Recursion

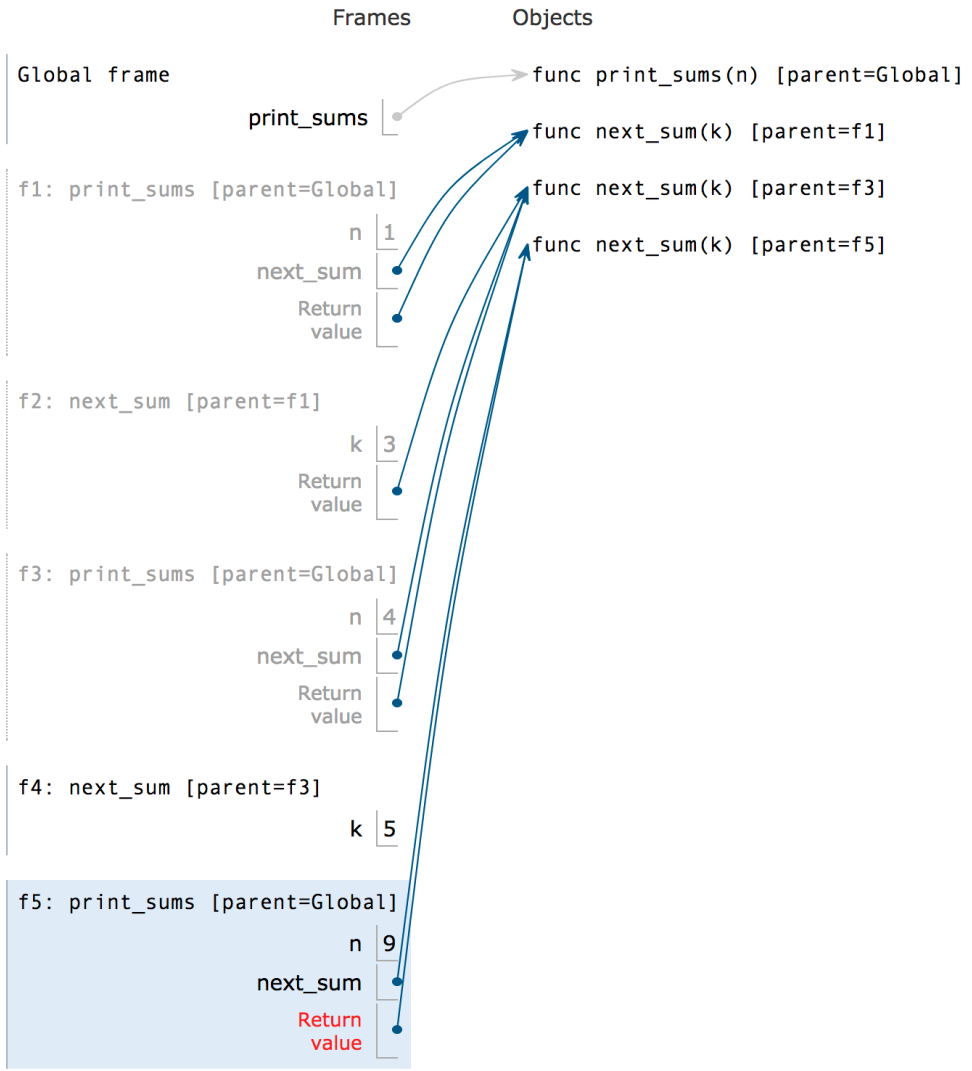
Announcements

Self-Reference

(Demo)

Returning a Function Using Its Own Name

```
1  def print_sums(n):
2      print(n)
3      def next_sum(k):
4          return print_sums(n+k)
5      return next_sum
6
7  print_sums(1)(3)(5)
```



http://pythontutor.com/composingprograms.html#code=def%20print_all%28k%29%3A%0A%20%20%20%20print%28k%29%0A%20%20%20%20return%20print_all%0A%20%20%20%20%0Aprint_all%281%29%283%29%285%29&cumulative=true&curInstr=0&mode=display&origin=composingprograms.js&py=3&rawInputLstJSON=%5B%5D

