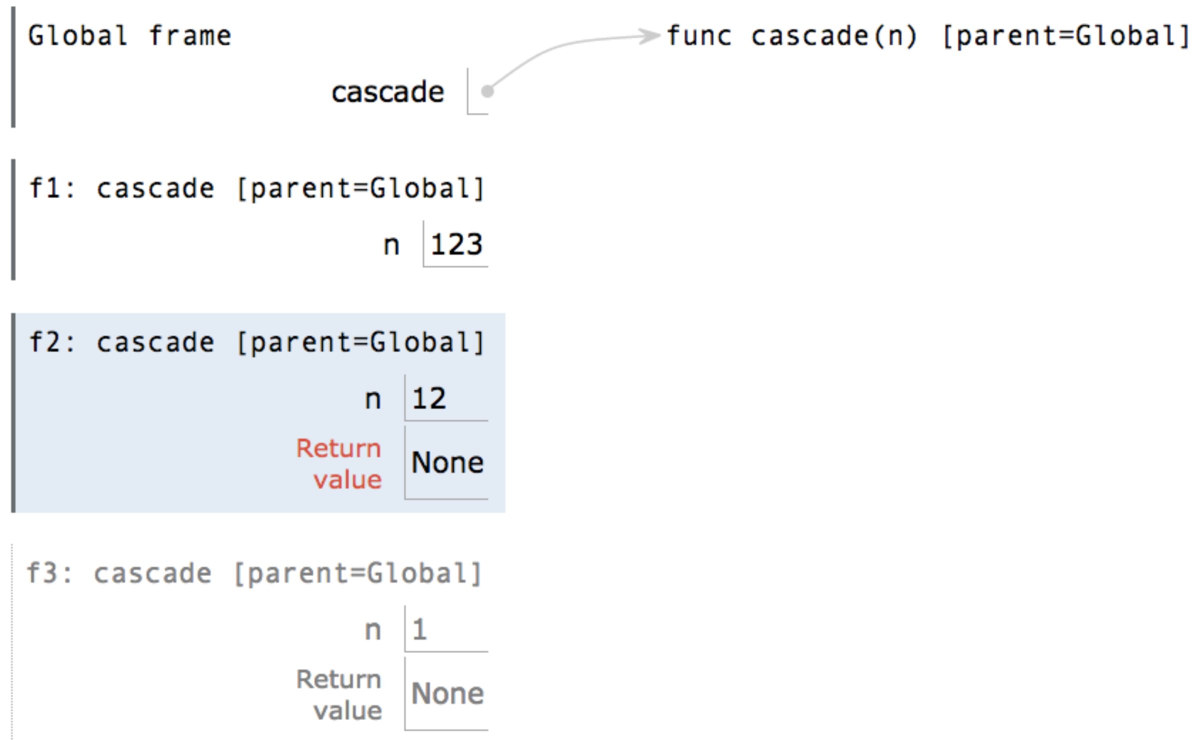

Program output:

```
123
12
1
12
```

The Cascade Function

(Demo)

```
def cascade(n):
    if n < 10:
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    else:
        print(n)
        cascade(n//10)
        print(n)

cascade(123)
```
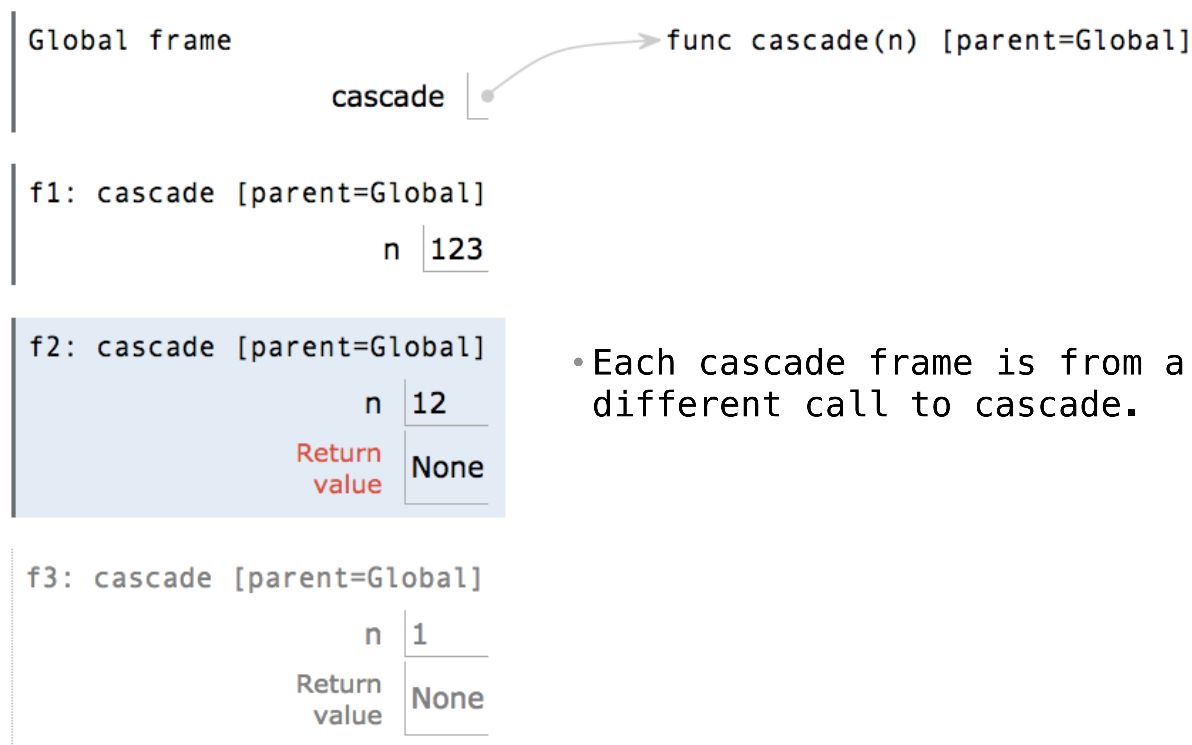

Program output:

```
123
12
1
12
```



- Each cascade frame is from a different call to cascade.

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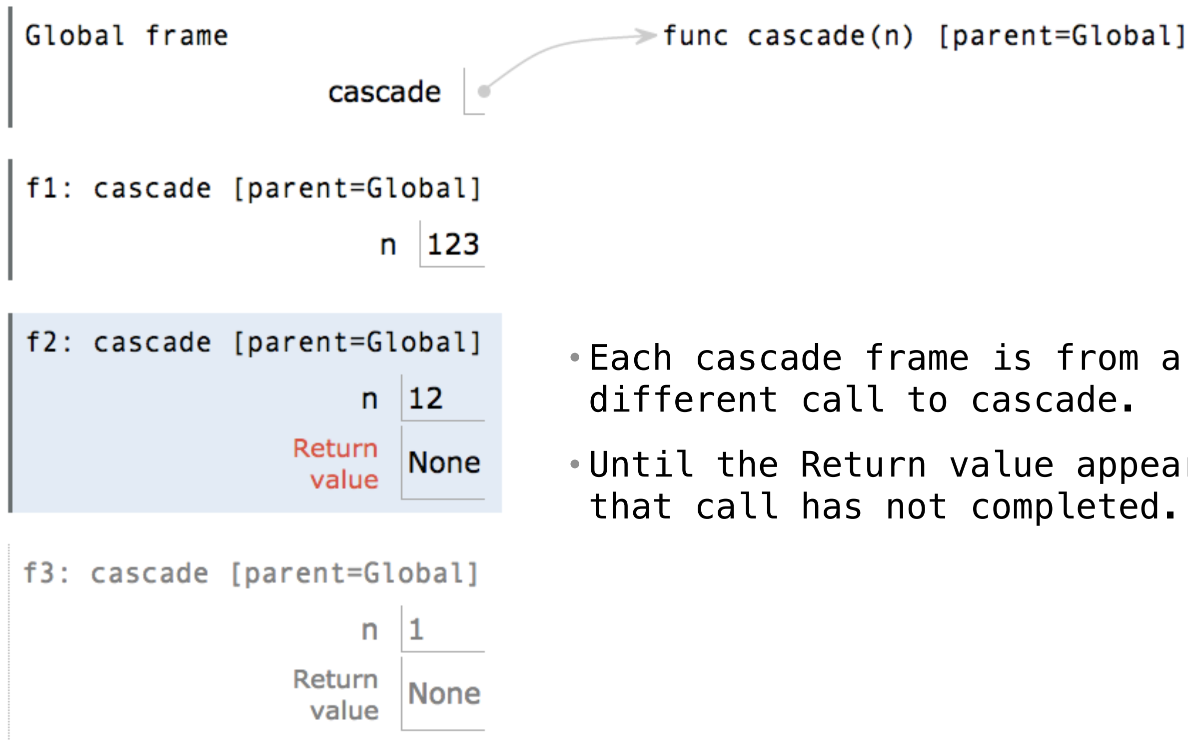

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10%29%20%20%20%20%20%20%20%20%0A%20%20%20%20%20%20%20%20print%28n%29%20%20%20%20%20%20%20%20%0A%20%20%20%20%20%20%20%20%0Acascade%28123%29&cumulative=true&curInstr=0&mode=display&origin=composingprograms.js&py=3&rawInputLstJSON=%5B%5D

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Program output:

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12
1
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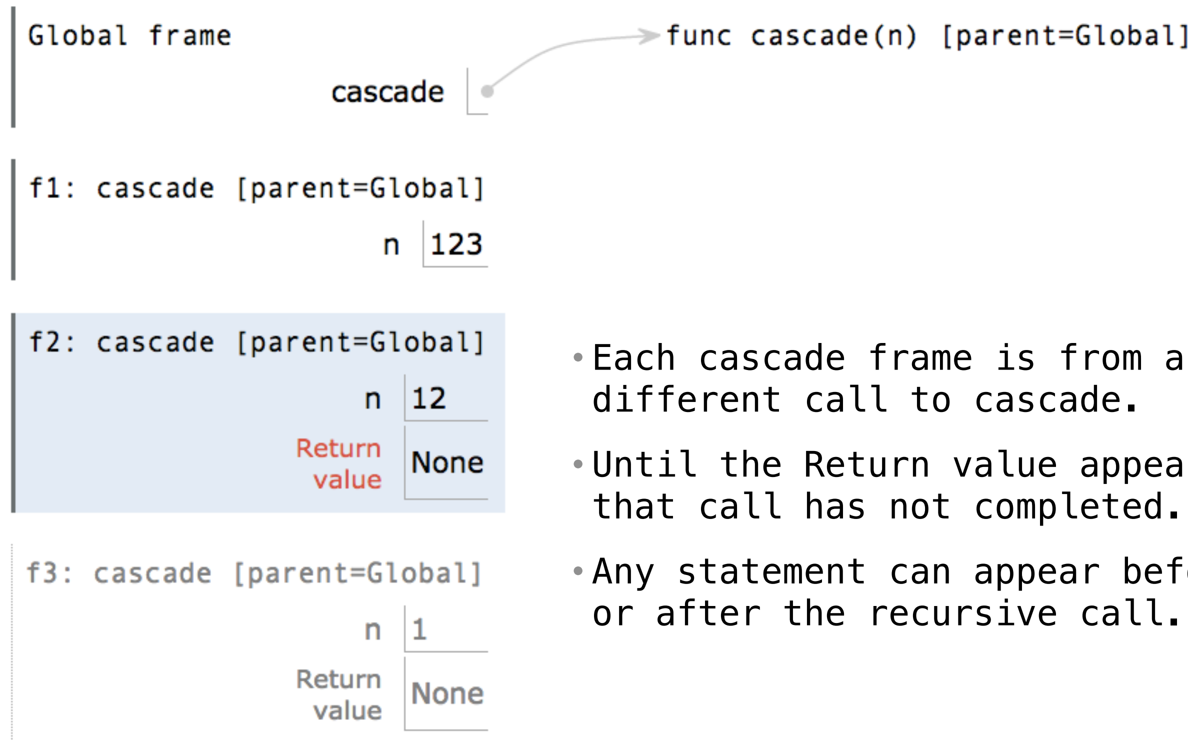

- Each cascade frame is from a different call to cascade.
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- Any statement can appear before or after the recursive call.

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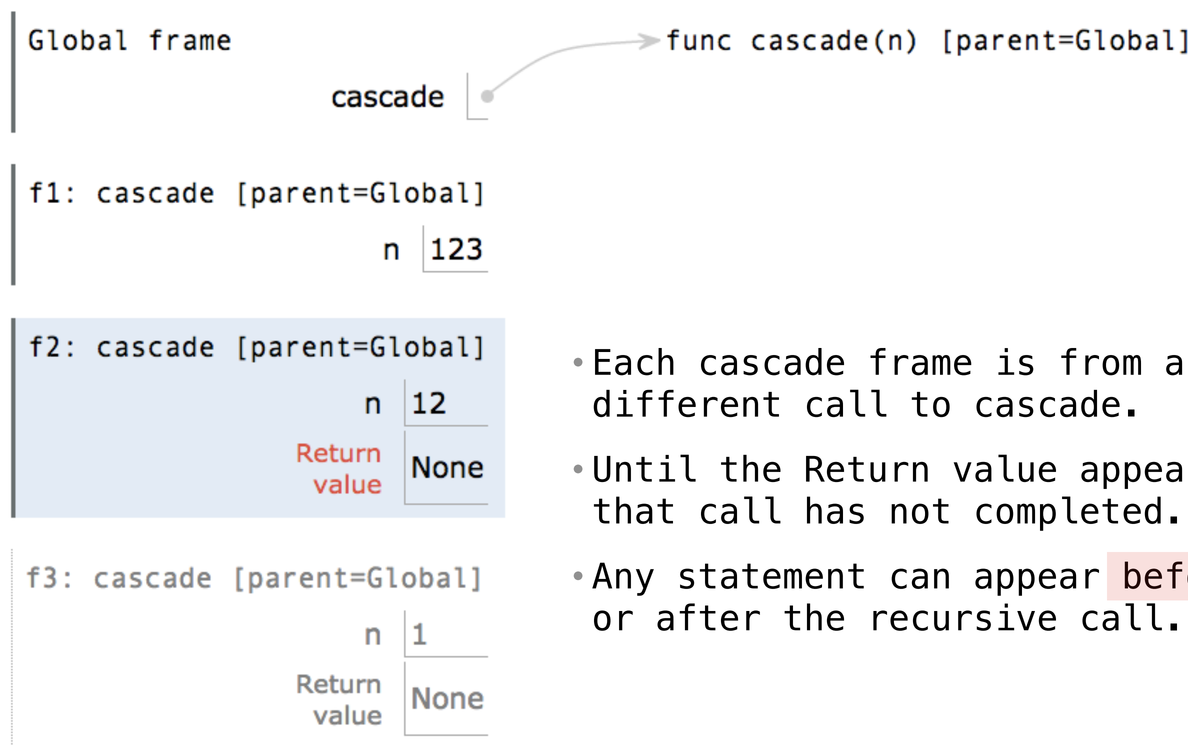

Program output:

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cascade(123)
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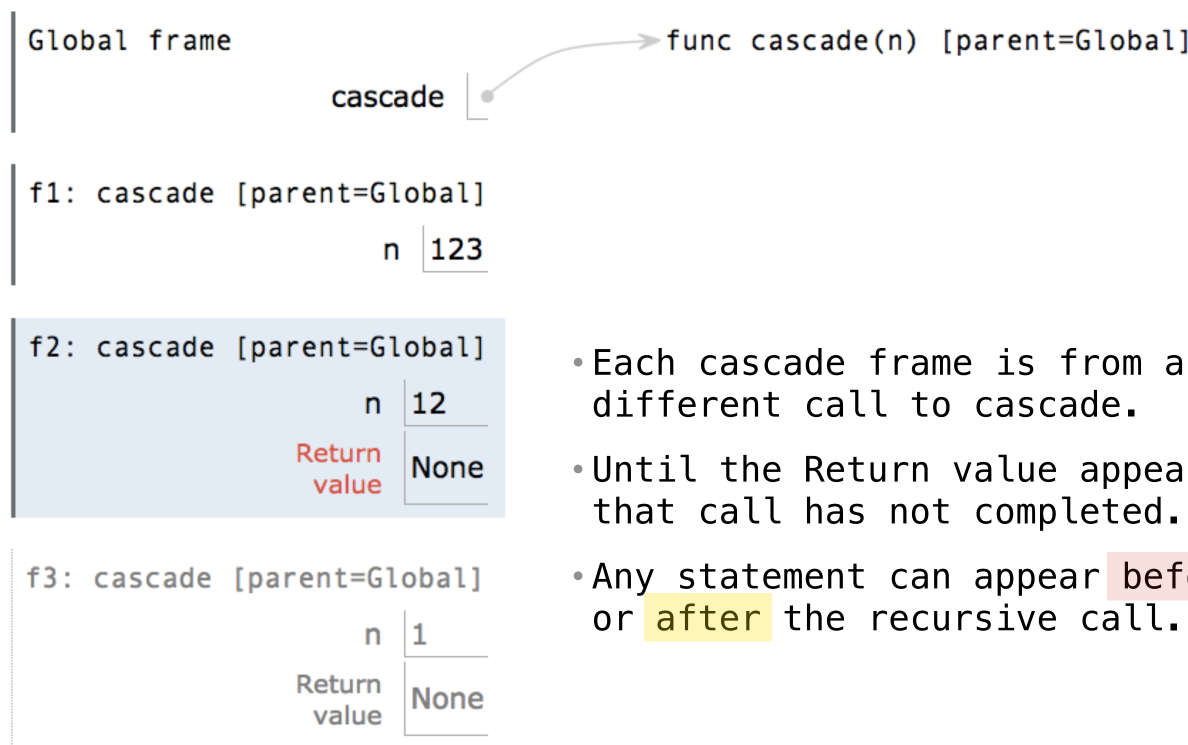
Program output:

```
123
12
1
12
```



