Environments

Announcements

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Environment diagrams describe how higher-order functions work!

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Environment diagrams describe how higher-order functions work!

(Demo)

Names can be Bound to Functional Arguments

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2      return f(f(x))
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4  def square(x):
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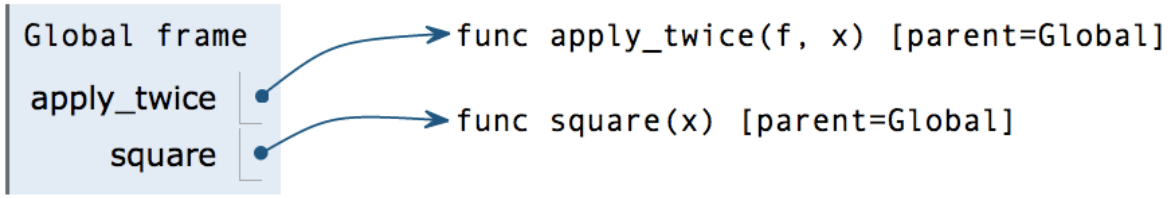

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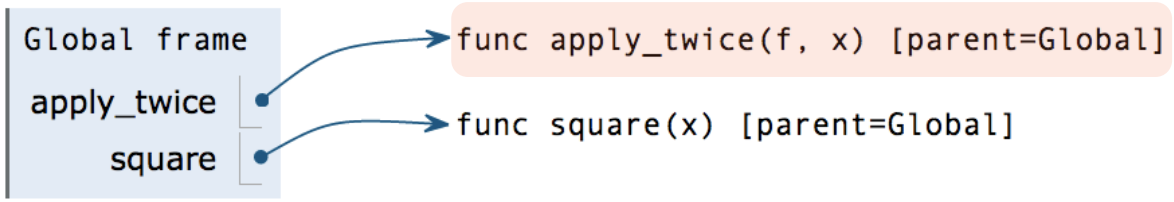

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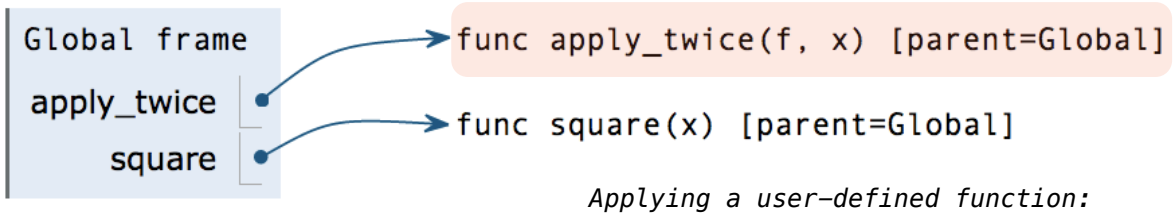

Applying a user-defined function:

- Create a new frame
- Bind formal parameters (f & x) to arguments
- Execute the body: return f(f(x))

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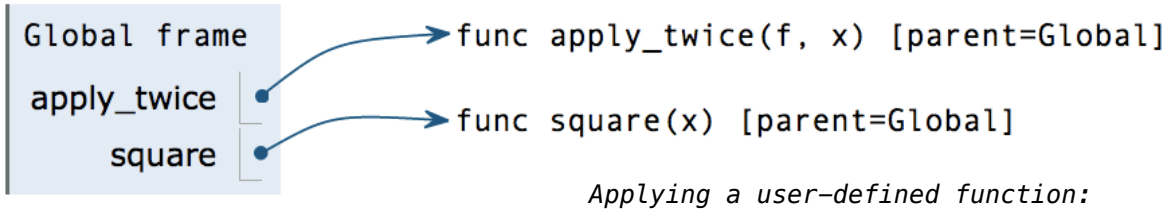

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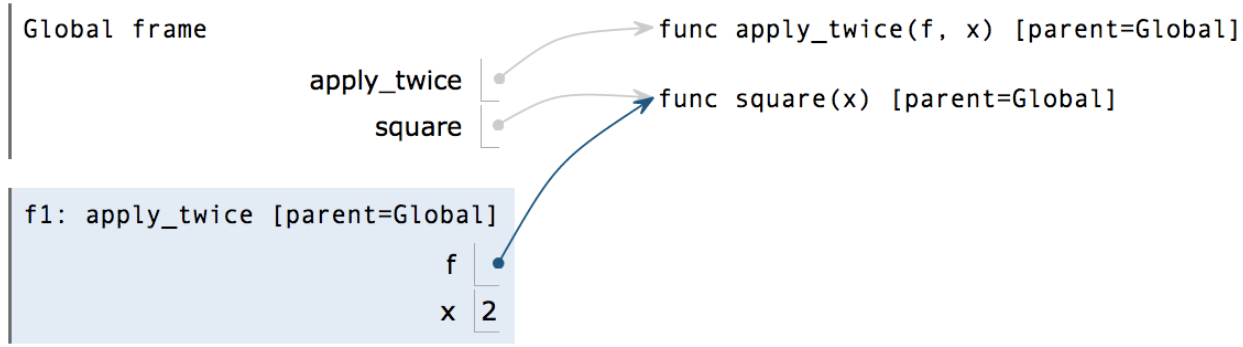

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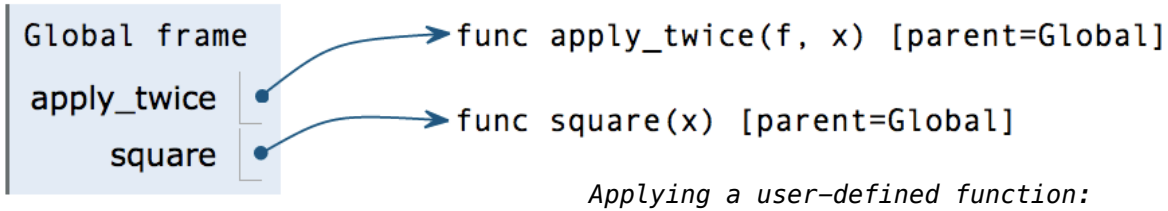

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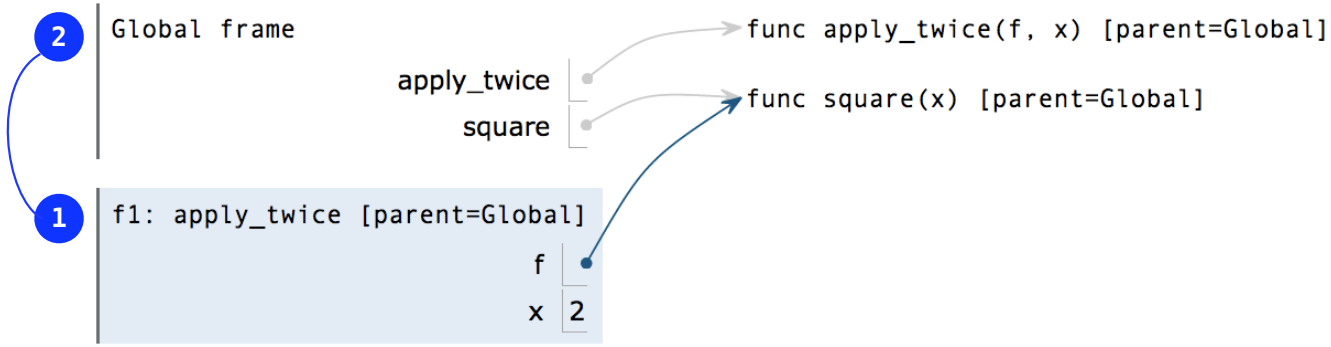

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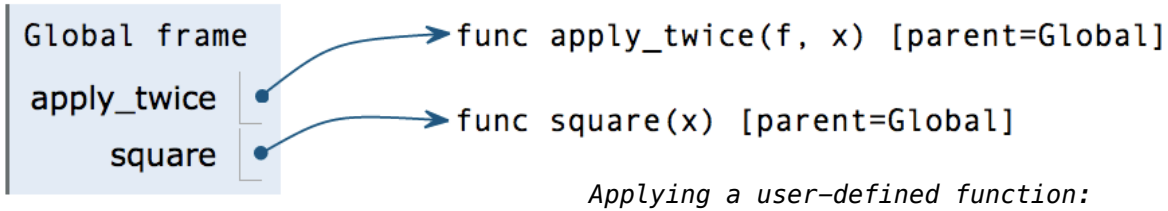

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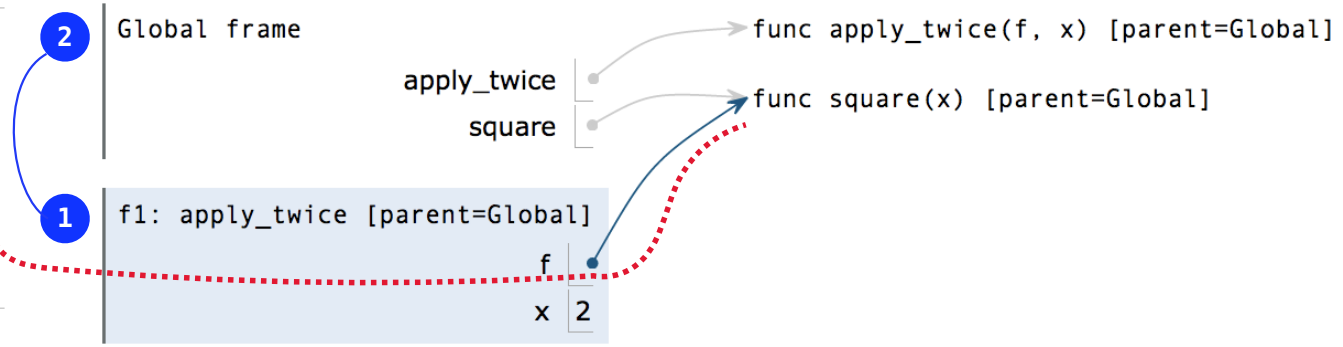