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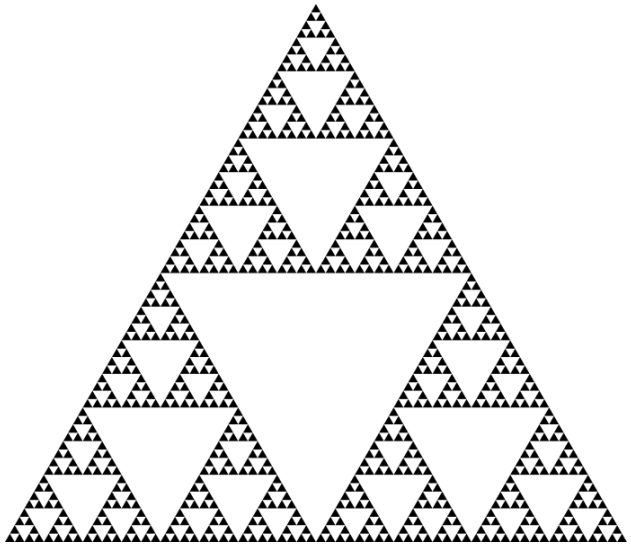

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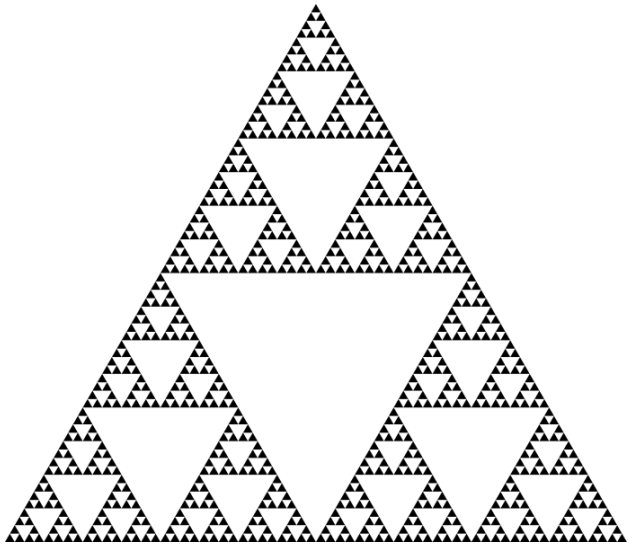

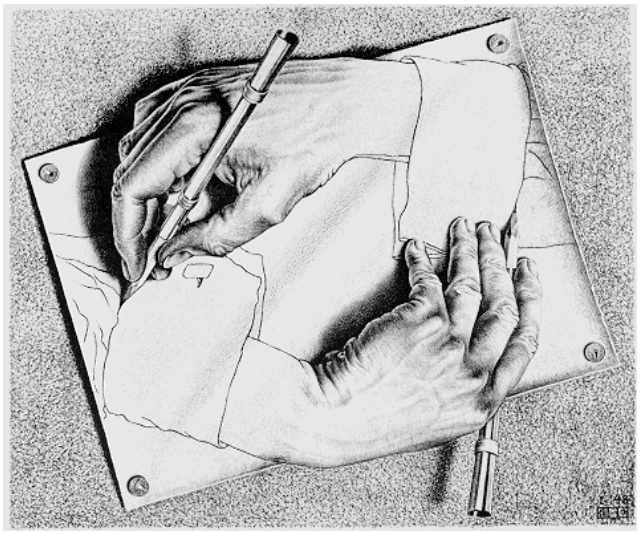

Drawing Hands, by M. C. Escher (lithograph, 1948)

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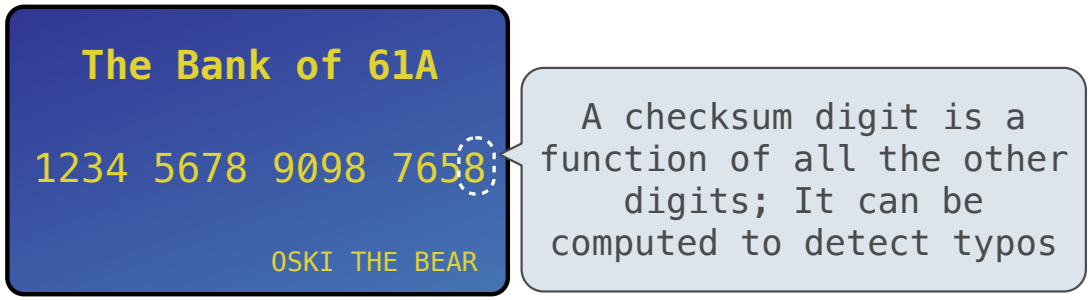