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        cascade(n//10)
        print(n)

cascade(123)
```

Global frame

cascade → func cascade(n) [parent=Global]

f1: cascade [parent=Global]

n 123

f2: cascade [parent=Global]

n 12

Return value None

f3: cascade [parent=Global]

n 1

Return value None

Program output:

```
123
12
1
12
```

- Each cascade frame is from a different call to cascade.
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10%29%20%20%20%20%20%20%20%20%0A%20%20%20%20%20%20%20%20print%28n%29%20%20%20%20%20%20%20%20%0A%20%20%20%20%20%20%20%20%0Acascade%28123%29&cumulative=true&curInstr=0&mode=display&origin=composingprograms.js&py=3&rawInputLstJSON=%5B%5D

The Cascade Function

(Demo)

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cascade(123)
```

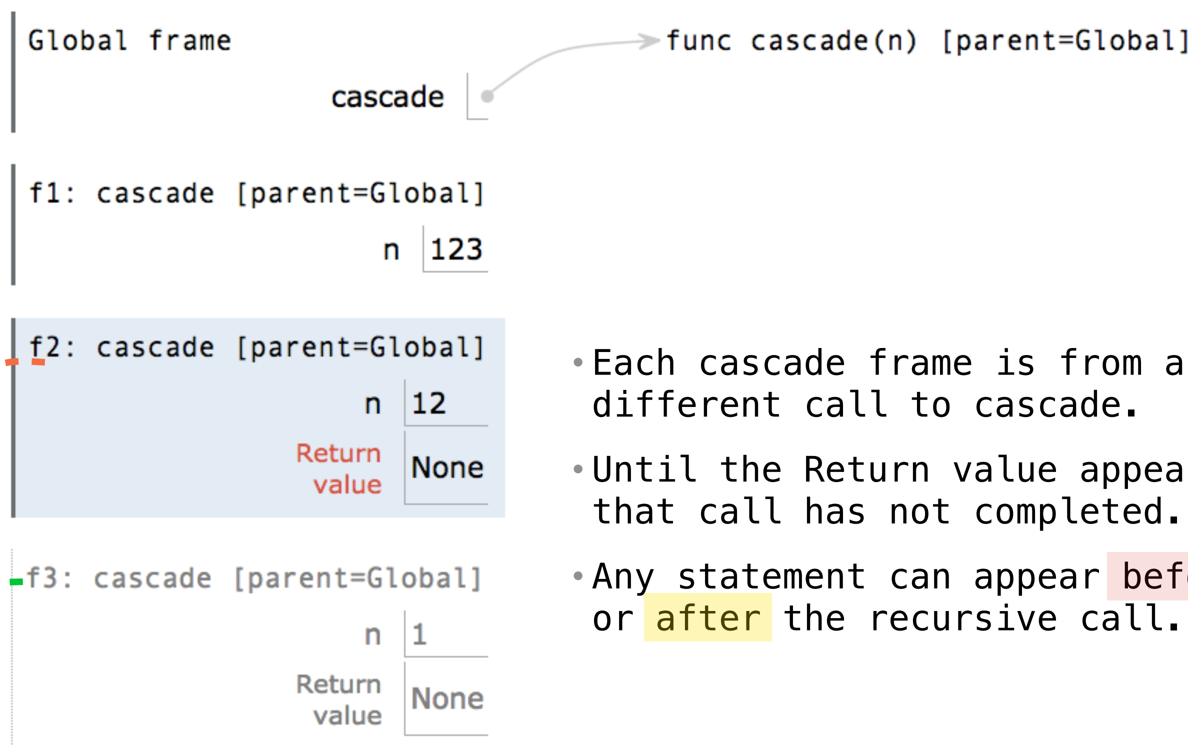

Program output:

```
123
12
1
12
```

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Two Definitions of Cascade

`(Demo)`

Two Definitions of Cascade

(Demo)

```python
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
        print(n)
```

```python
def cascade(n):
    print(n)
    if n >= 10:
        cascade(n//10)
        print(n)
```

Two Definitions of Cascade

(Demo)

```
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
        print(n)
```

```
def cascade(n):
    print(n)
    if n >= 10:
        cascade(n//10)
        print(n)
```

- If two implementations are equally clear, then shorter is usually better

Two Definitions of Cascade

(Demo)

```
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
        print(n)
```

```
def cascade(n):
    print(n)
    if n >= 10:
        cascade(n//10)
        print(n)
```

- If two implementations are equally clear, then shorter is usually better
- In this case, the longer implementation is more clear (at least to me)

Two Definitions of Cascade

(Demo)

```
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
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```

```
def cascade(n):
    print(n)
    if n >= 10:
        cascade(n//10)
        print(n)
```

- If two implementations are equally clear, then shorter is usually better
- In this case, the longer implementation is more clear (at least to me)
- When learning to write recursive functions, put the base cases first

Two Definitions of Cascade

(Demo)

```
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
        print(n)
```

```
def cascade(n):
    print(n)
    if n >= 10:
        cascade(n//10)
        print(n)
```

- If two implementations are equally clear, then shorter is usually better
- In this case, the longer implementation is more clear (at least to me)
- When learning to write recursive functions, put the base cases first
- Both are recursive functions, even though only the first has typical structure

Example: Inverse Cascade

Inverse Cascade

Write a function that prints an inverse cascade:

Inverse Cascade

```
Write a function that prints an inverse cascade:
```

```
1
12
123
1234
123
12
1
```

Inverse Cascade

Write a function that prints an inverse cascade:

```
1
12
123
1234
123
12
1
```

```
def inverse_cascade(n):
    grow(n)
    print(n)
    shrink(n)
```

Inverse Cascade

Write a function that prints an inverse cascade:

```
1
12
123
1234
123
12
1
```

```
def inverse_cascade(n):
    grow(n)
    print(n)
    shrink(n)

def f_then_g(f, g, n):
    if n:
        f(n)
        g(n)
```

Inverse Cascade

Write a function that prints an inverse cascade:

```
1
12
123
1234
123
12
1
```

```
def inverse_cascade(n):
    grow(n)
    print(n)
    shrink(n)

def f_then_g(f, g, n):
    if n:
        f(n)
        g(n)

grow =   lambda n: f_then_g(                    )
shrink = lambda n: f_then_g(                    )
```

Inverse Cascade

Write a function that prints an inverse cascade:

```
1
12
123
1234
123
12
1
```

```
def inverse_cascade(n):
    grow(n)
    print(n)
    shrink(n)

def f_then_g(f, g, n):
    if n:
        f(n)
        g(n)

grow =   lambda n: f_then_g(grow,  print,  n//10)
shrink = lambda n: f_then_g(print, shrink, n//10)
```

Tree Recursion

Tree Recursion

Tree-shaped processes arise whenever executing the body of a recursive function makes more than one recursive call

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http://en.wikipedia.org/wiki/File:Fibonacci.jpg

Tree Recursion

Tree-shaped processes arise whenever executing the body of a recursive function makes more than one recursive call

n: 0, 1, 2, 3, 4, 5, 6, 7, 8,

http://en.wikipedia.org/wiki/File:Fibonacci.jpg

Tree Recursion

Tree-shaped processes arise whenever executing the body of a recursive function makes more than one recursive call

n:	0, 1, 2, 3, 4, 5, 6, 7, 8,
fib(n):	0, 1, 1, 2, 3, 5, 8, 13, 21,

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Tree Recursion

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n:	0,	1,	2,	3,	4,	5,	6,	7,	8,	... ,	35
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```
def fib(n):
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```
def fib(n):
    if n == 0:
```

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fib(n):	0,	1,	1,	2,	3,	5,	8,	13,	21,	... ,	9,227,465

```
def fib(n):
    if n == 0:
        return 0
```