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Environments for Nested Definitions

(Demo)

Environment Diagrams for Nested Def Statements

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3          return k + n
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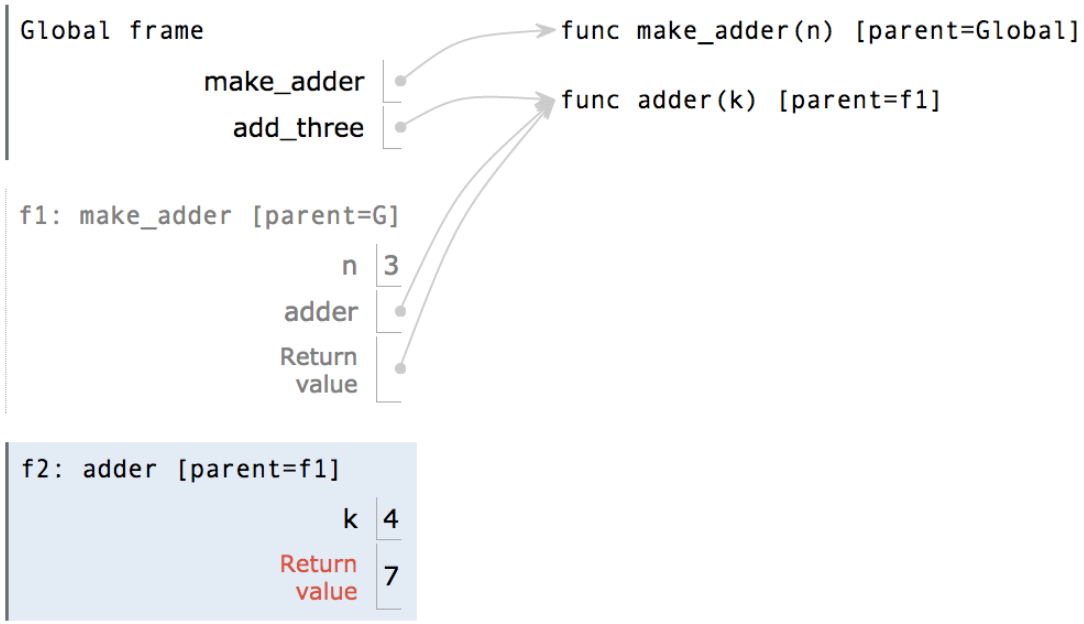

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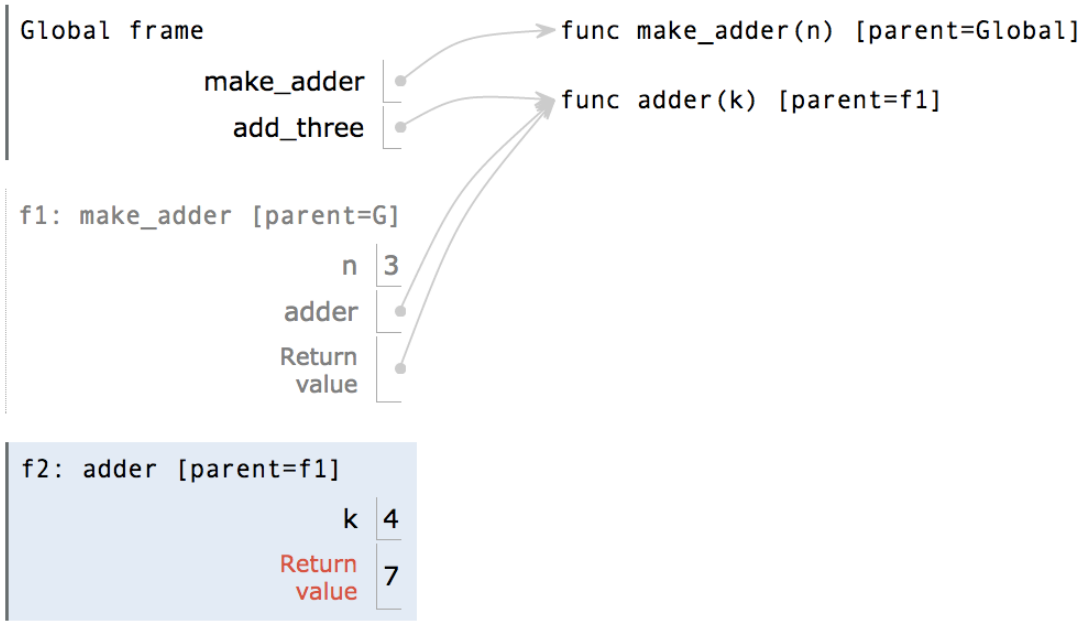

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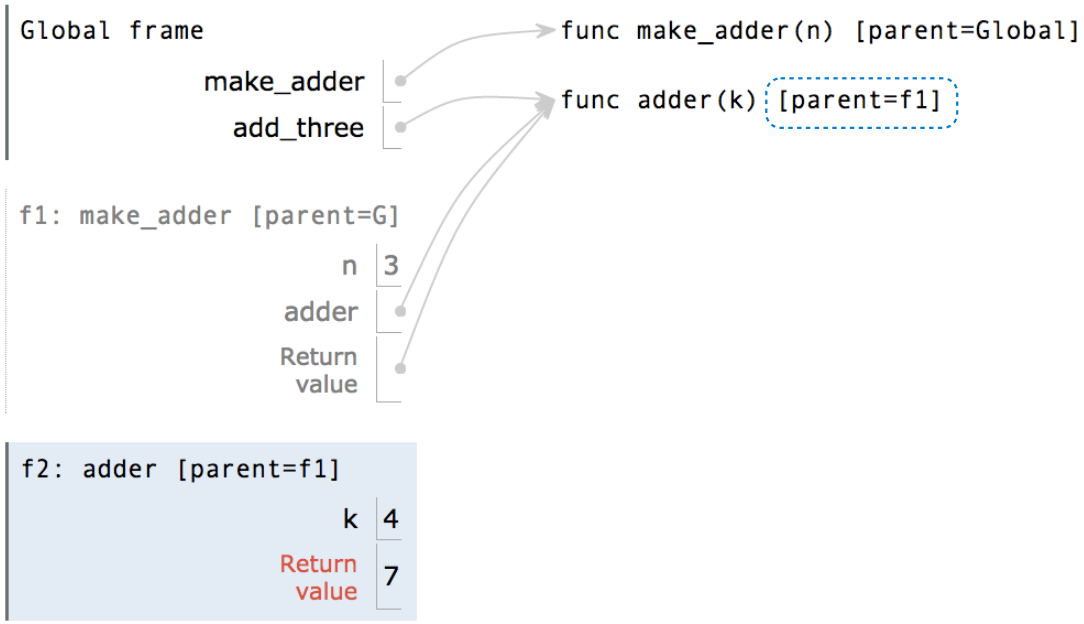

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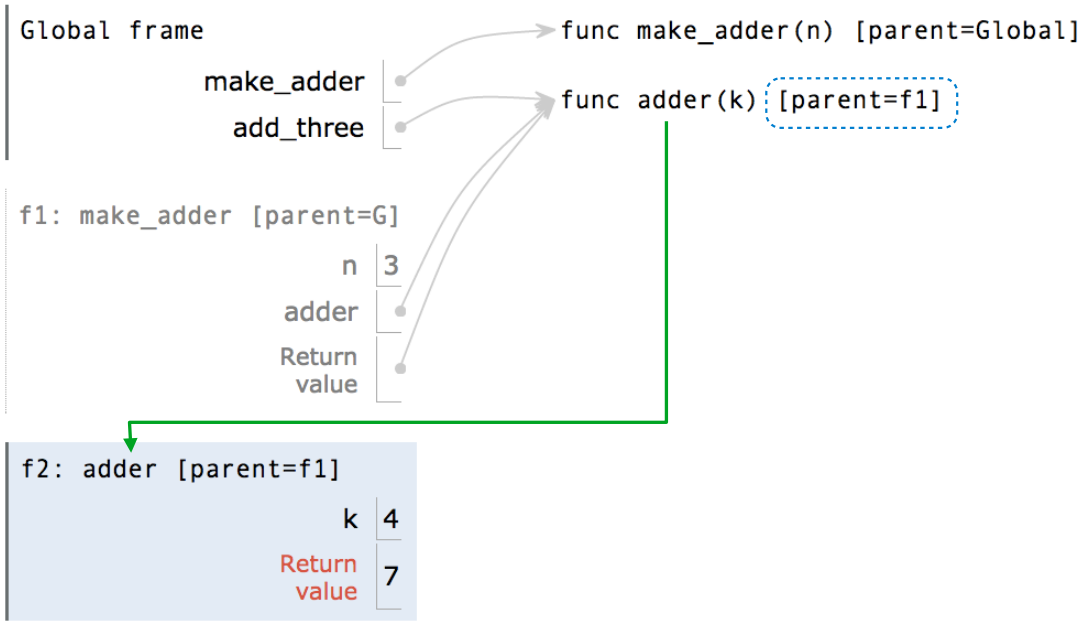