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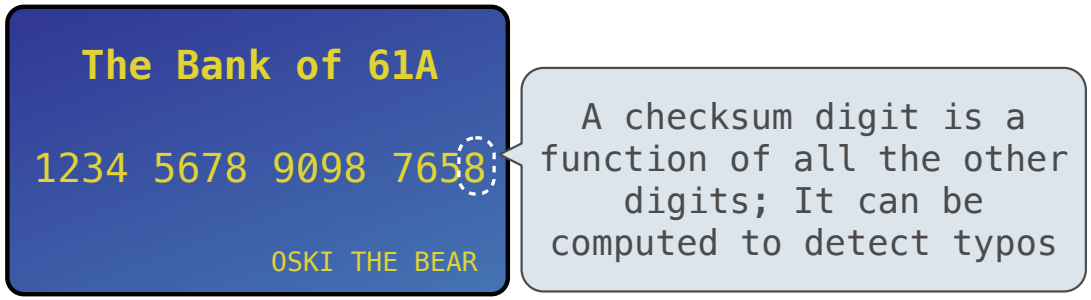


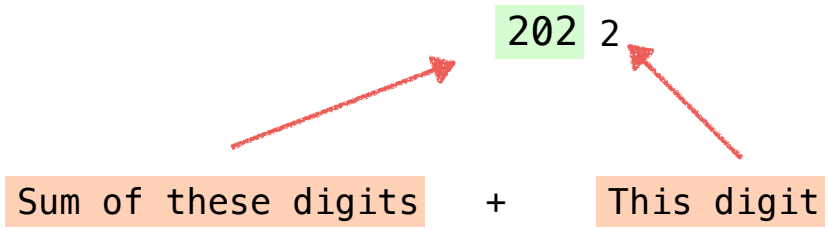
- Credit cards actually use the Luhn algorithm, which we'll implement after `sum_digits`

The Problem Within the Problem

The sum of the digits of 6 is 6.

Likewise for any one-digit (non-negative) number (i.e., < 10).

The sum of the digits of 2022 is



That is, we can break the problem of summing the digits of 2022 into a smaller instance of the same problem, plus some extra stuff.

We call this recursion.

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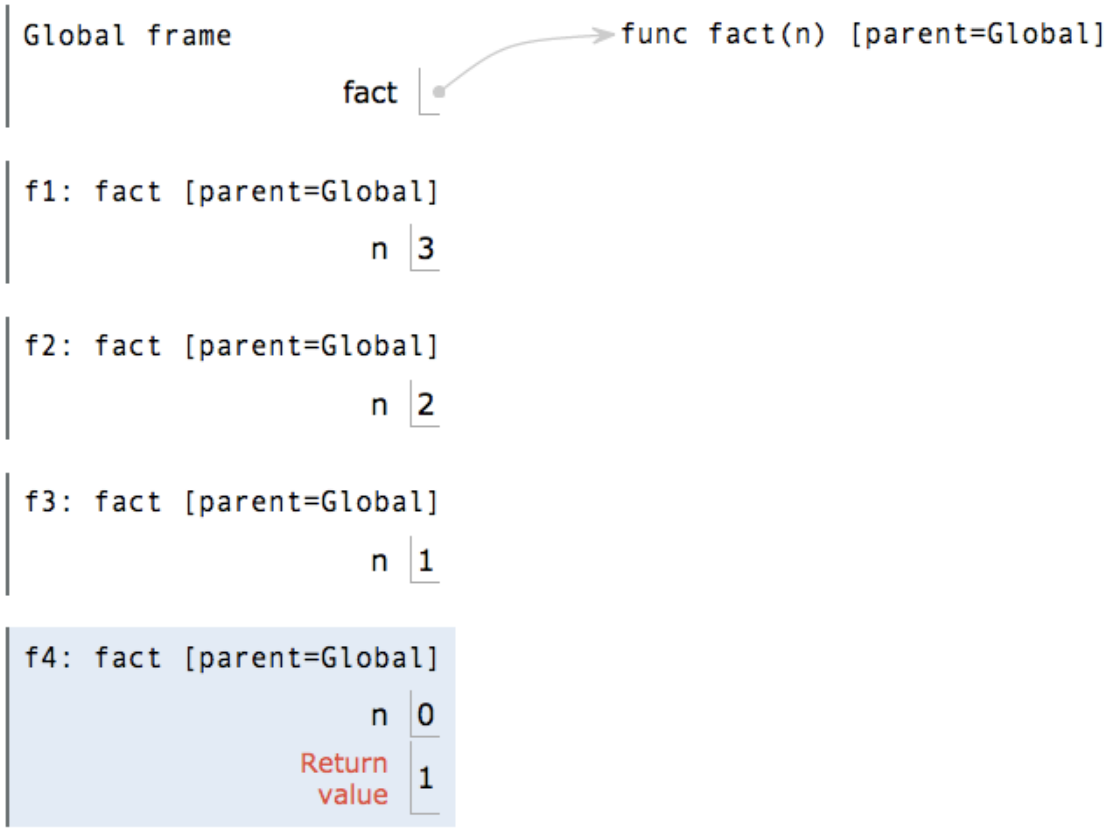