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```
def fib(n):
    if n == 0:
        return 0
    elif n == 1:
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```
def fib(n):
    if n == 0:
        return 0
    elif n == 1:
        return 1
    else:
        return fib(n-2) + fib(n-1)
```

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A Tree-Recursive Process

The computational process of fib evolves into a tree structure

A Tree-Recursive Process

The computational process of fib evolves into a tree structure

fib(5)

A Tree-Recursive Process

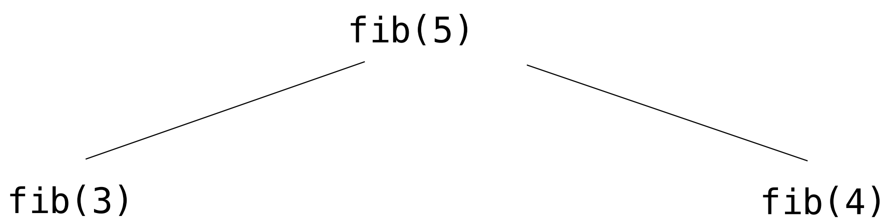
The computational process of fib evolves into a tree structure

fib(5)

fib(3)

A Tree-Recursive Process

The computational process of fib evolves into a tree structure



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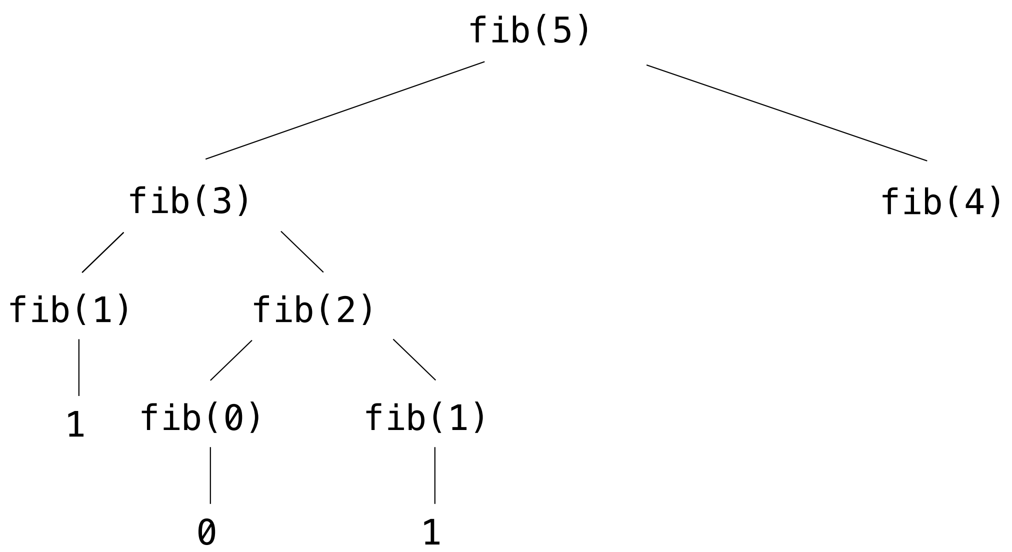

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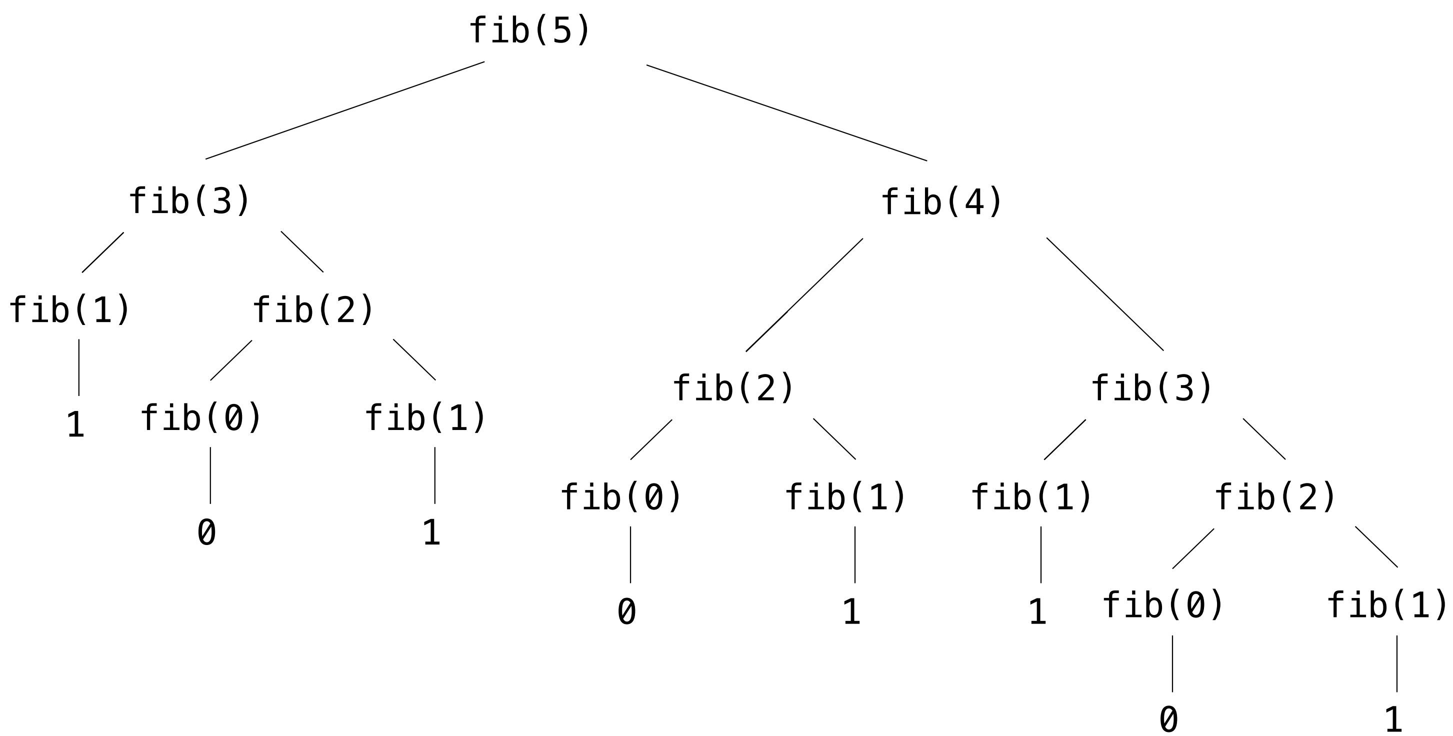