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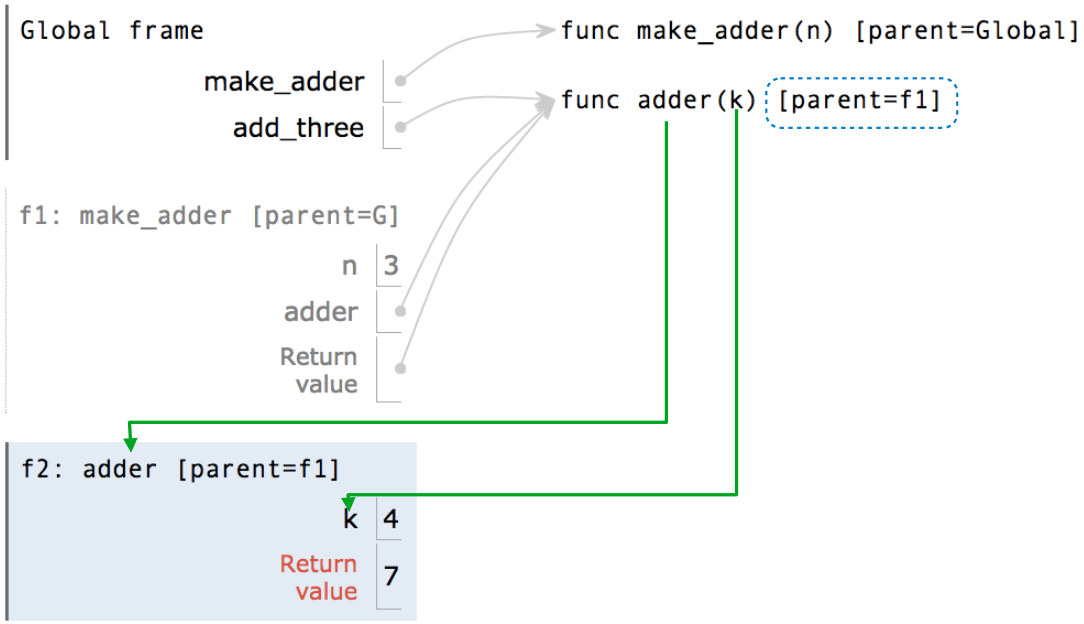

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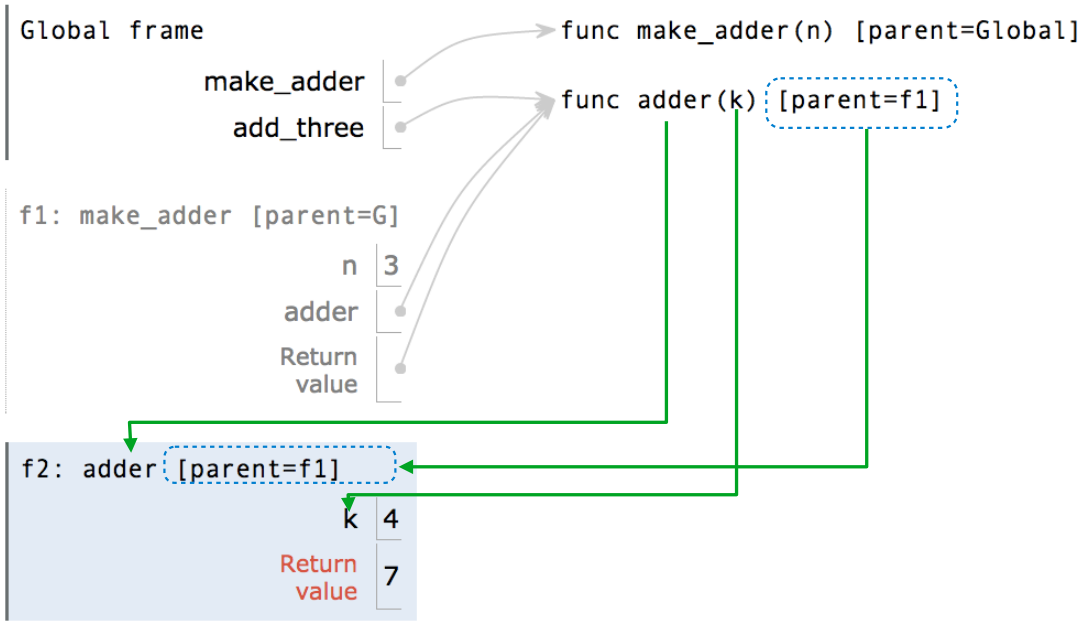

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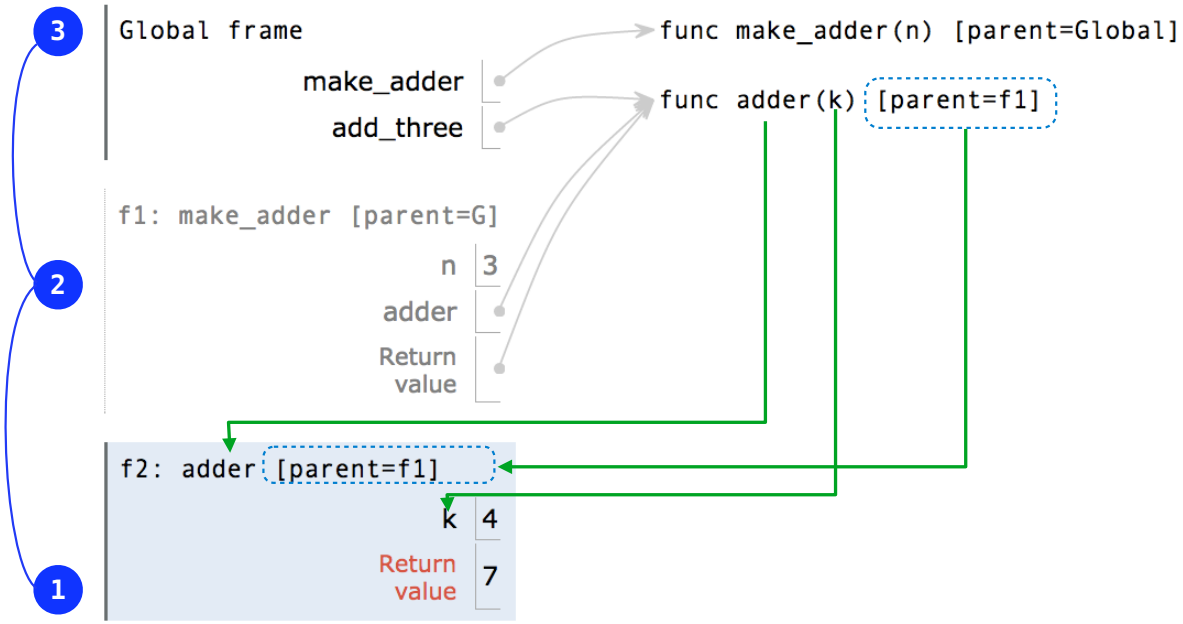

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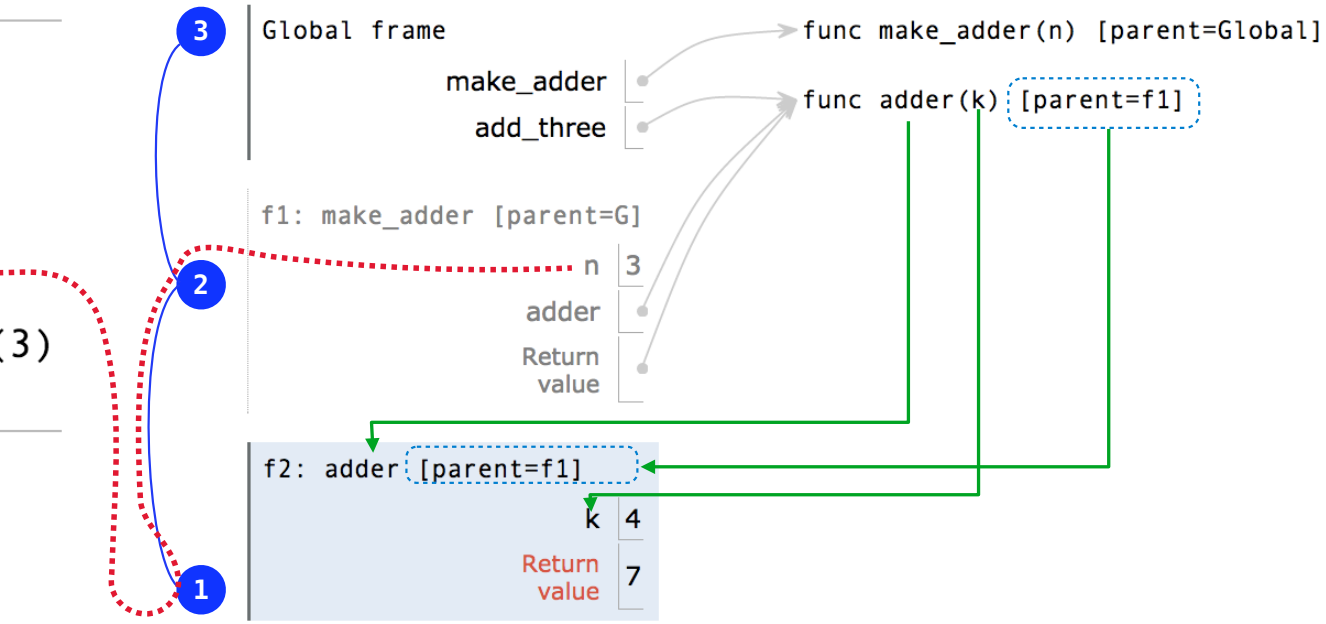