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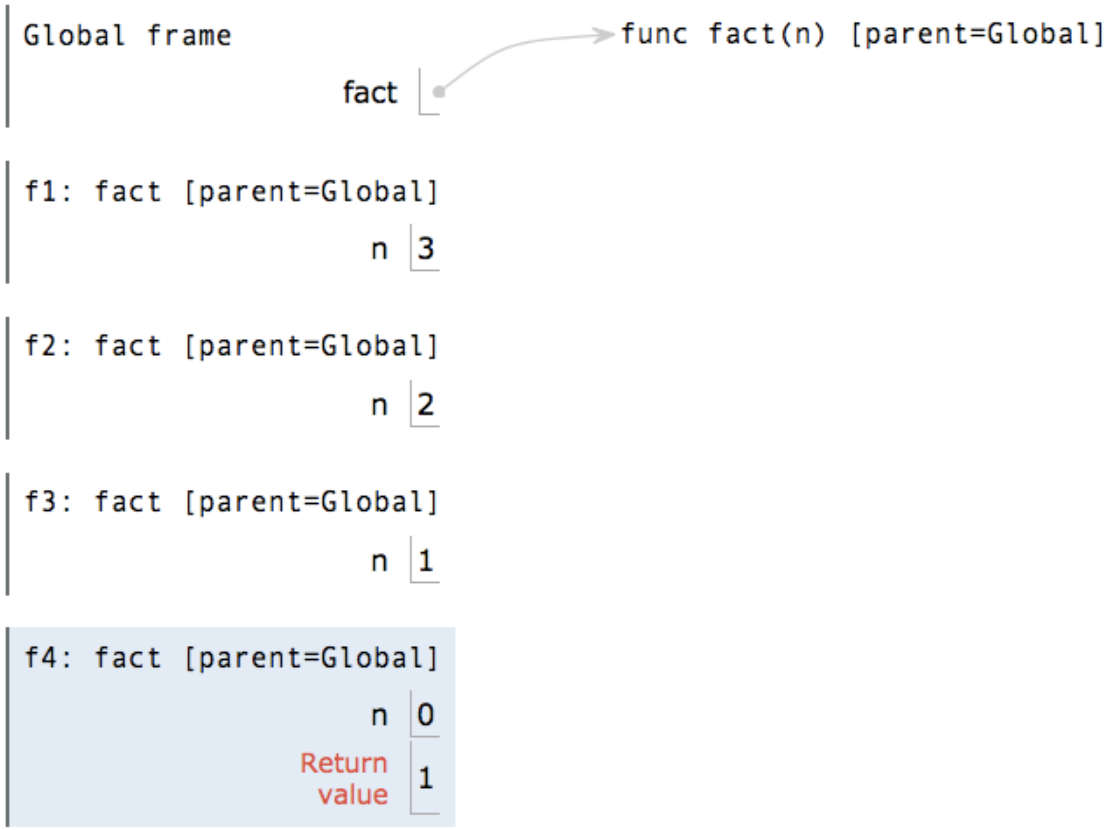