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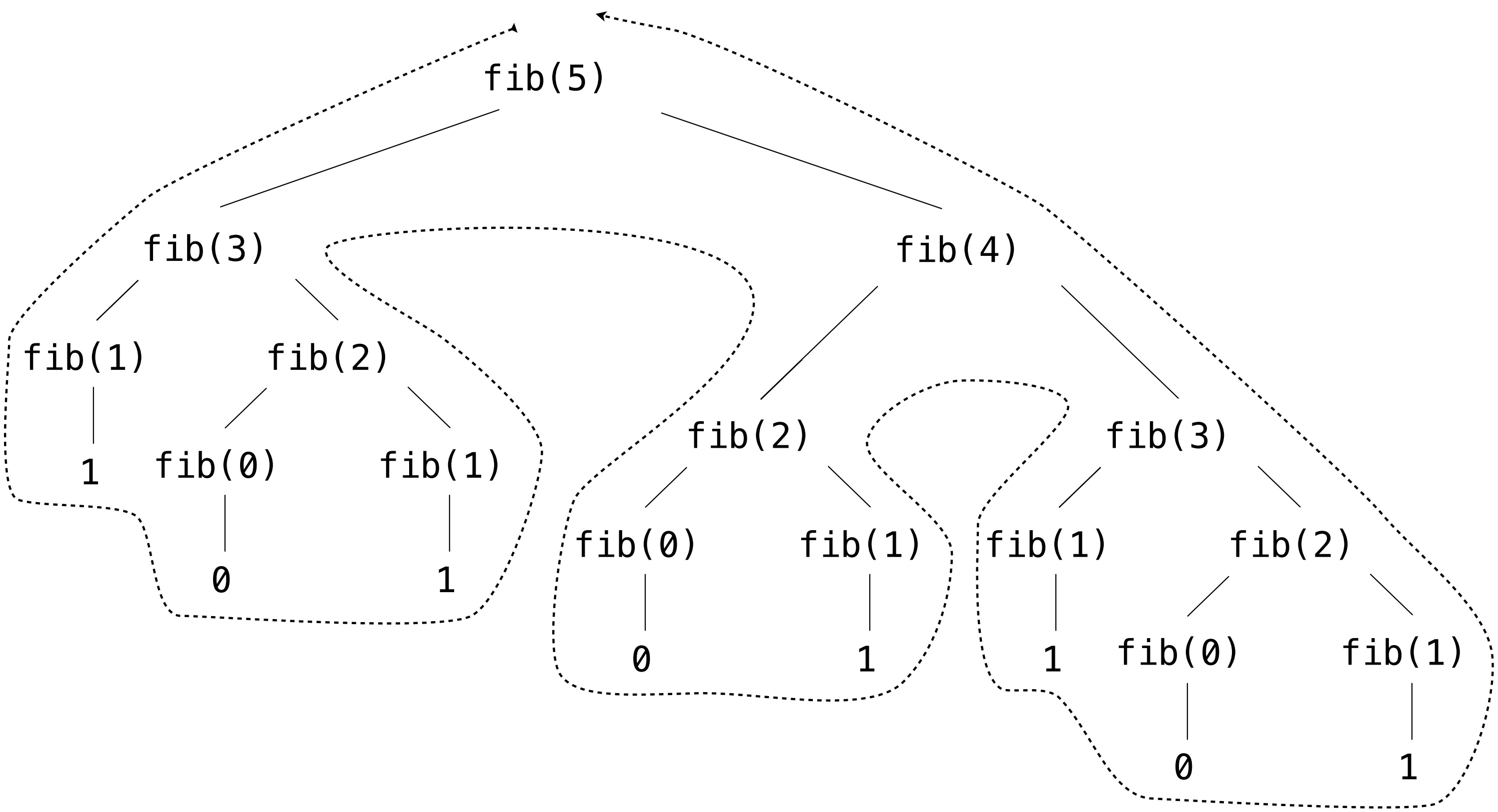

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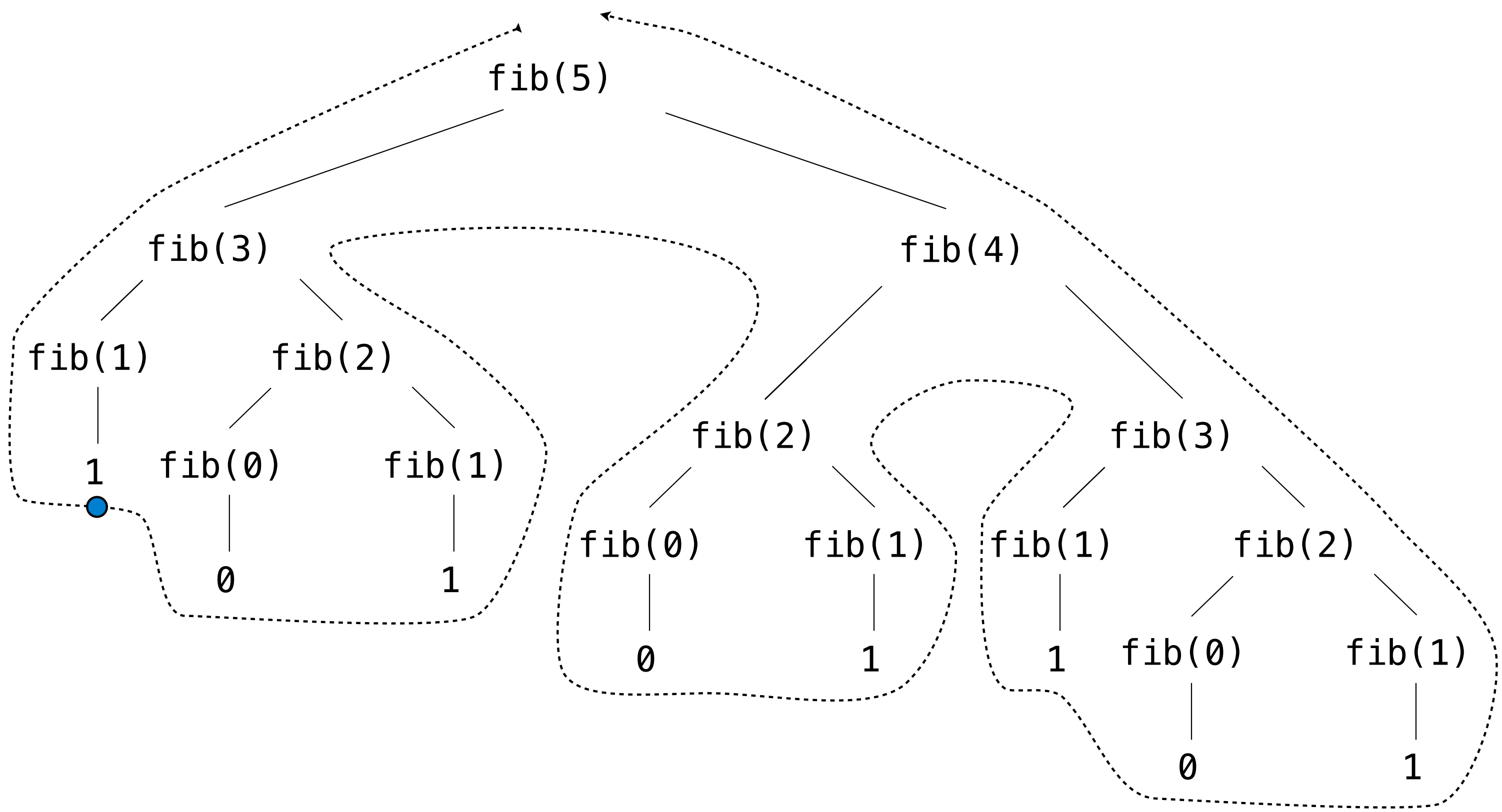
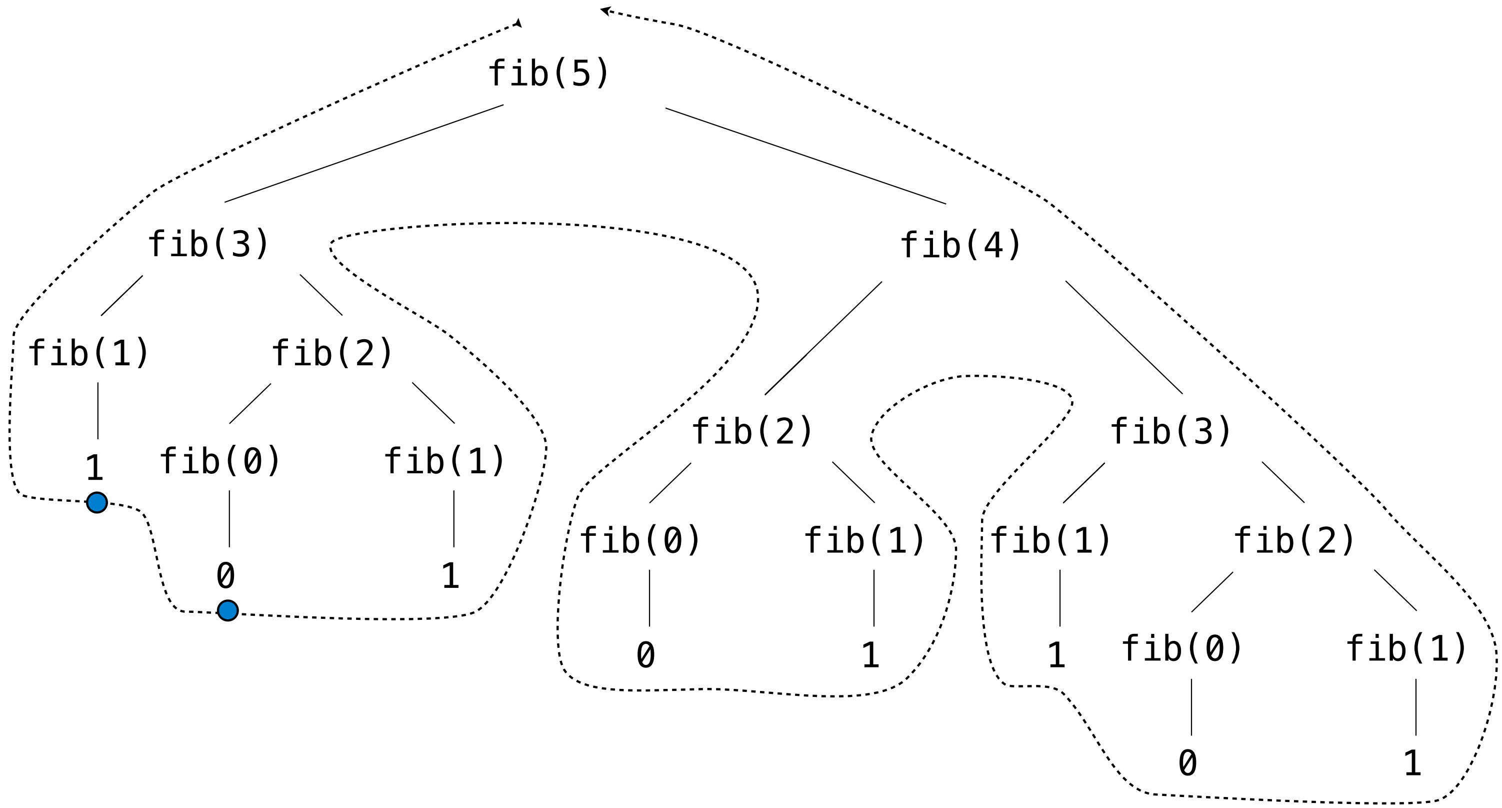


A Tree-Recursive Process

The computational process of fib evolves into a tree structure



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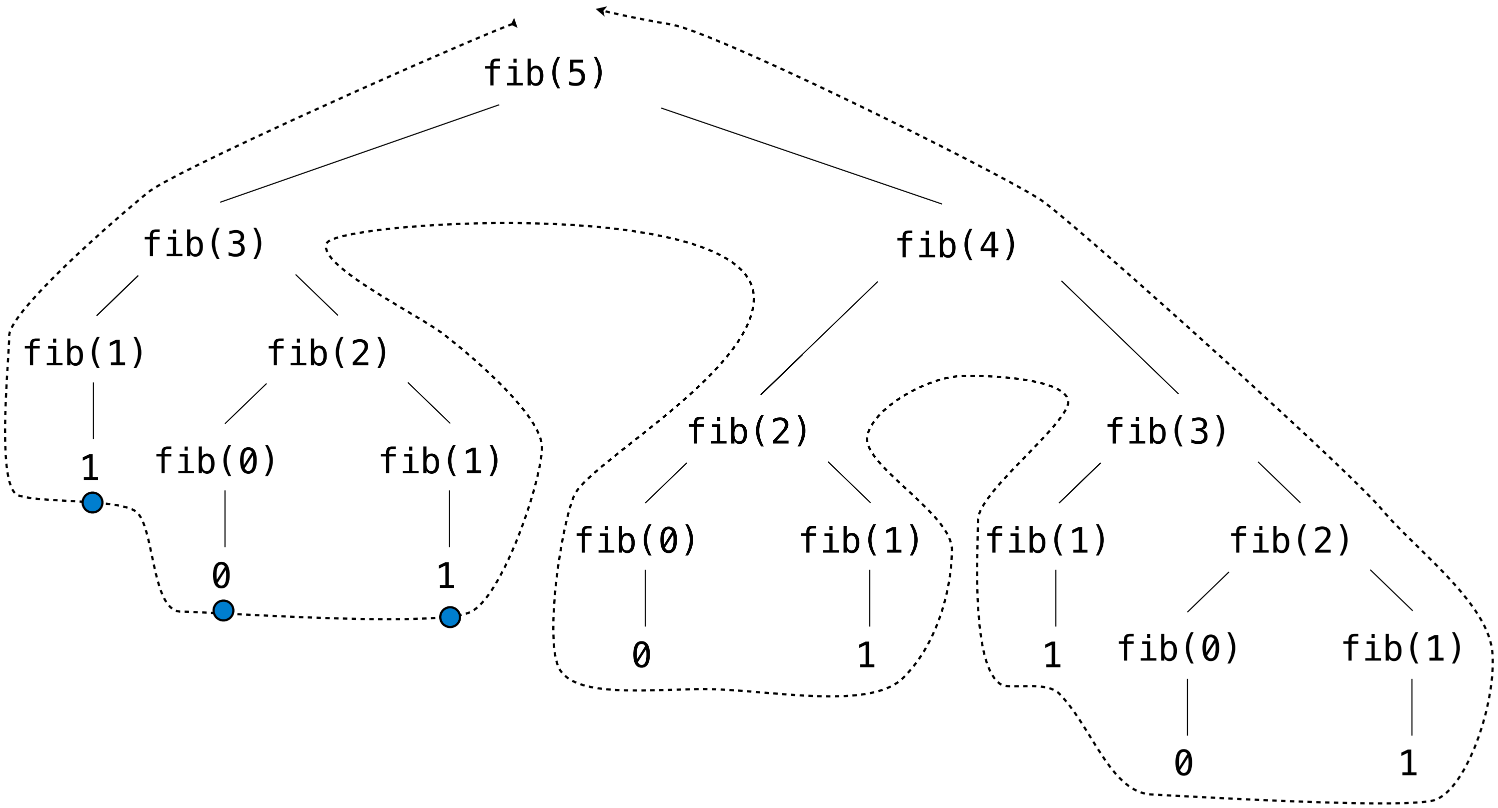

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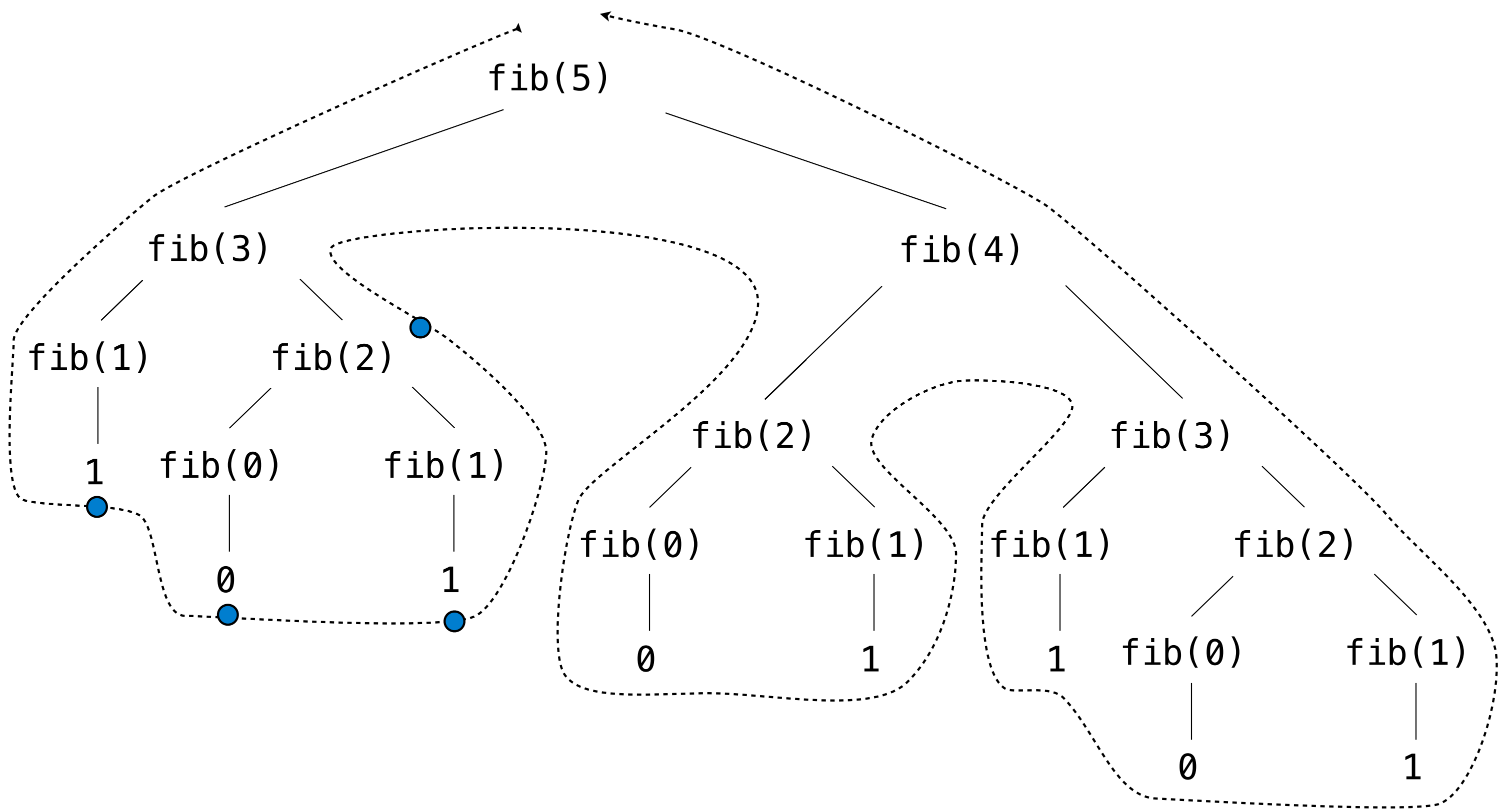

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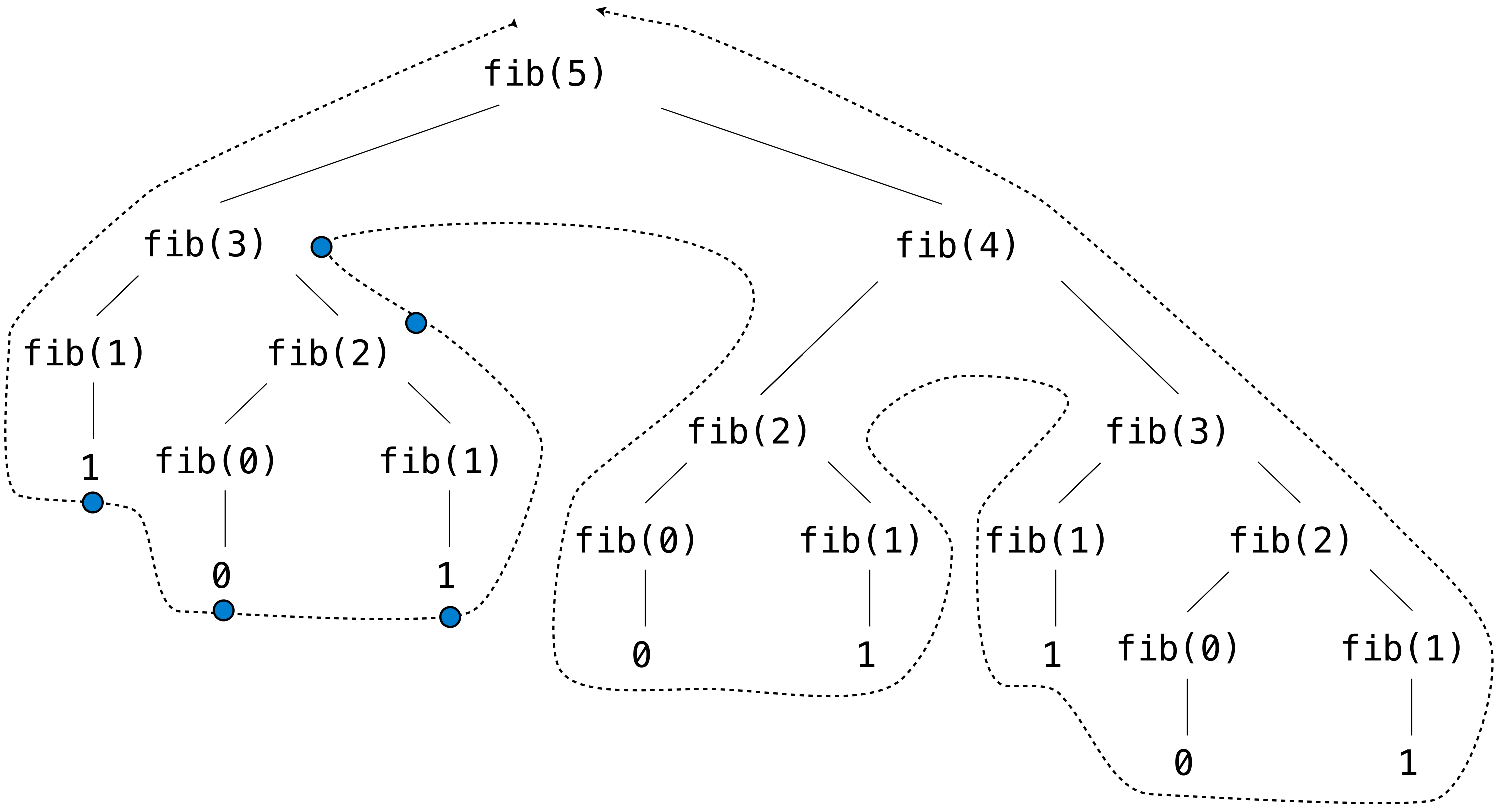

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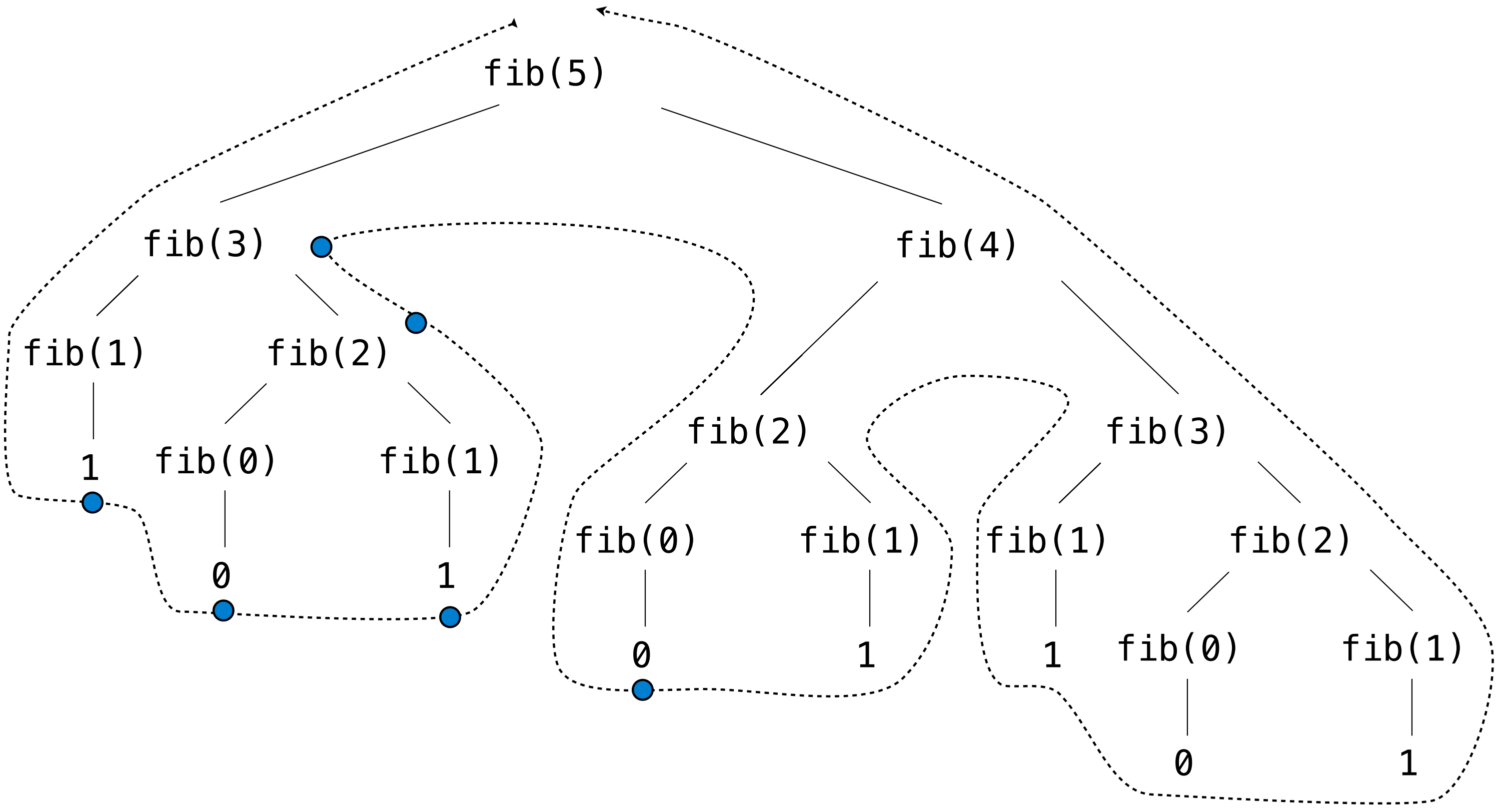

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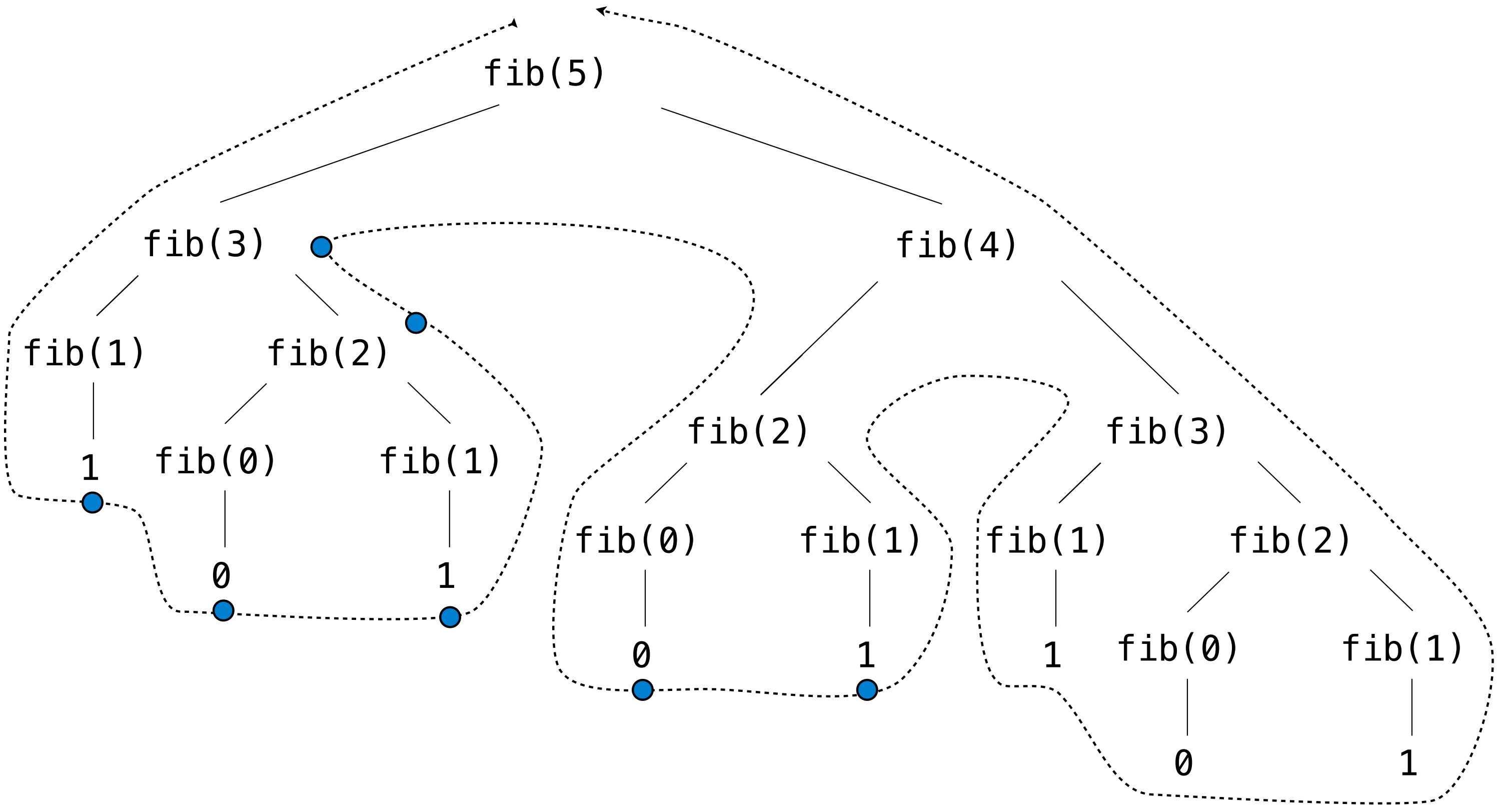

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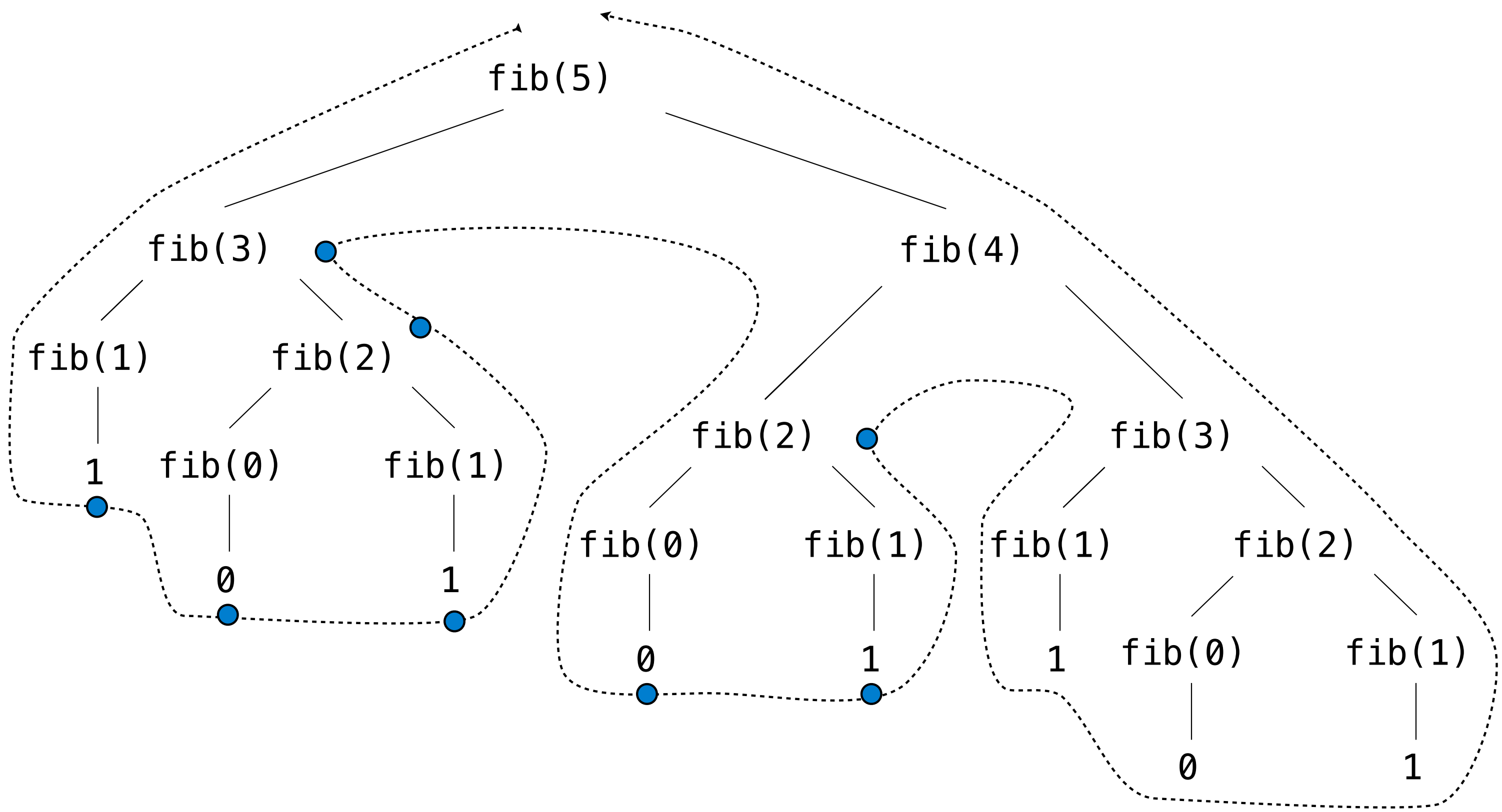

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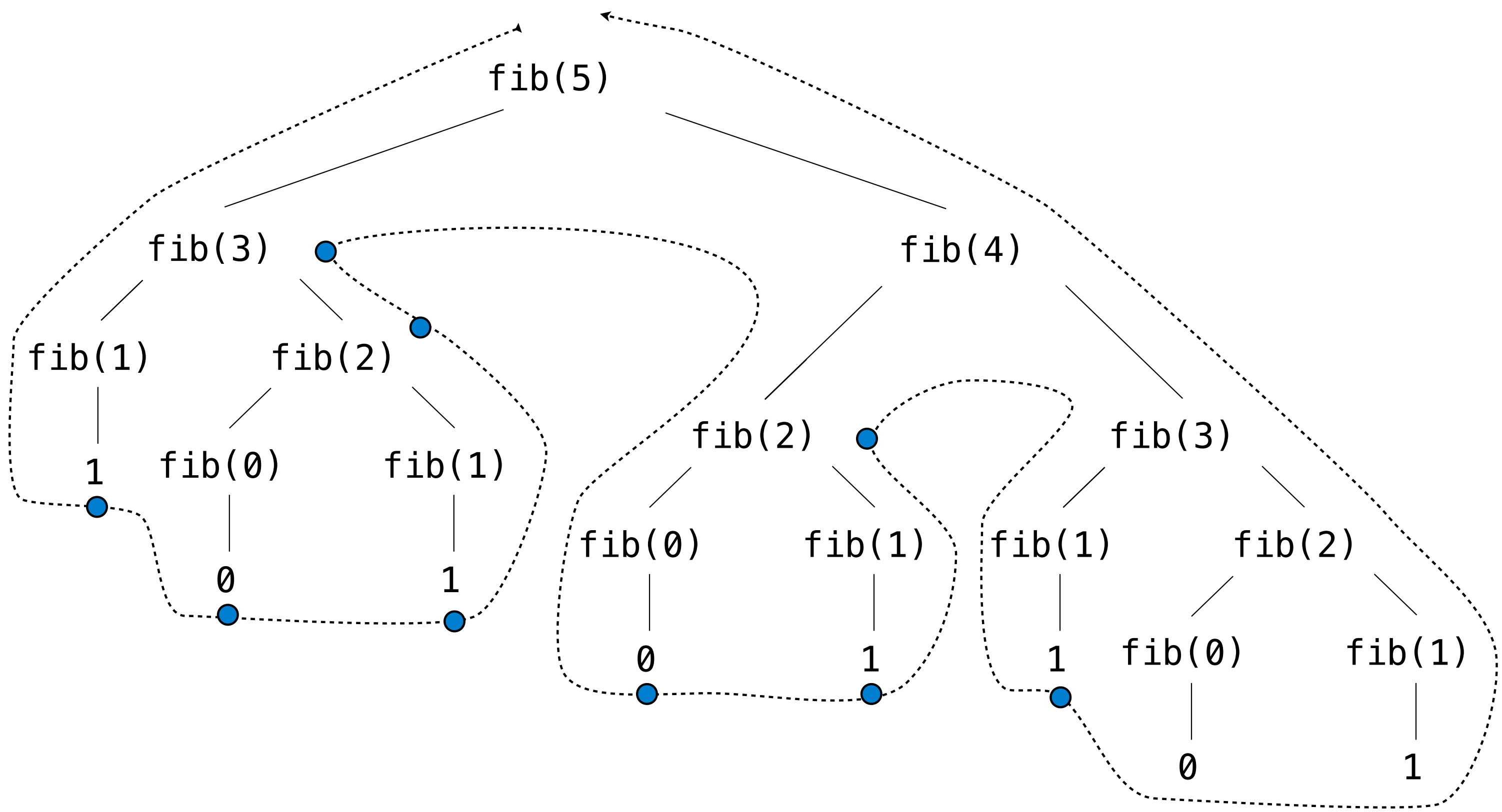