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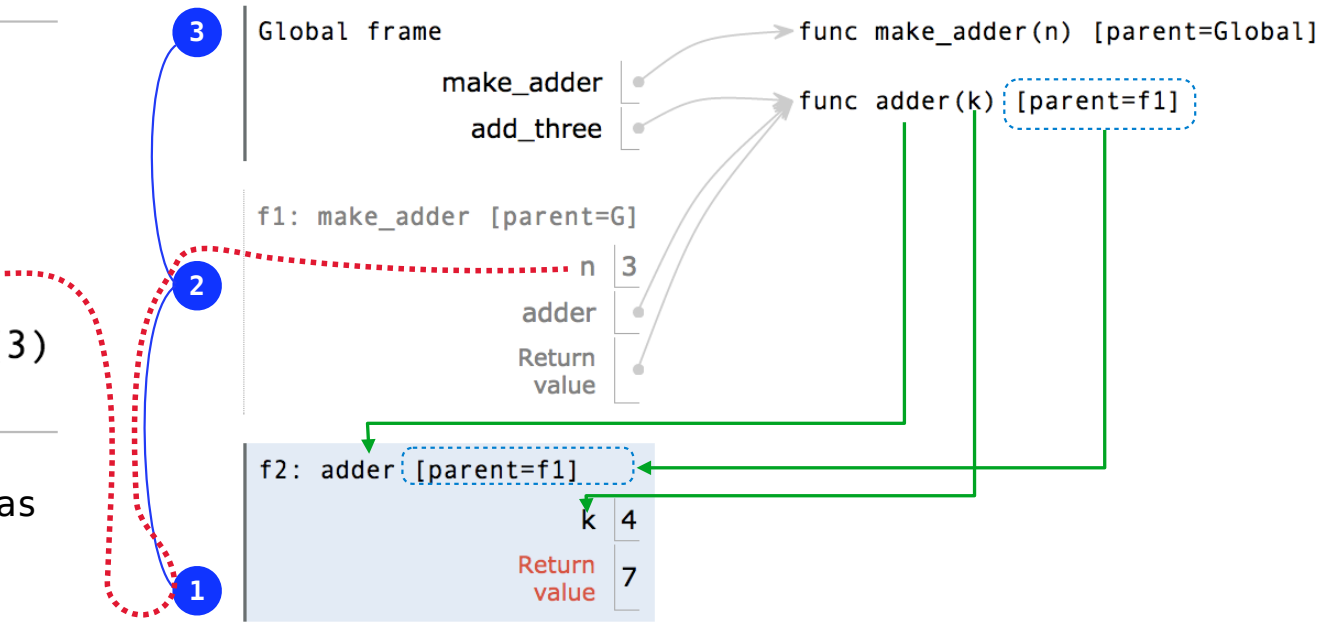

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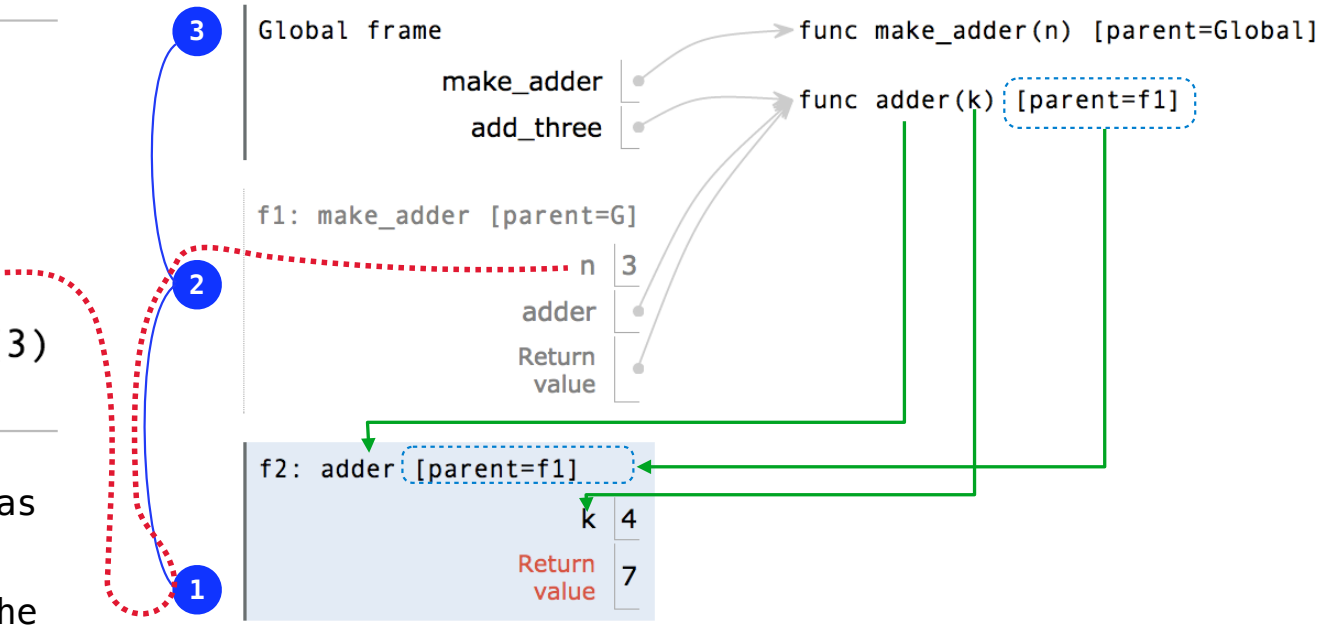

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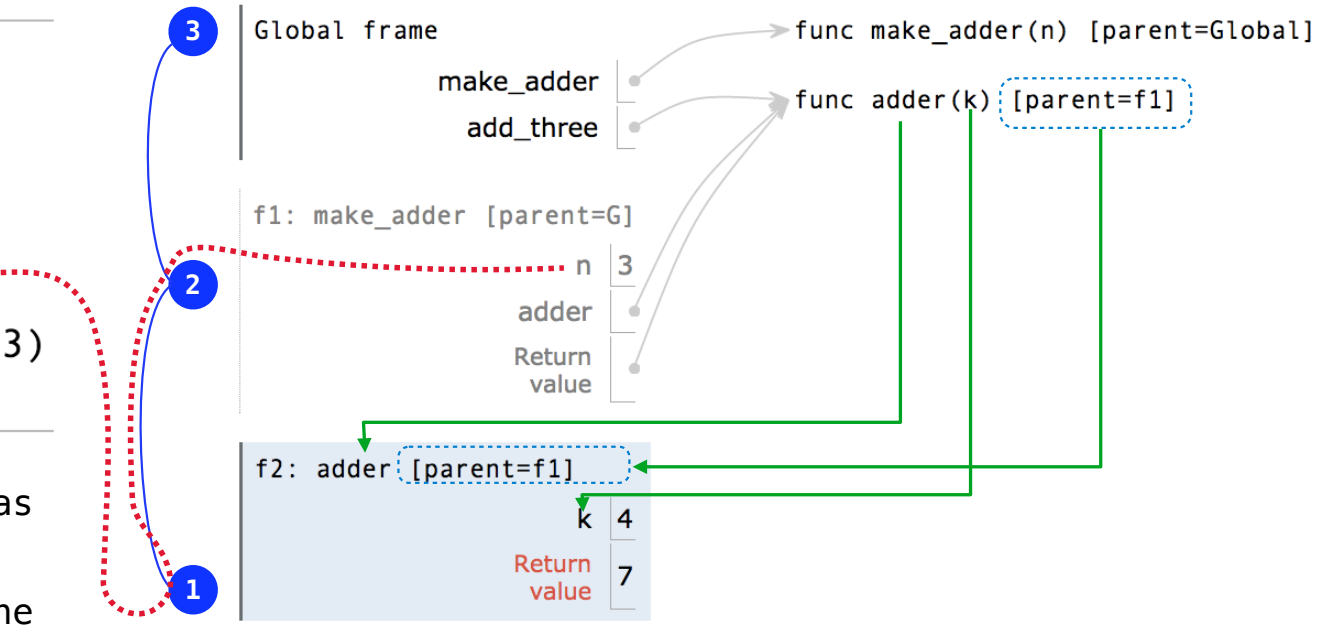

- Every user-defined function has a parent frame (often global)
- The parent of a function is the frame in which it was defined
- Every local frame has a parent frame (often global)