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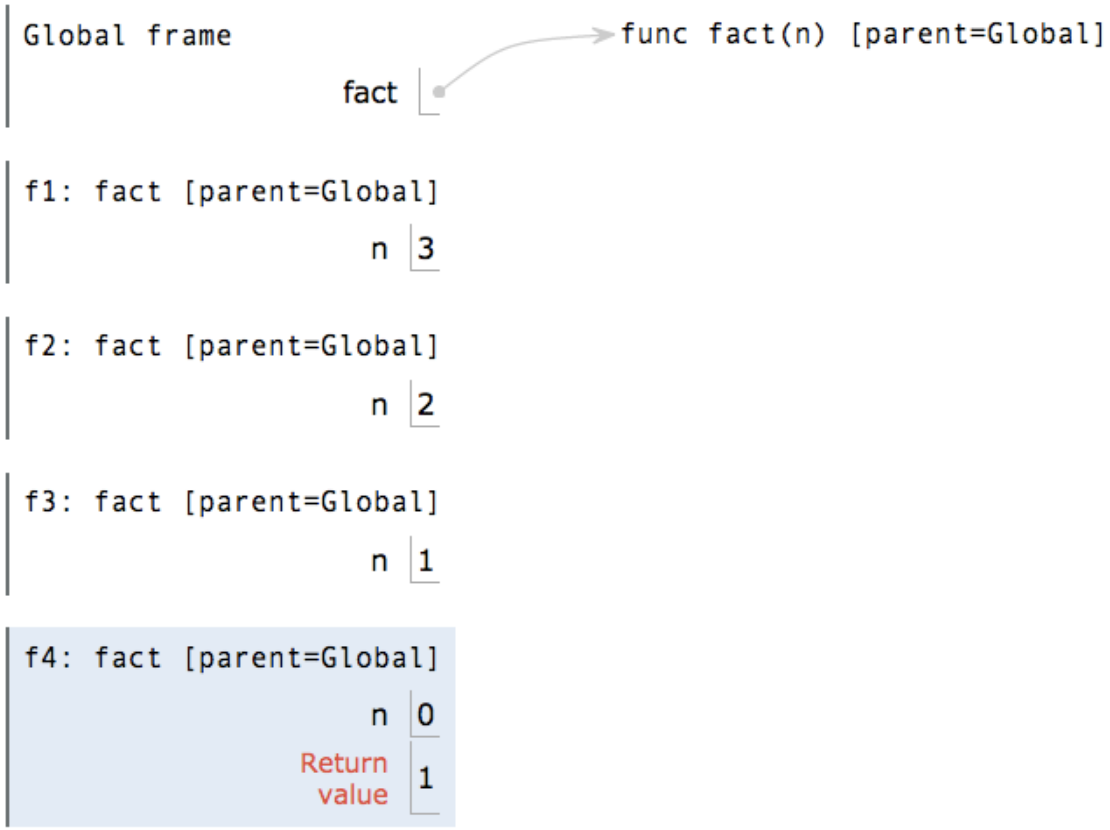