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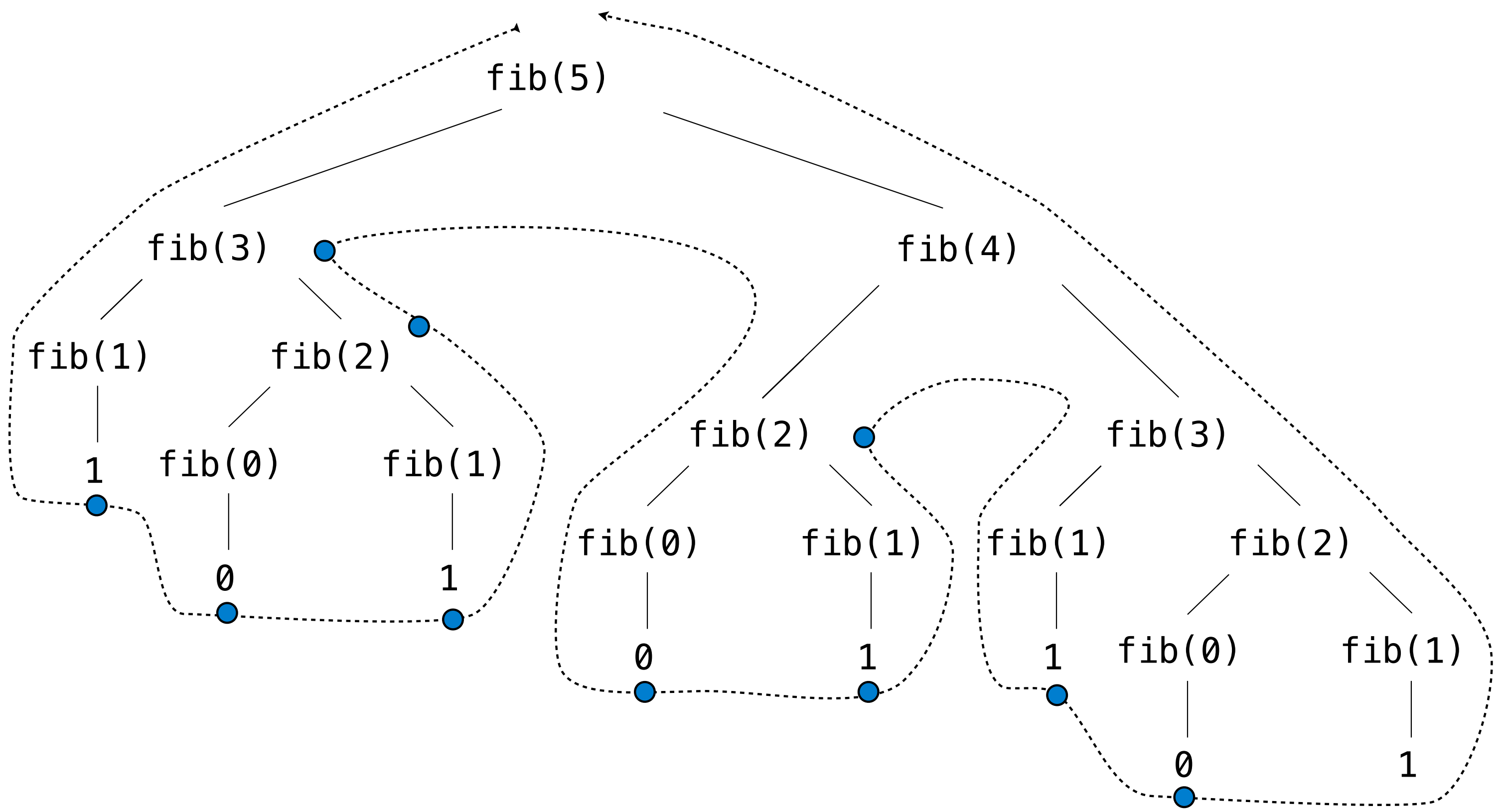

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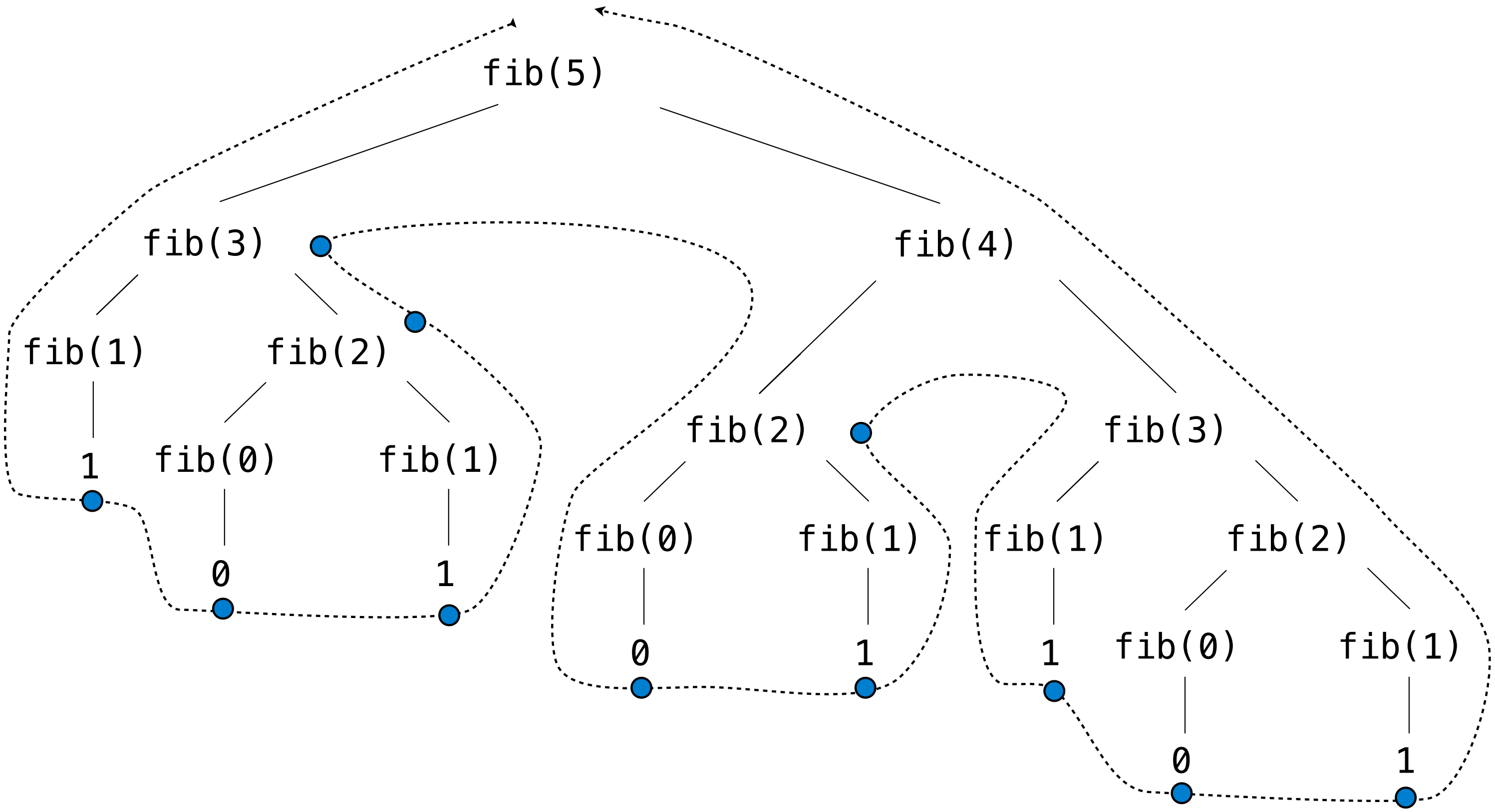

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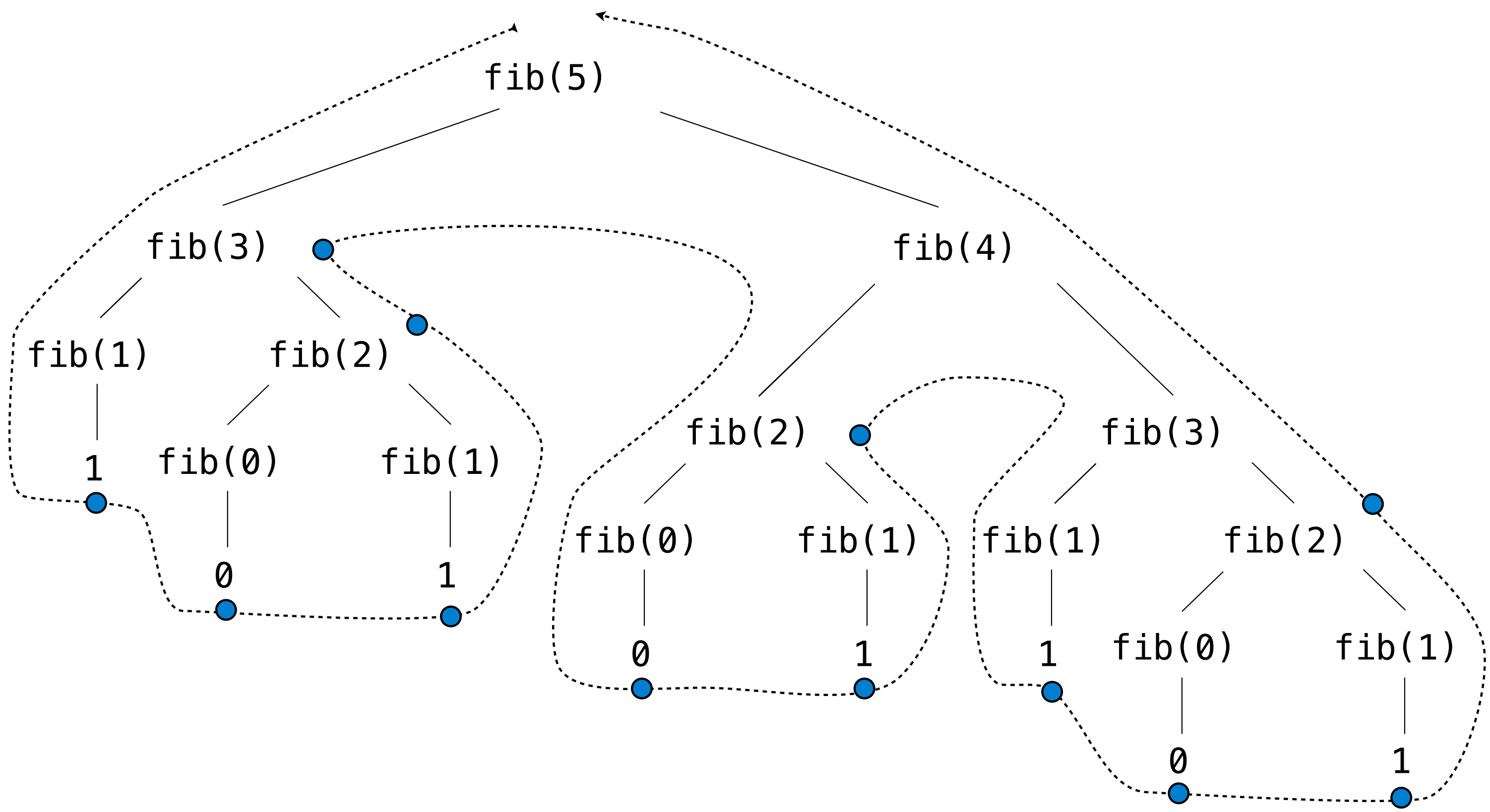

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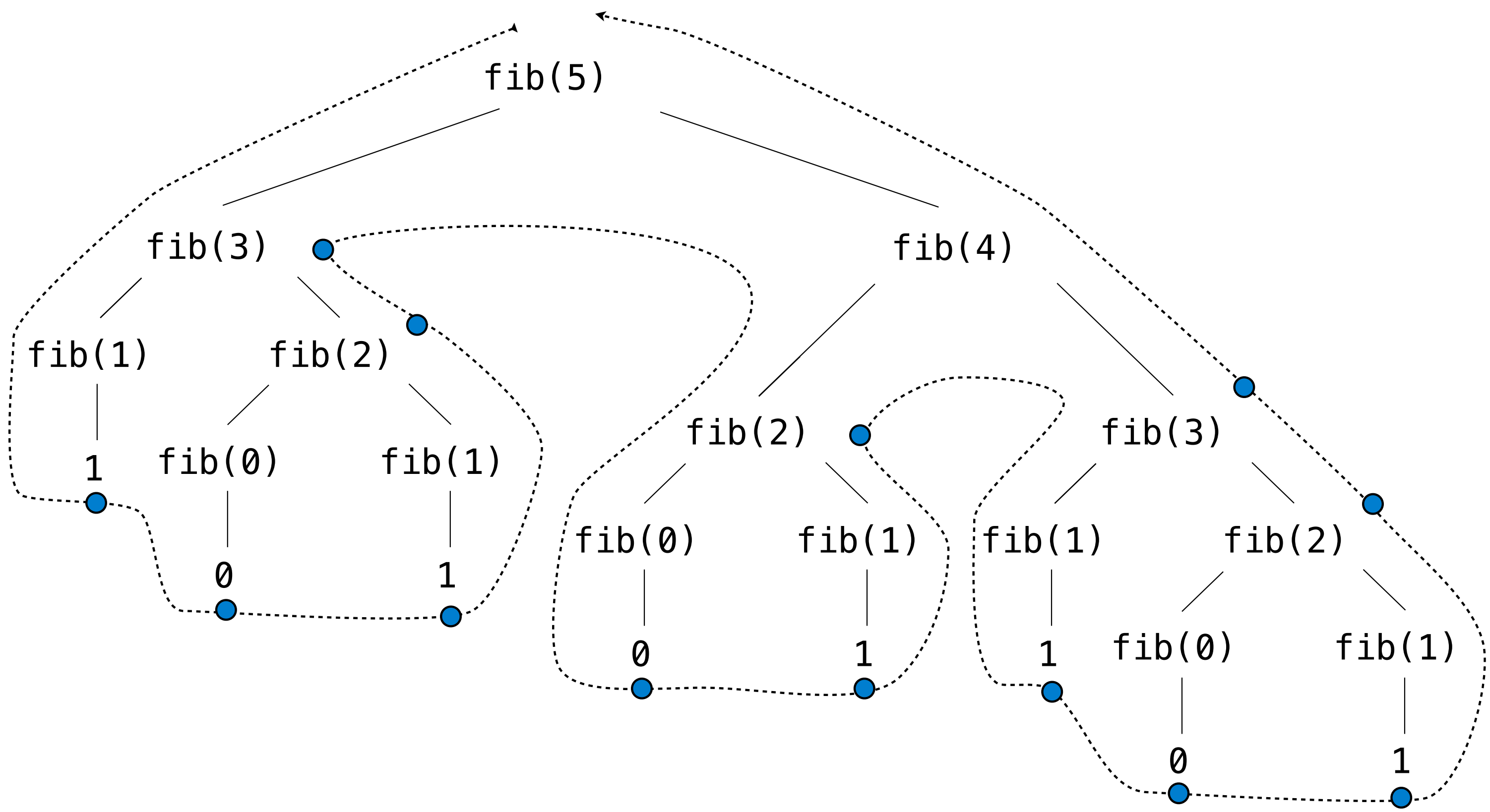

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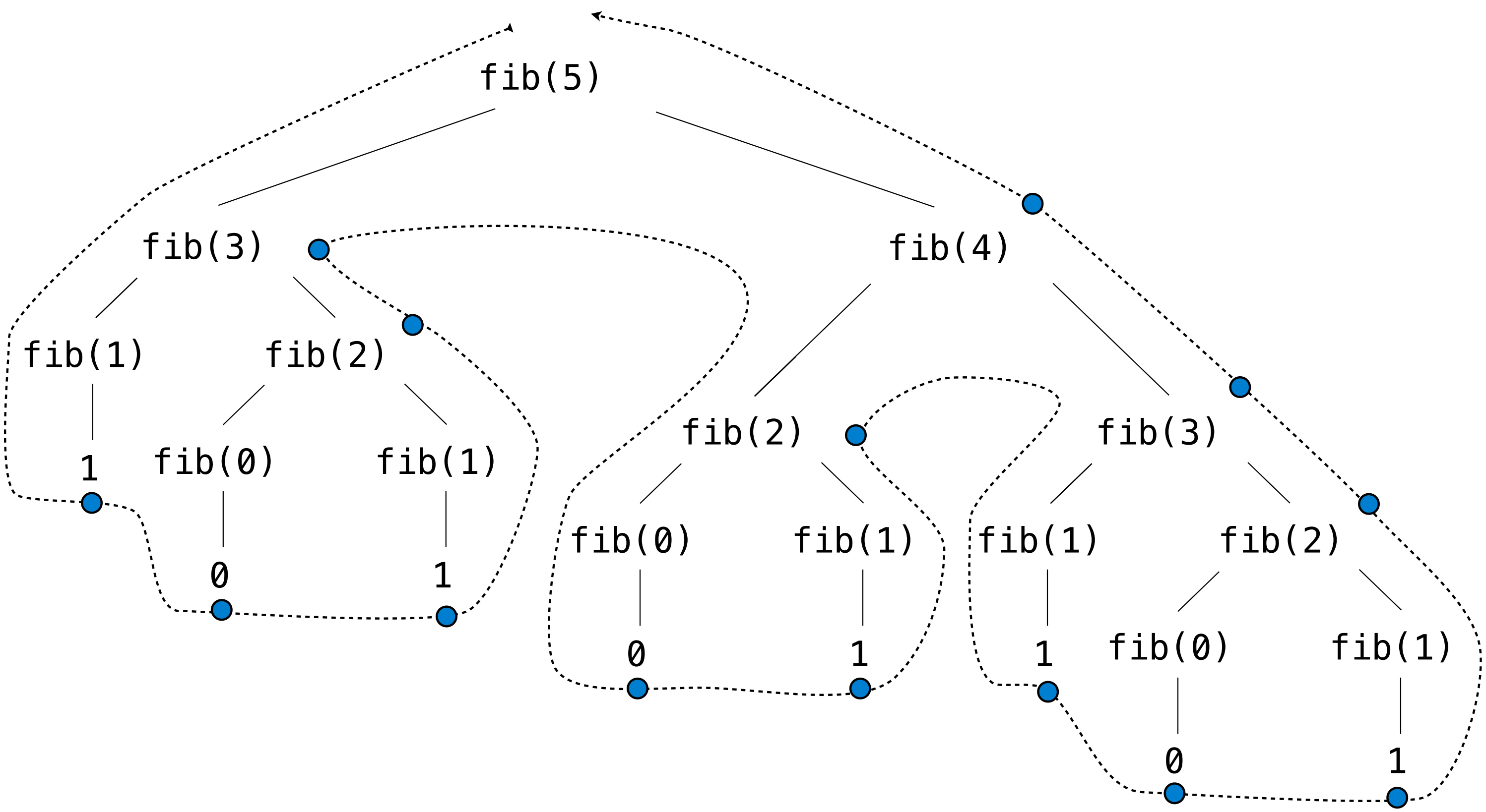

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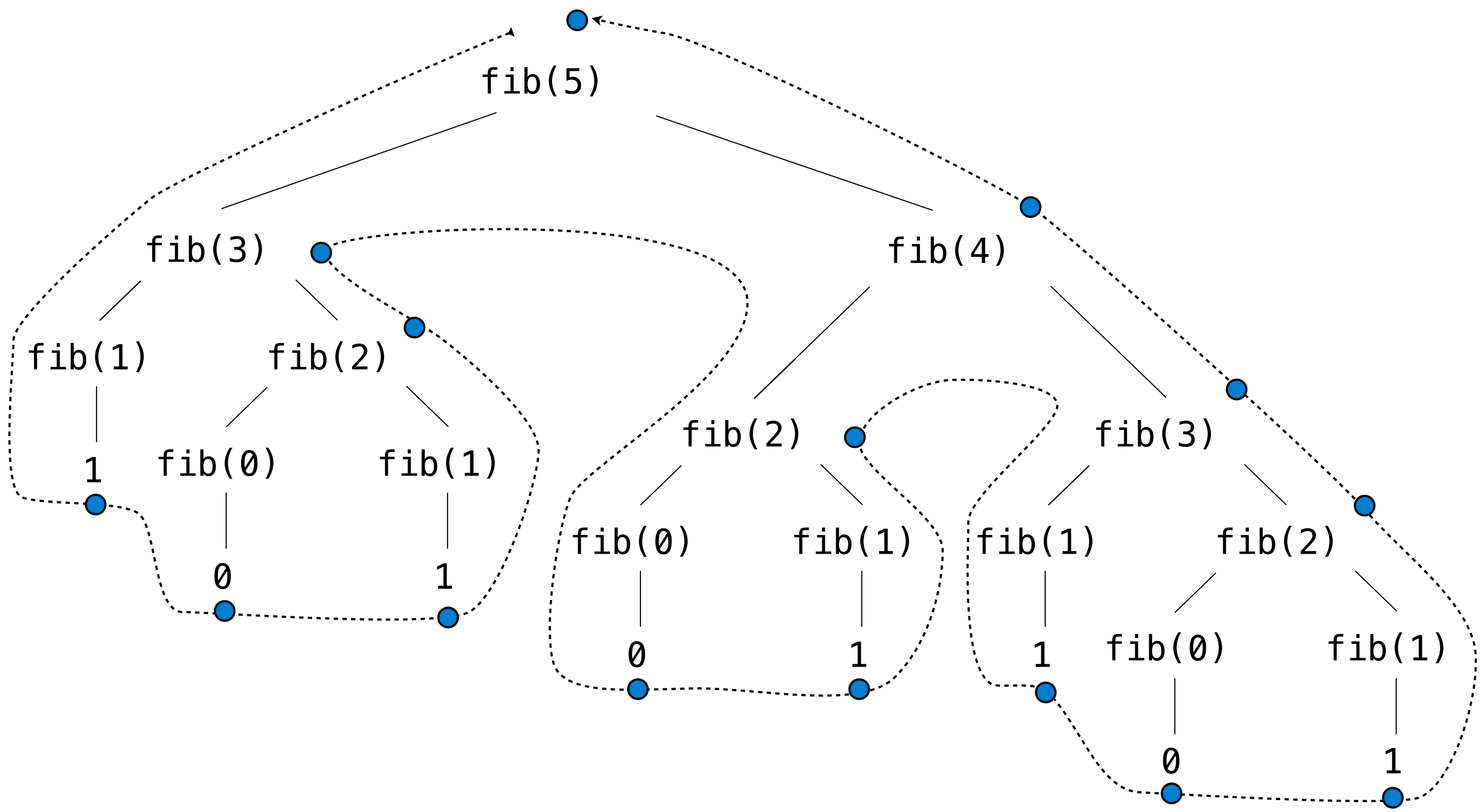

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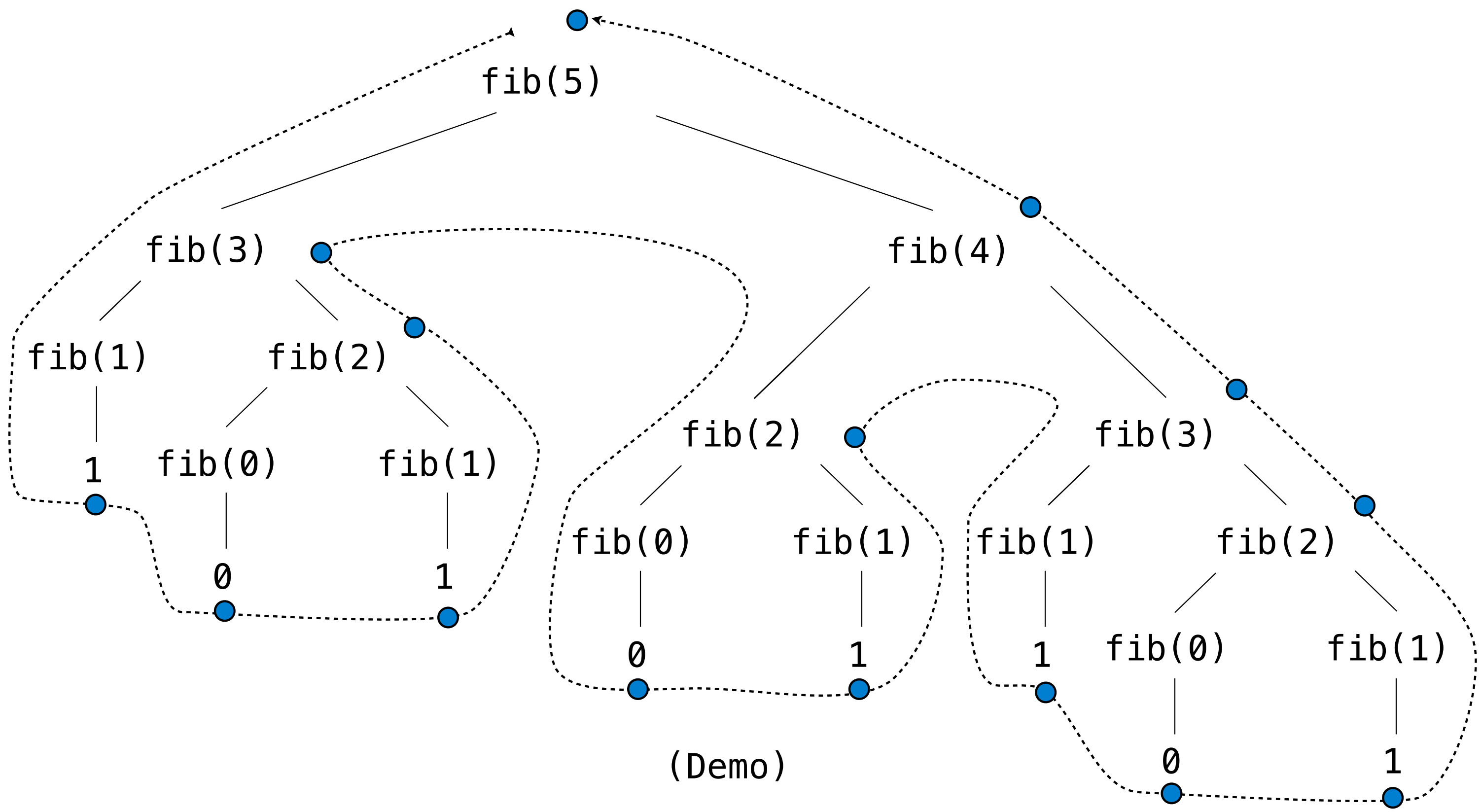

(Demo)

Repetition in Tree-Recursive Computation

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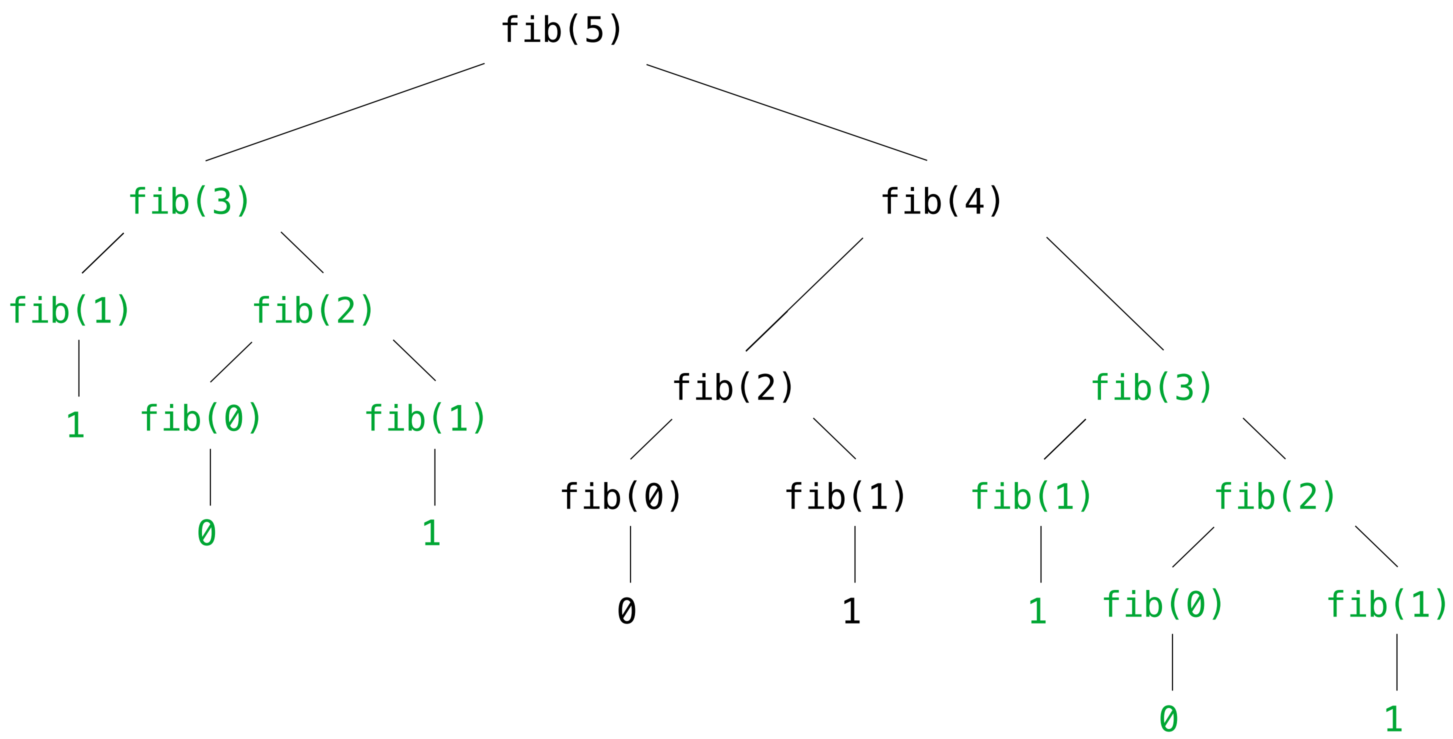

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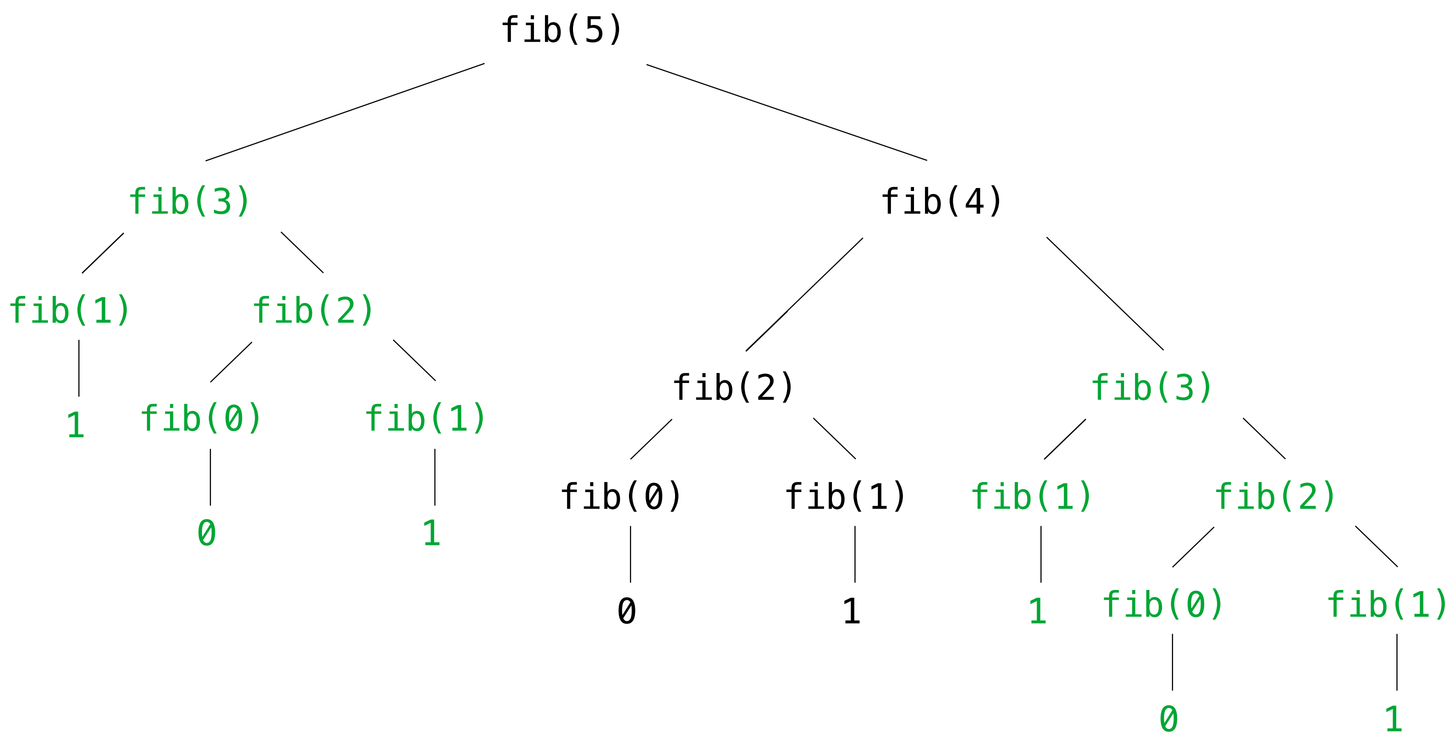

(We will speed up this computation dramatically in a few weeks by remembering results)

Example: Counting Partitions