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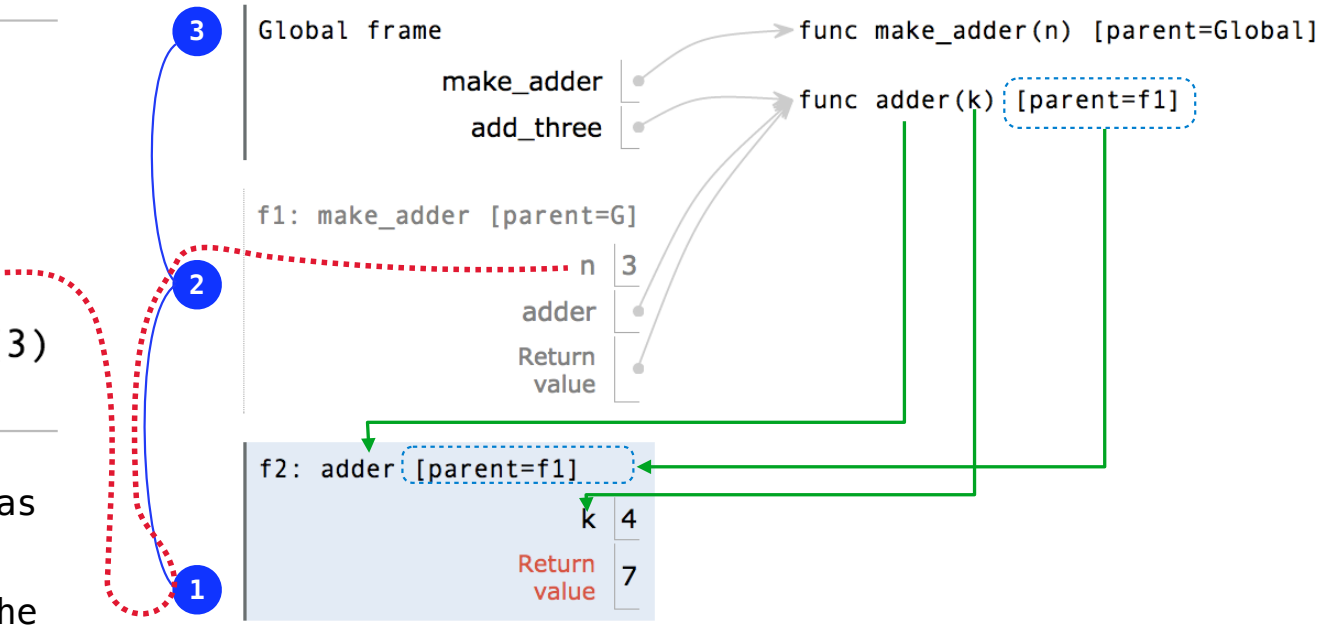

- Every user-defined function has a parent frame (often global)
- The parent of a function is the frame in which it was defined
- Every local frame has a parent frame (often global)
- The parent of a frame is the parent of the function called

http://pythontutor.com/composingprograms.html#code=def%20make_adder%28n%29%3A%0A%20%20%20%20def%20adder%28k%29%3A%0A%20%20%20%20%20%20%20%20return%20k%20%2B%20n%0A%20%20%20%20return%20adder%0A%20%20%20%20%0Athree_more_than%20%3D%20make_adder%283%29%0Aresult%20%3D%20three_more_than%284%29&cumulative=false&curInstr=0&mode=display&origin=composingprograms.js&py=3&rawInputLstJSON=%5B%5D

How to Draw an Environment Diagram

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Create a function value: `func <name>(<formal parameters>) [parent=<label>]`

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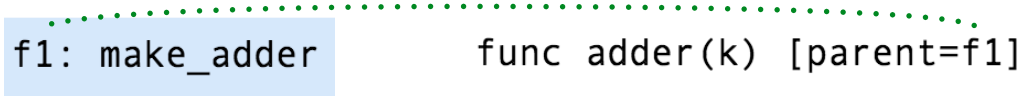

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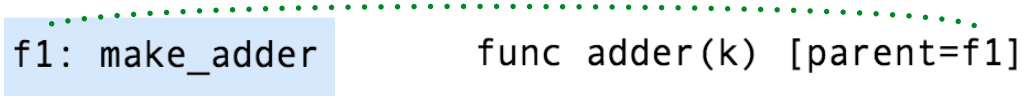

Bind <name> to the function value in the current frame

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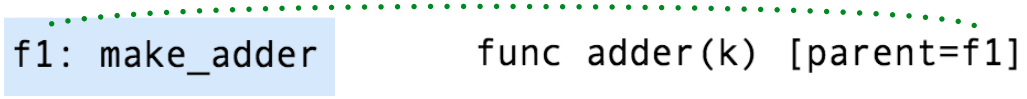

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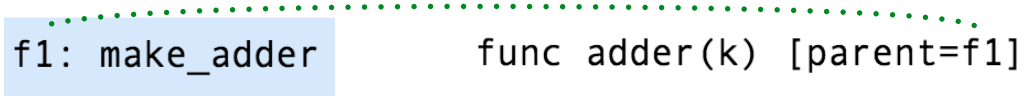

Bind `<name>` to the function value in the current frame

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1. Add a local frame, titled with the `<name>` of the function being called.

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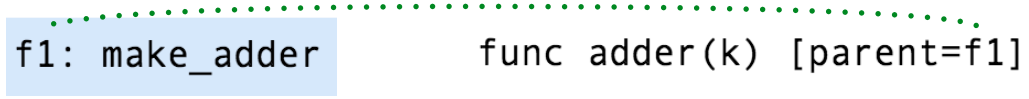
Its parent is the current frame.



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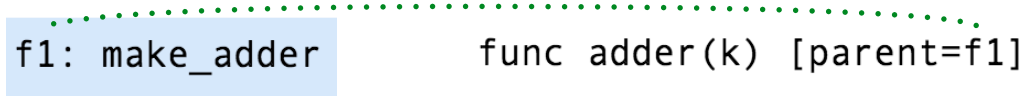

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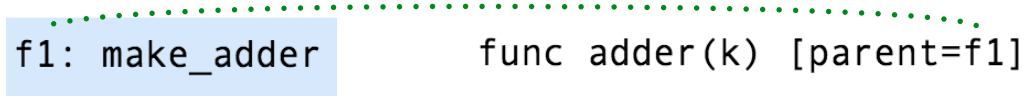


Bind `<name>` to the function value in the current frame

When a function is called:

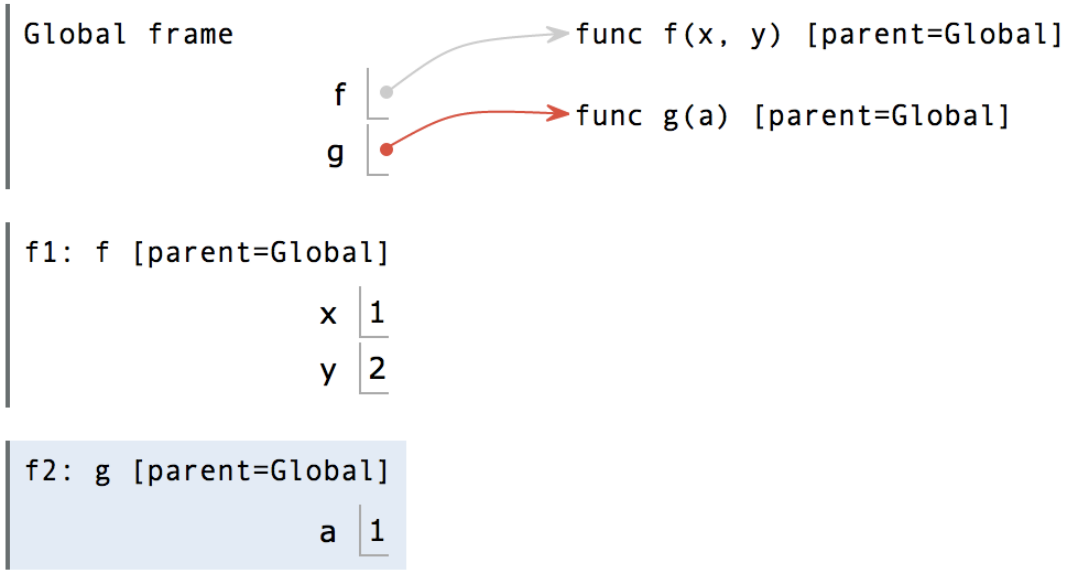
1. Add a local frame, titled with the `<name>` of the function being called.
2. Copy the parent of the function to the local frame: `[parent=<label>]`
3. Bind the `<formal parameters>` to the arguments in the local frame.
4. Execute the body of the function in the environment that starts with the local frame.