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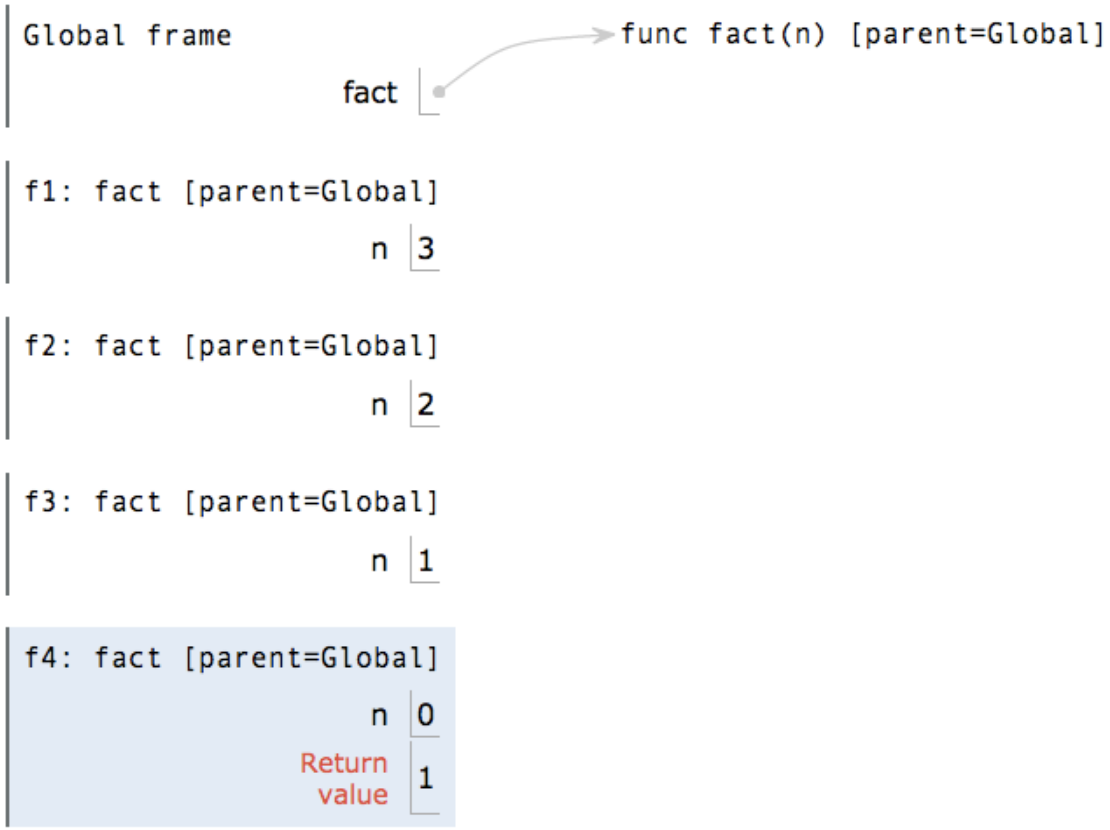

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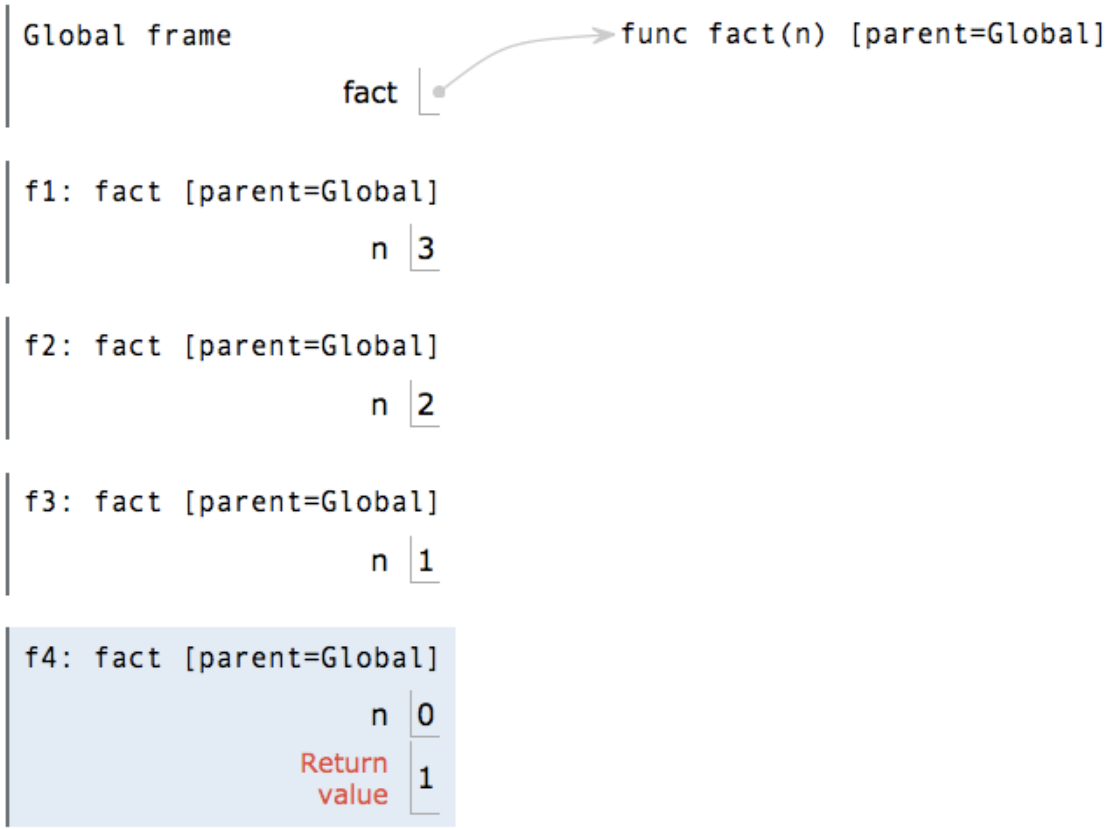