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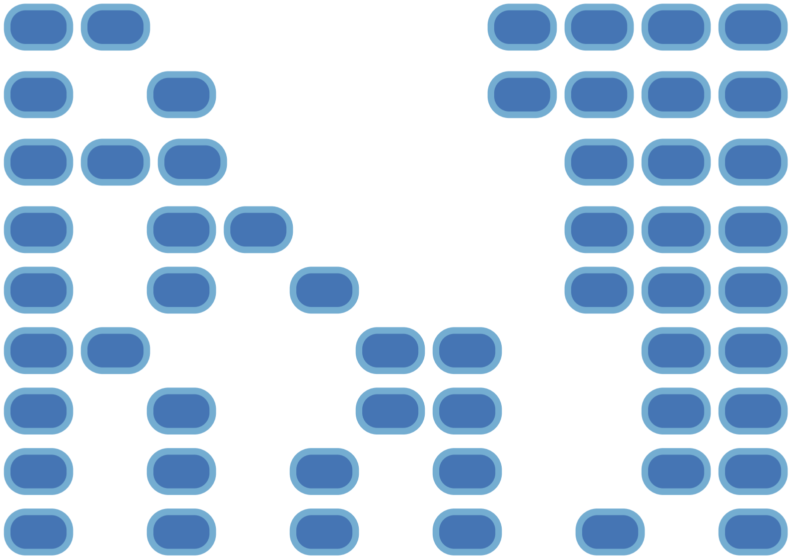

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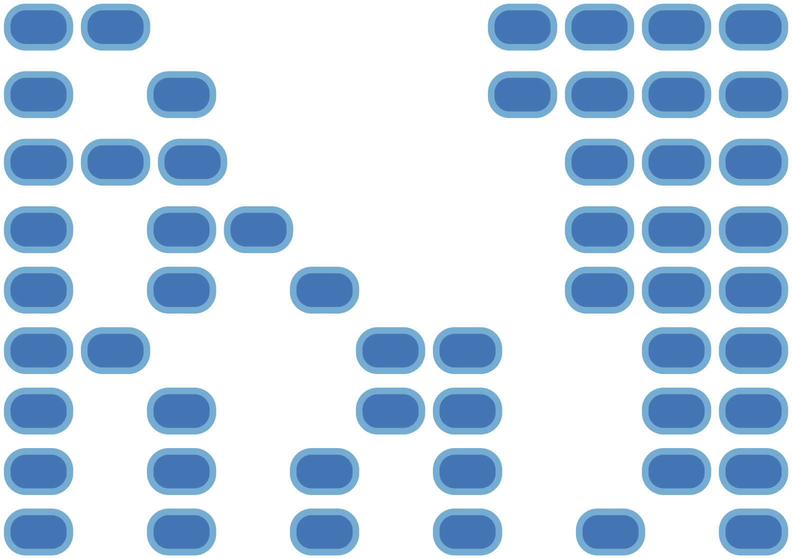

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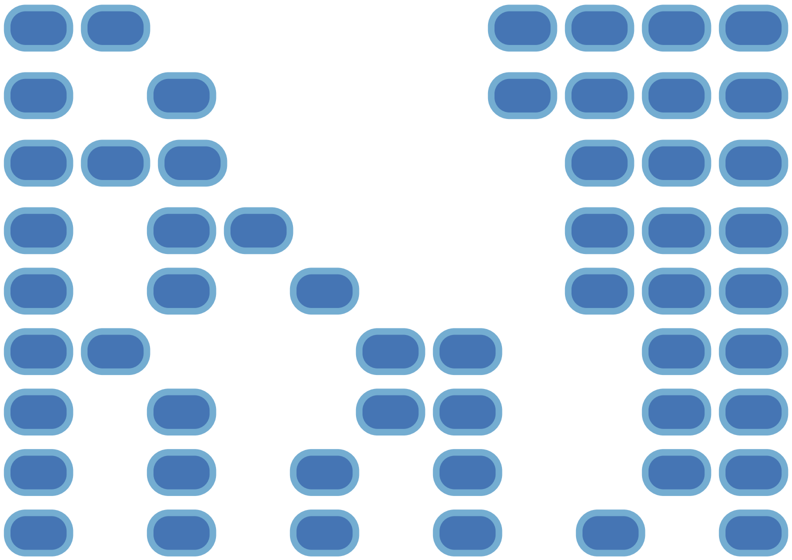

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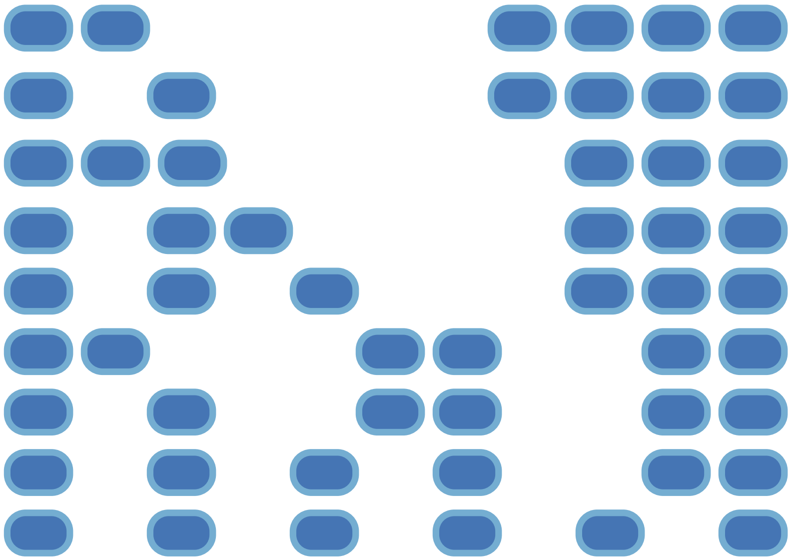

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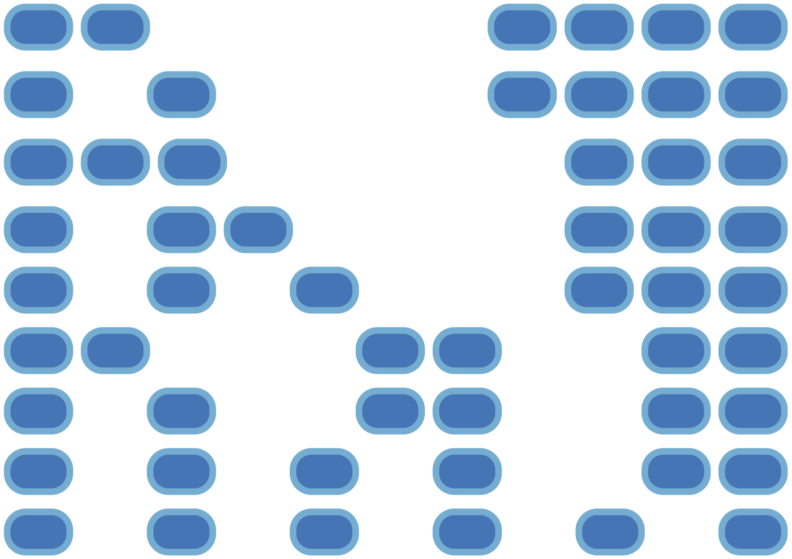

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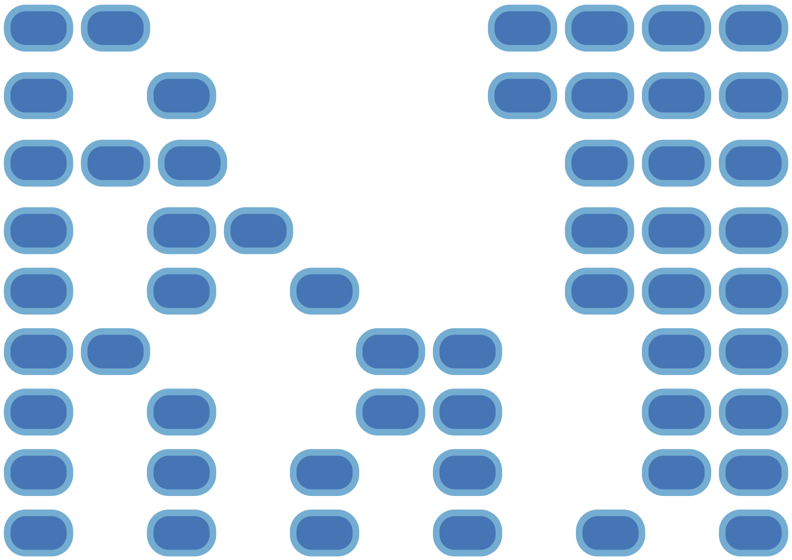

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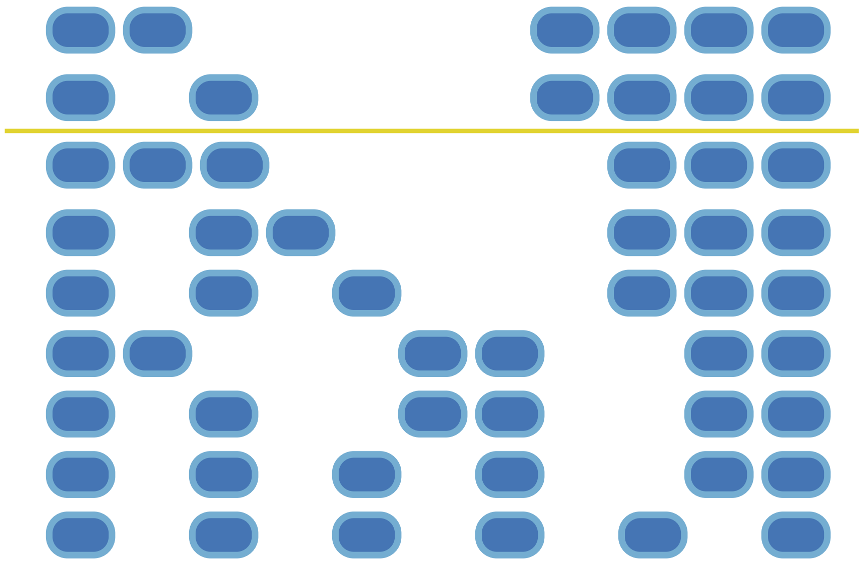

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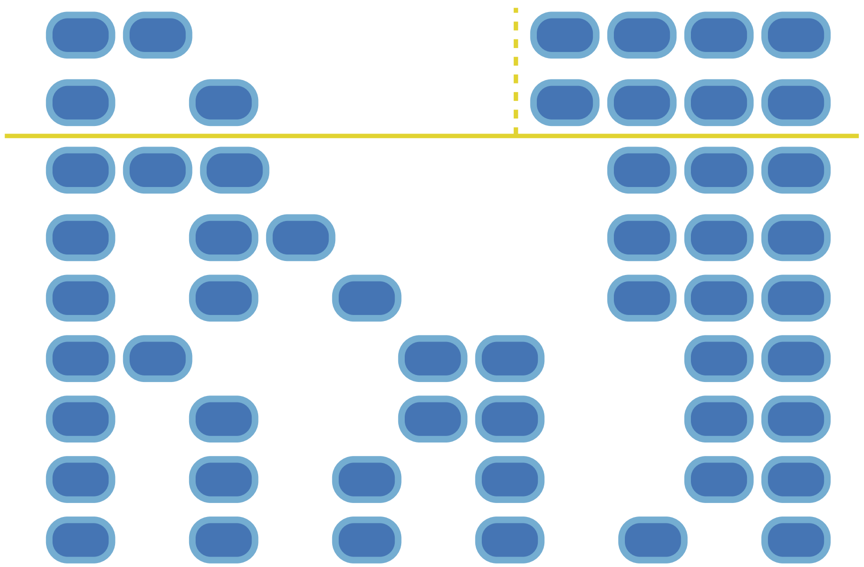

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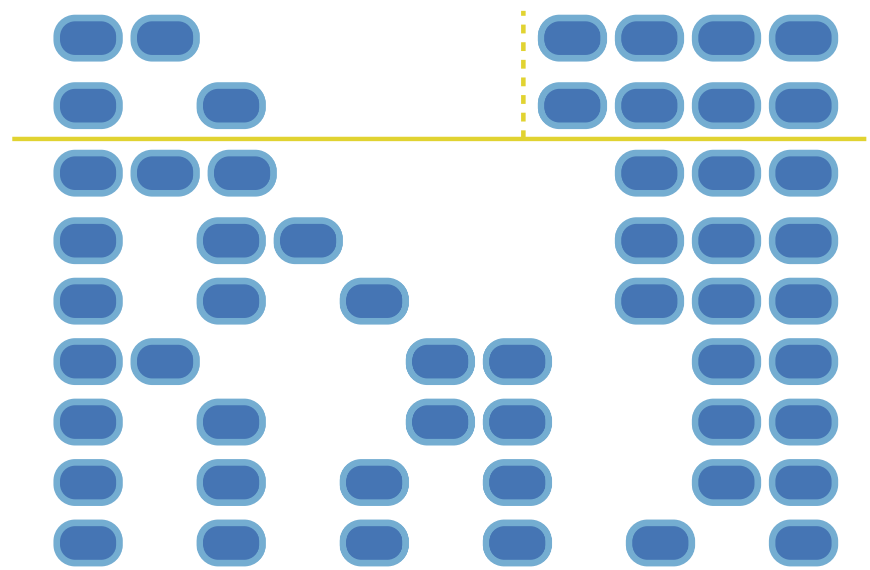

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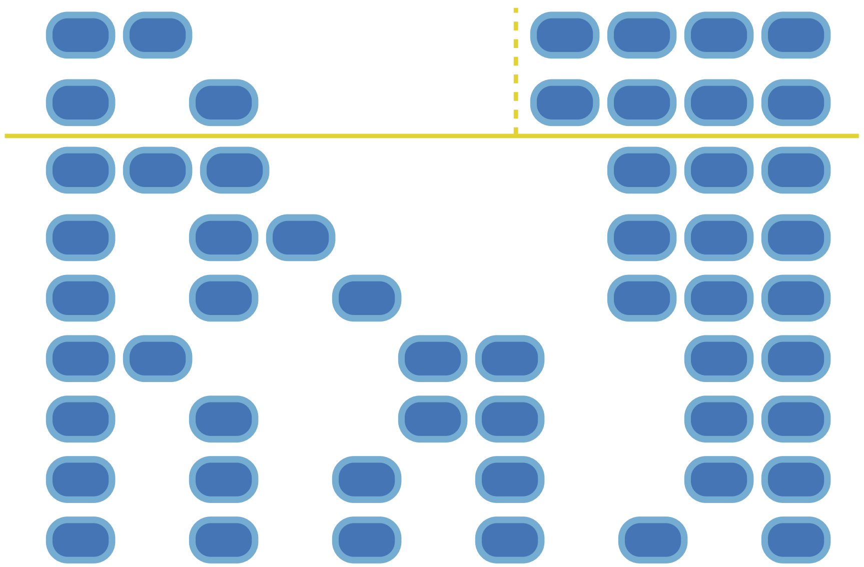