Local Names

(Demo)

Local Names are not Visible to Other (Non-Nested) Functions

```
1  def f(x, y):
2      return g(x)
3
4  def g(a):
5      return a + y
6
7  result = f(1, 2)
```



http://pythontutor.com/composingprograms.html#code=def%20f%28x,%20y%29%3A%0A%20%20%20%20return%20g%28x%29%0A%0Adef%20g%28a%29%3A%0A%20%20%20%20return%20a%20%2B%20y%0A%20%20%20%20%0Aresult%20%3D%20f%281,%202%29&cumulative=true&curInstr=0&mode=display&origin=composingprograms.js&py=3&rawInputLstJSON=%5B%5D

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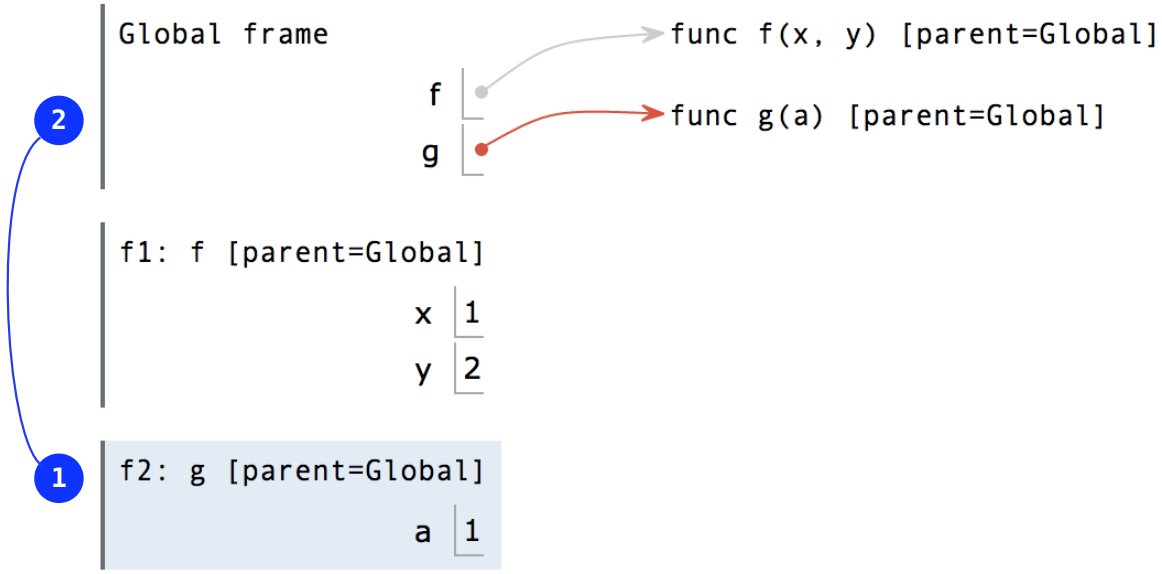

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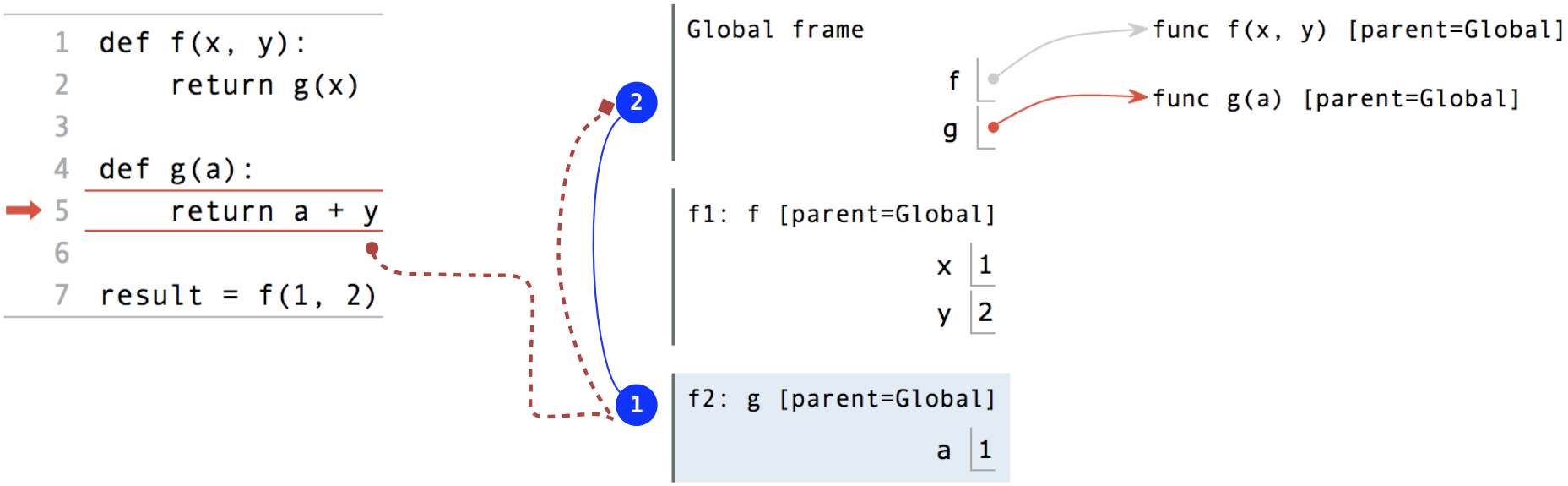

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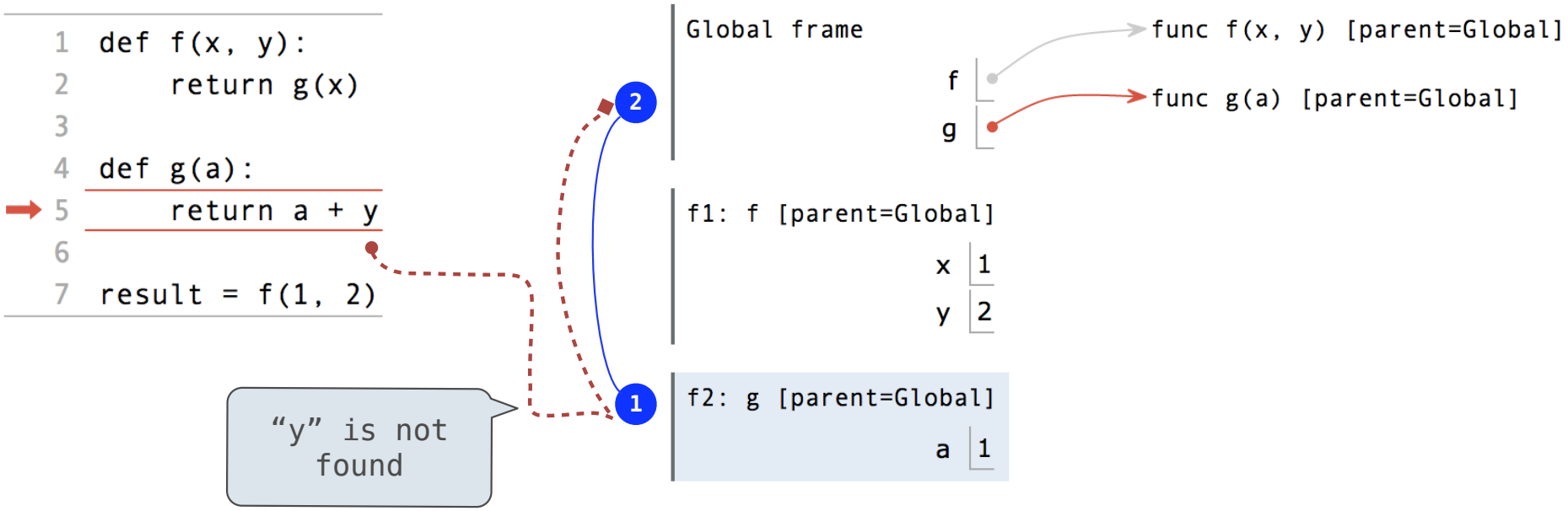

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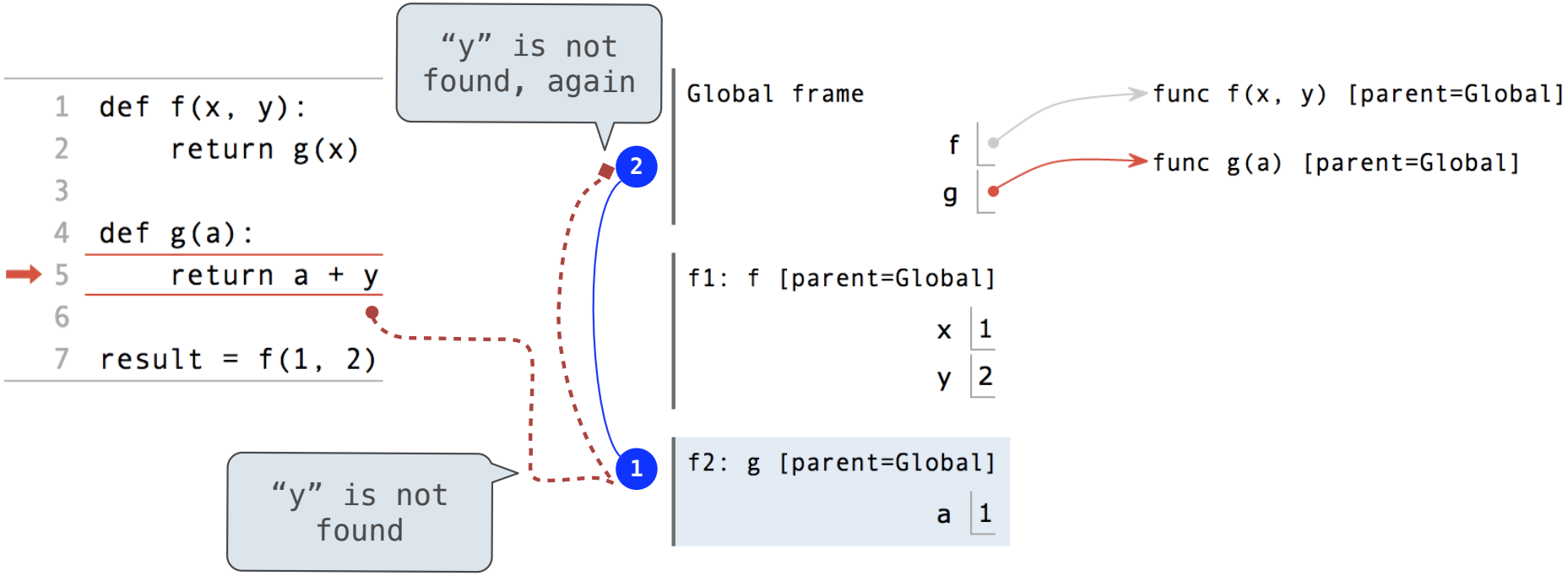