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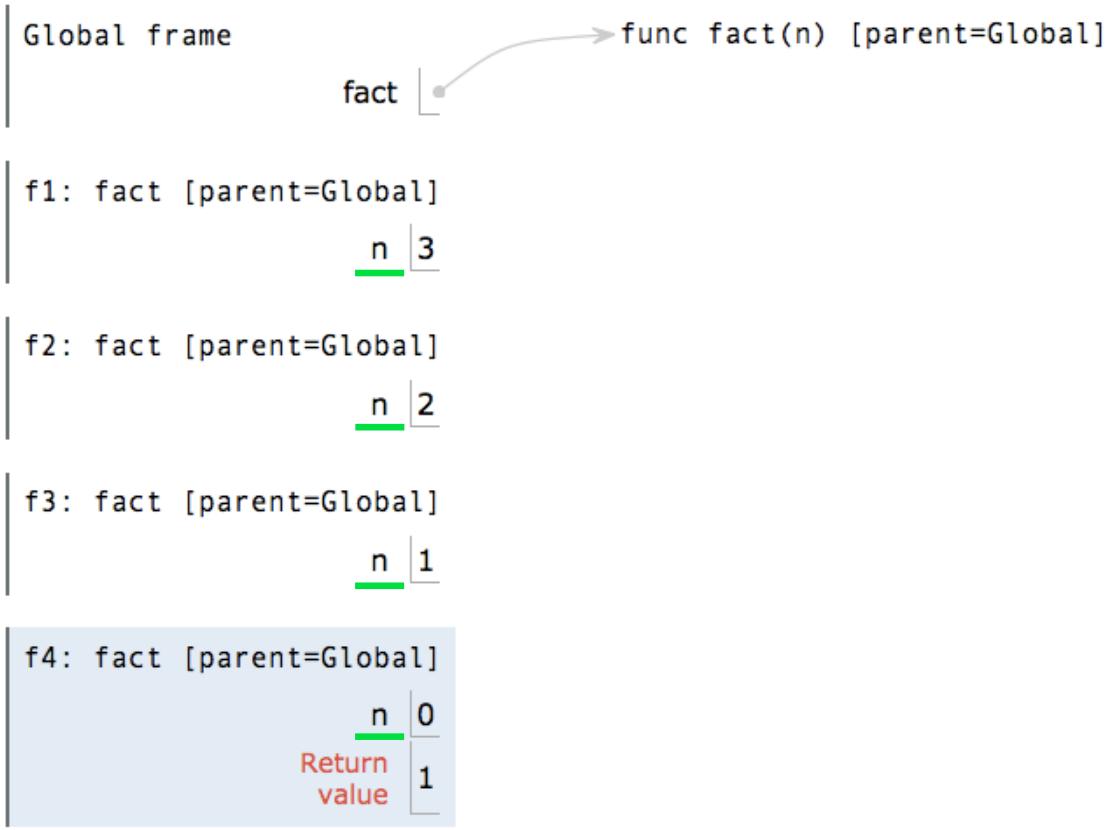

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- Each call to **fact** solves a simpler problem than the last: smaller **n**