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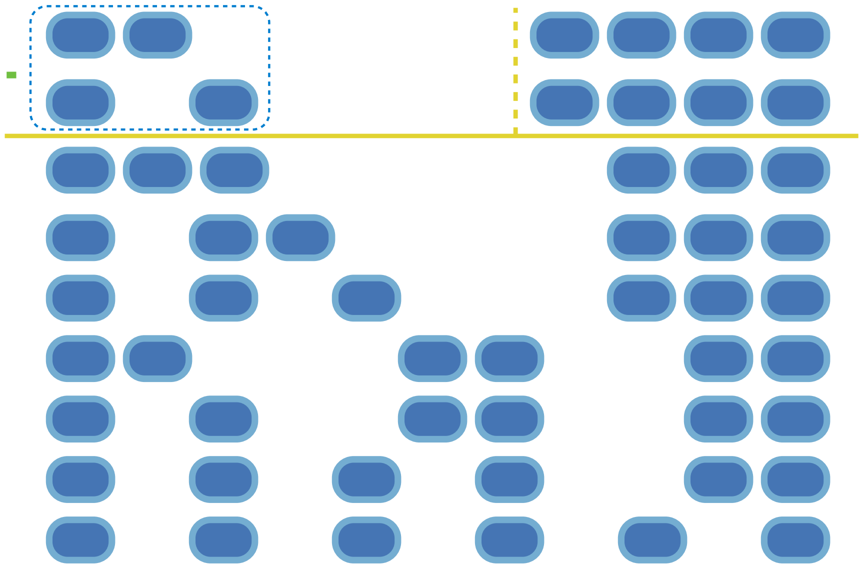

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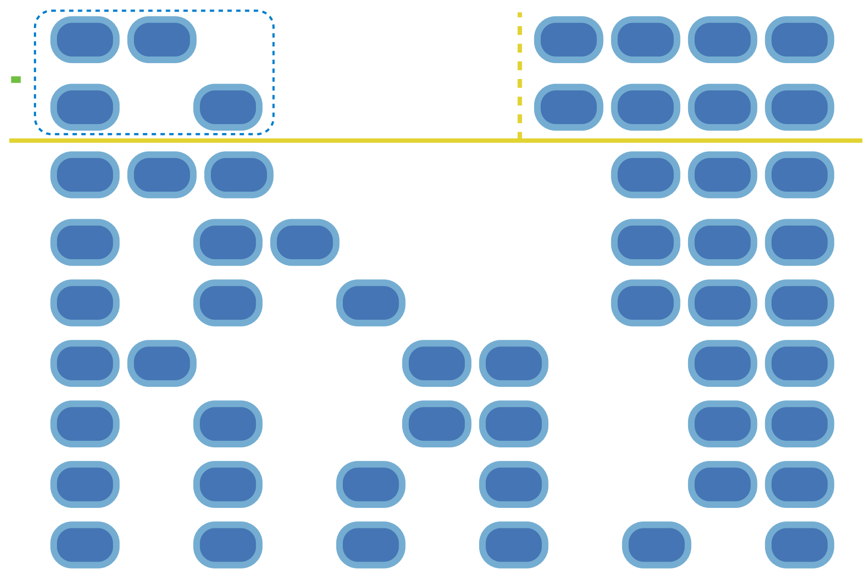

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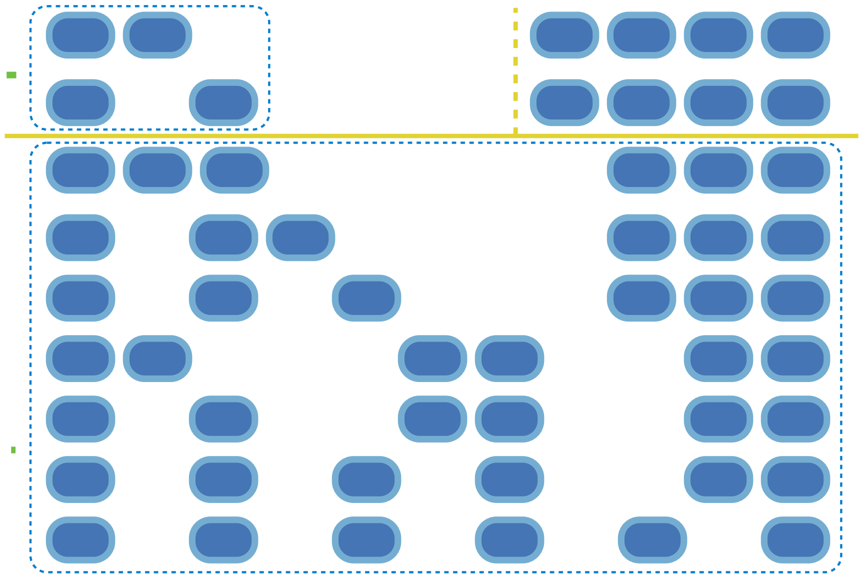

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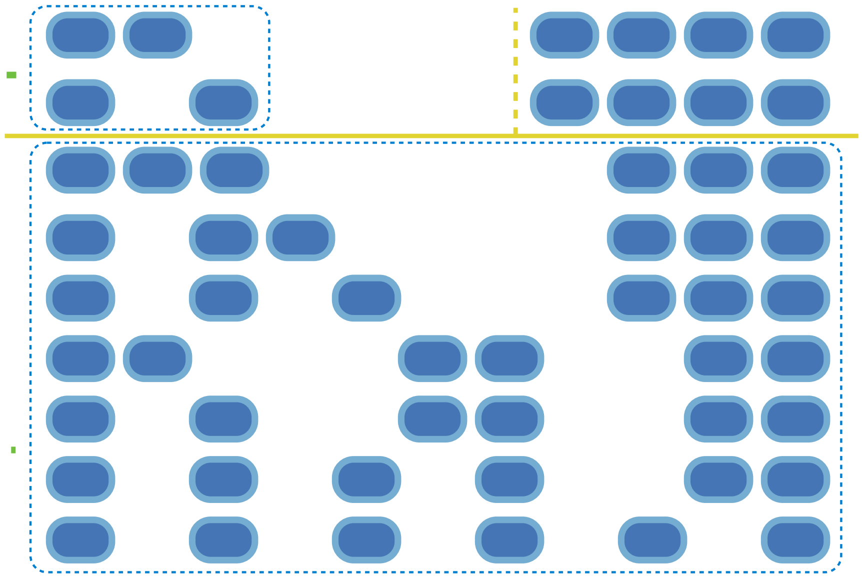

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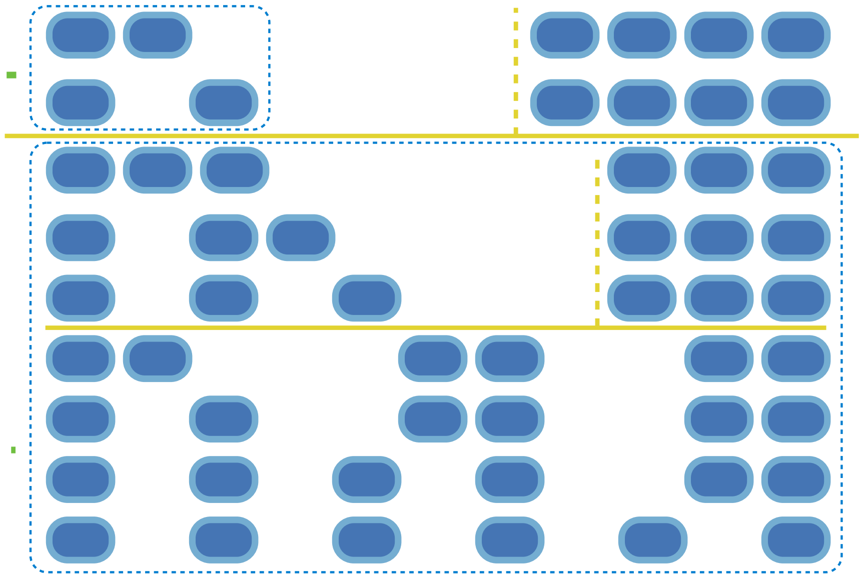

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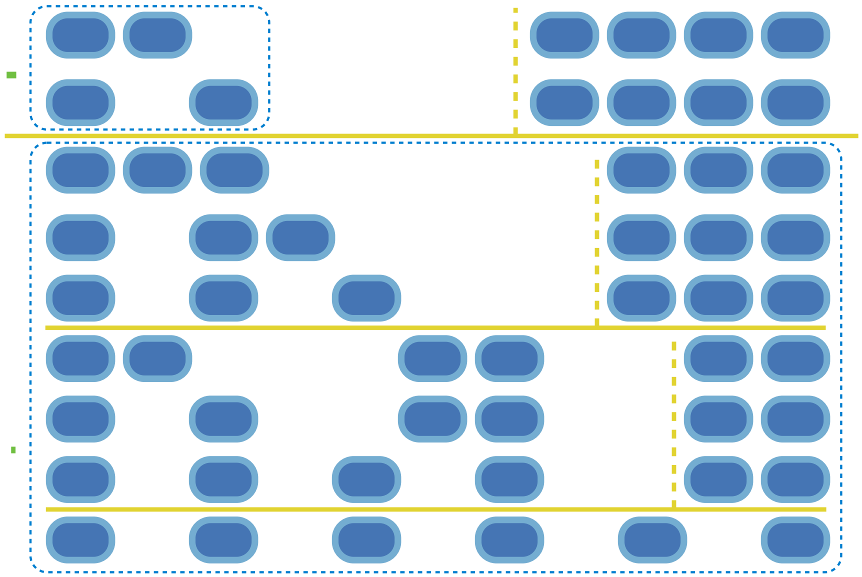

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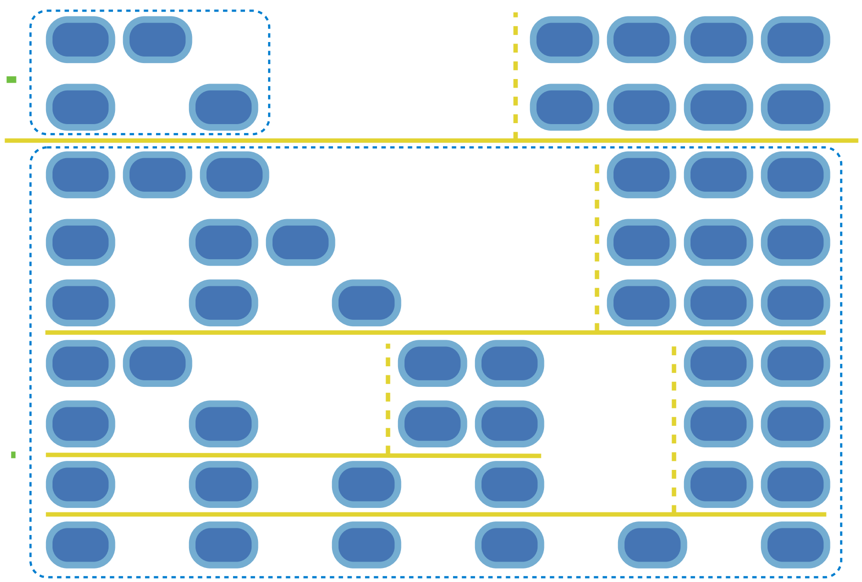

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  - Use at least one 4
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- Tree recursion often involves exploring different choices.

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def count_partitions(n, m):
    if n == 0:



    else:
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(Demo)