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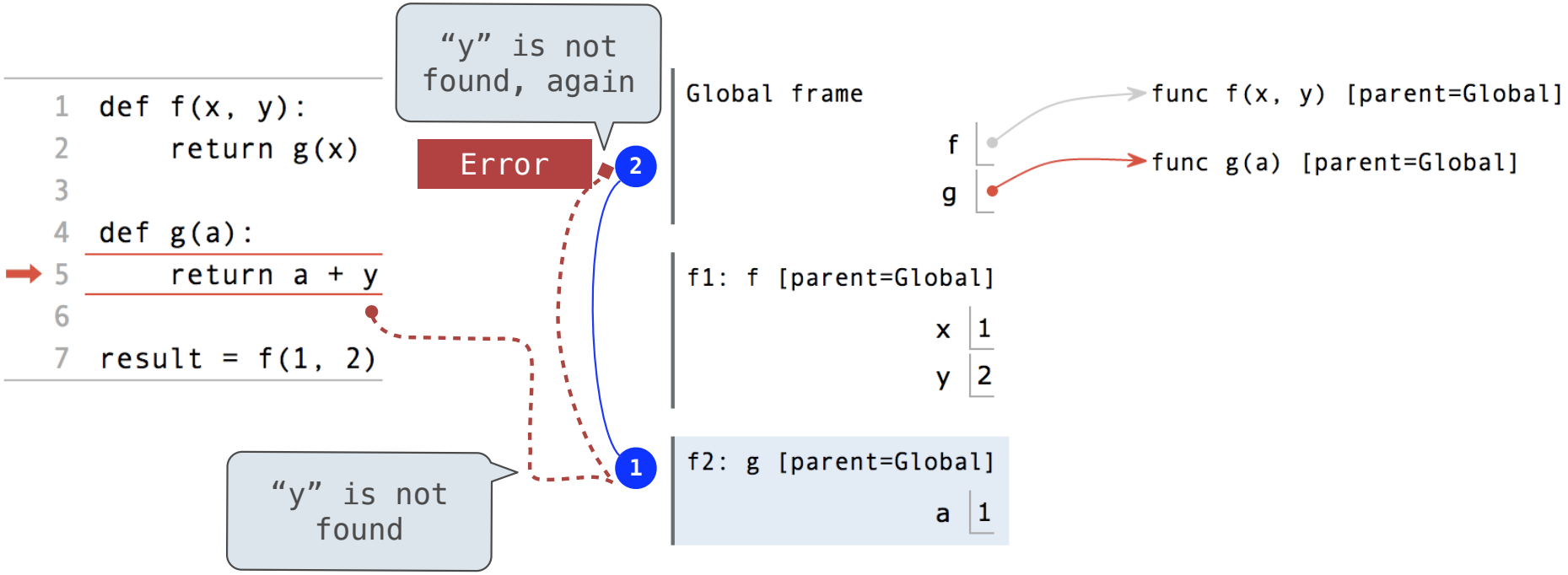

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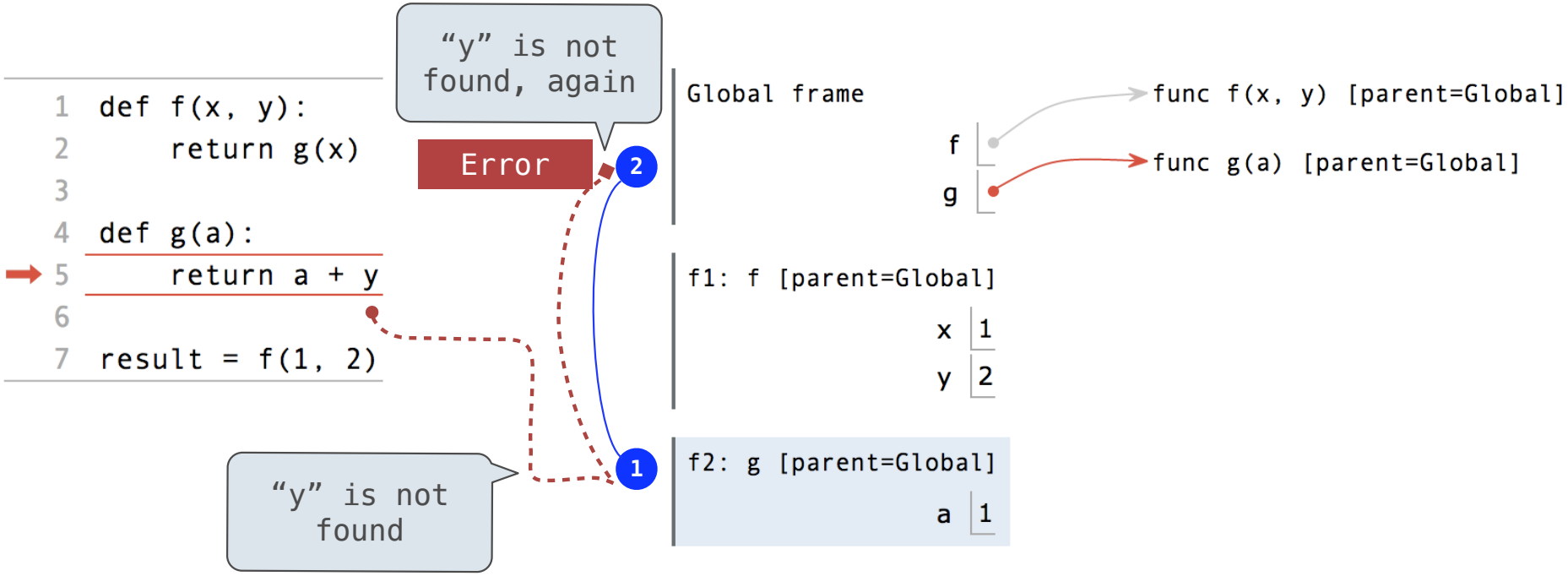

- An environment is a sequence of frames.

http://pythontutor.com/composingprograms.html#code=def%20f%28x,%20y%29%3A%0A%20%20%20%20return%20g%28x%29%0A%0Adef%20g%28a%29%3A%0A%20%20%20%20return%20a%20%2B%20y%0A%20%20%20%20%0Aresult%20%3D%20f%281,%202%29&cumulative=true&curInstr=0&mode=display&origin=composingprograms.js&py=3&rawInputLstJSON=%5B%5D

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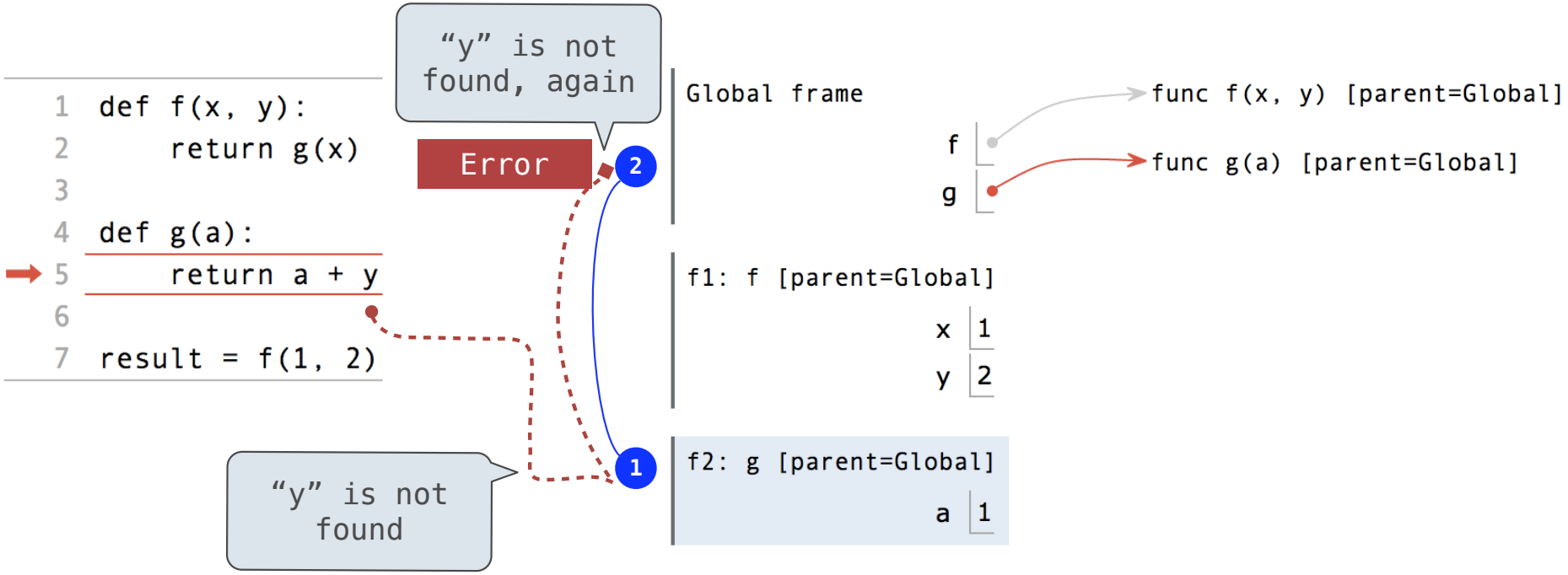