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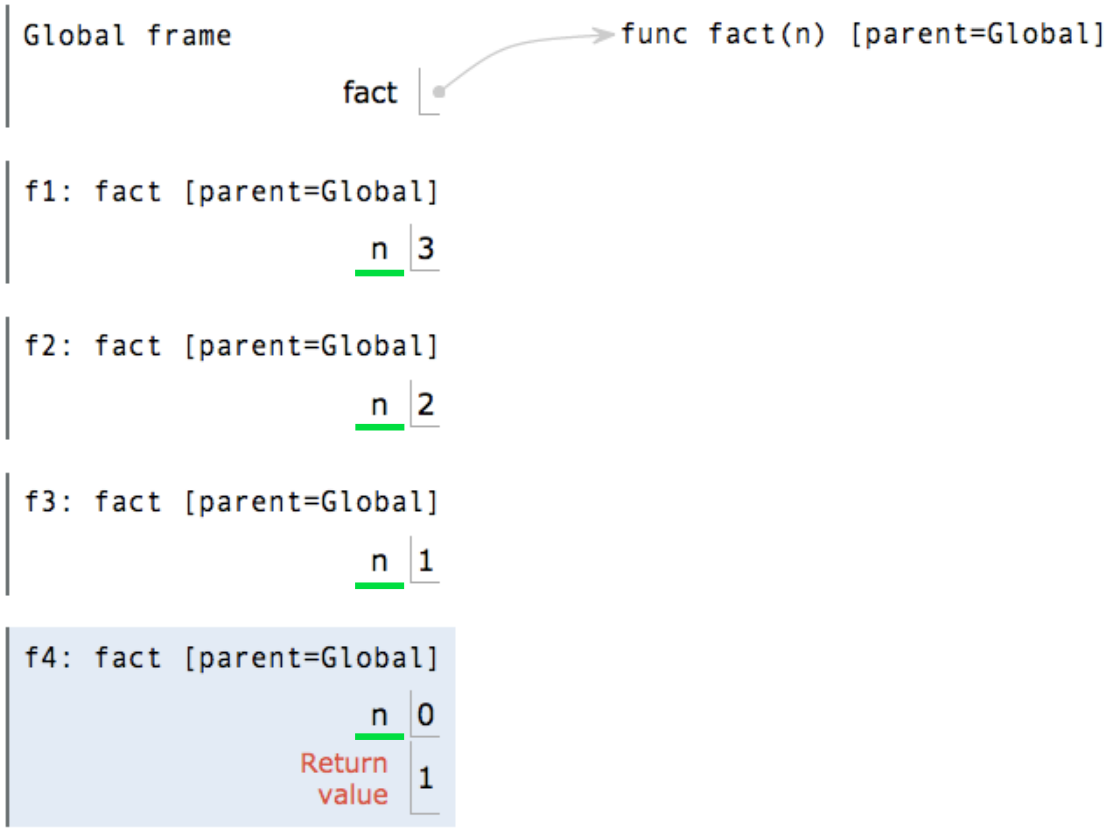

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Using recursion:

```
def fact(n):
    if n == 0:
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```

Math:

$$n! = \prod_{k=1}^{n} k$$

$$n! = \begin{cases} 1 & \text{if } n = 0 \\ n \cdot (n-1)! & \text{otherwise} \end{cases}$$

Names:

Iteration vs Recursion

Iteration is a special case of recursion

$$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$$

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Verifying Recursive Functions

The Recursive Leap of Faith

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Photo by Kevin Lee, Preikestolen, Norway

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Photo by Kevin Lee, Preikestolen, Norway

Mutual Recursion

The Luhn Algorithm

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Used to verify credit card numbers

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- **First:** From the rightmost digit, which is the check digit, moving left, double the value of every second digit; if product of this doubling operation is greater than 9 (e.g., 7 * 2 = 14), then sum the digits of the products (e.g., 10: 1 + 0 = 1, 14: 1 + 4 = 5)

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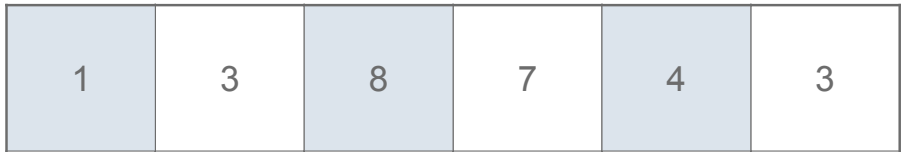

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The Luhn sum of a valid credit card number is a multiple of 10 (Demo)

Recursion and Iteration

Converting Recursion to Iteration

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```
def sum_digits(n):
    """Return the sum of the digits of positive integer n."""
    if n < 10:
        return n
    else:
        all_but_last, last = split(n)
        return sum_digits(all_but_last) + last
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def sum_digits_iter(n):
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        n, last = split(n)
        digit_sum = digit_sum + last
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```

```
def sum_digits_rec(n, digit_sum):
    if n > 0:
        n, last = split(n)
        return sum_digits_rec(n, digit_sum + last)
    else:
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```

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Updates via assignment become...

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```

Converting Iteration to Recursion

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def sum_digits_rec(n, digit_sum):
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        return sum_digits_rec(n,
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```

...arguments to a recursive call