pythontutor.com/composingprograms.html#code=def%20count_partitions%28n,%20m%29%3A%0A%20%20%20%20if%20n%20%3D%3D%200%3A%0A%20%20%20%20%20%20%20%20return%201%0A%20%20%20%20elif%20n%20<%200%3A%0A%20%20%20%20%20%20%20%20return%200%0A%20%20%20%20elif%20m%20%3D%3D%200%3A%0A%20%20%20%20%20%20%20%20return%200%0A%20%20%20%20else%3A%0A%20%20%20%20%20%20%20%20with_m%20%3D%20count_partitions%28n-m,
%20m%29%20%0A%20%20%20%20%20%20%20%20without_m%20%3D%20count_partitions%28n,
%20m-1%29%0A%20%20%20%20%20%20%20%20return%20with_m%20%2B%20without_m%0A%20%20%20%20%20%20%20%20%0Aresult%20%3D%20count_partitions%285,%203%29%0A%0A#%201%20%2B%201%20%2B%201%20%2B%201%20%2B%201%20%3D%205%0A#%201%20%2B%201%20%2B%201%20%2B%202%20%2B%20%20%20%3D%205%0A#%201%20%2B%202%20%2B%202%20%2B%20%20%20%20%20%20%20%3D%205%0A#%201%20%2B%201%20%2B%203%20%2B%20%20%20%20%20%20%20%3D%205%0A
20%2B%203%20%2B%20%20%20%20%20%20%20%20%20%20%20%3D%205&mode=display&origin=composingprograms.js&cumulative=false&py=3&rawInputLstJSON=[]&curInstr=0