- An environment is a sequence of frames.
- The environment created by calling a top-level function (no def within def) consists of one local frame, followed by the global frame.

http://pythontutor.com/composingprograms.html#code=def%20f%28x,
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Function Composition

(Demo)

The Environment Diagram for Function Composition

```
1  def square(x):
2      return x * x
3
4  def make_adder(n):
5      def adder(k):
6          return k + n
7      return adder
8
9  def compose1(f, g):
10     def h(x):
11         return f(g(x))
12     return h
13
14 compose1(square, make_adder(2))(3)
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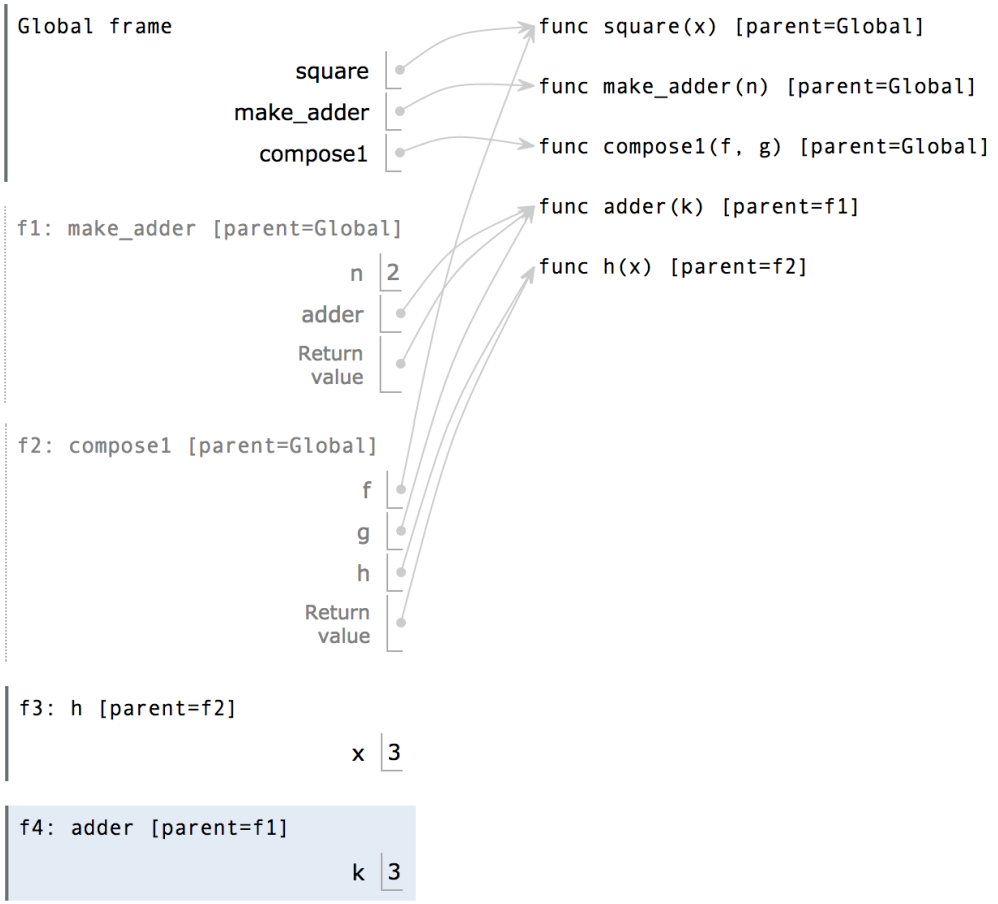

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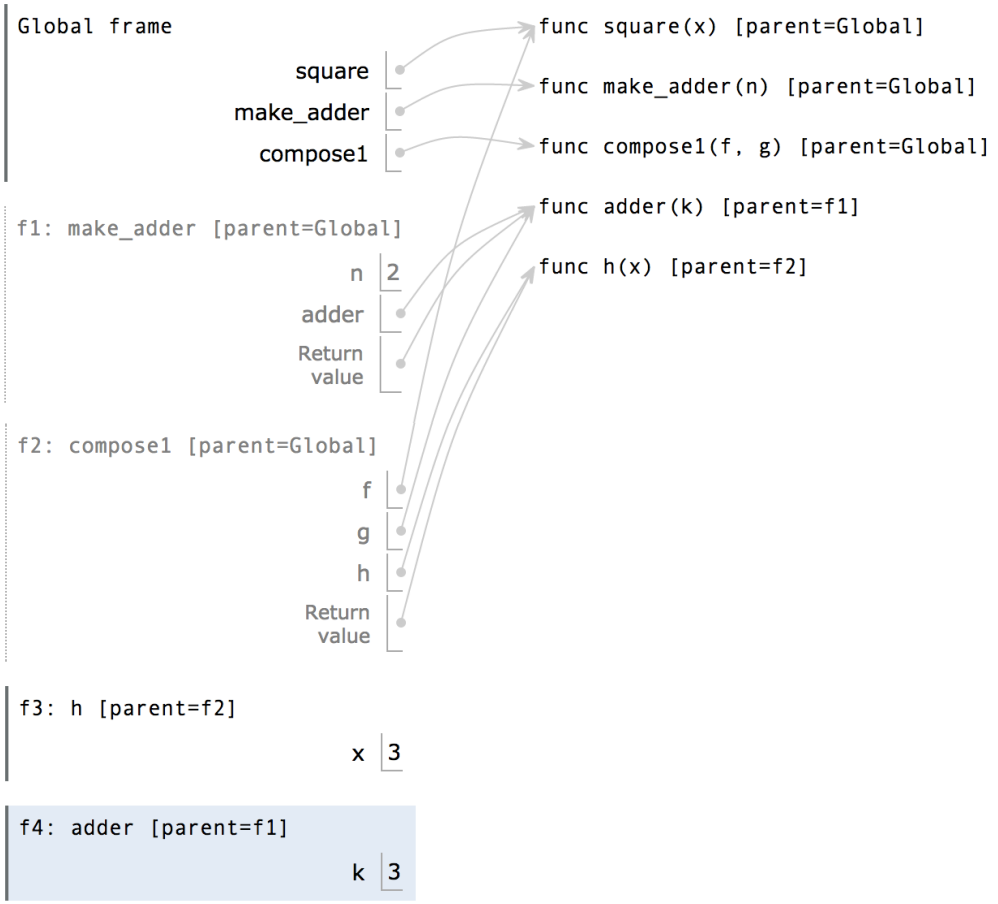

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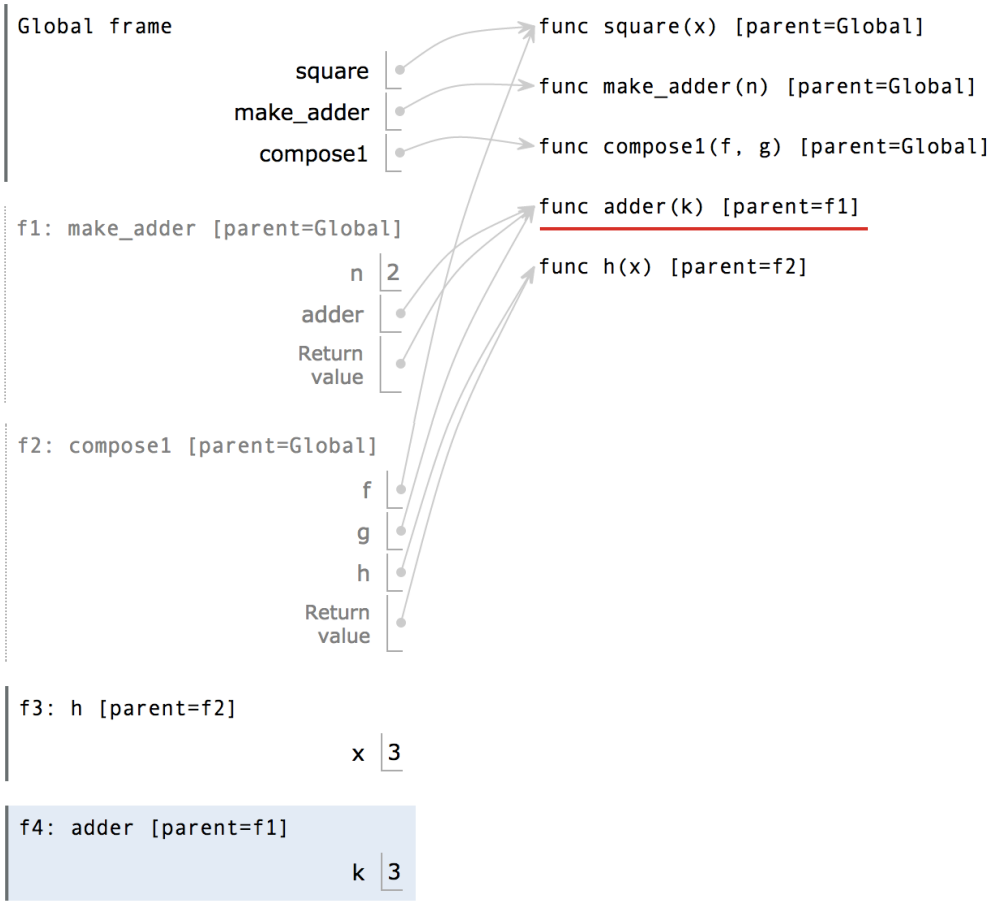

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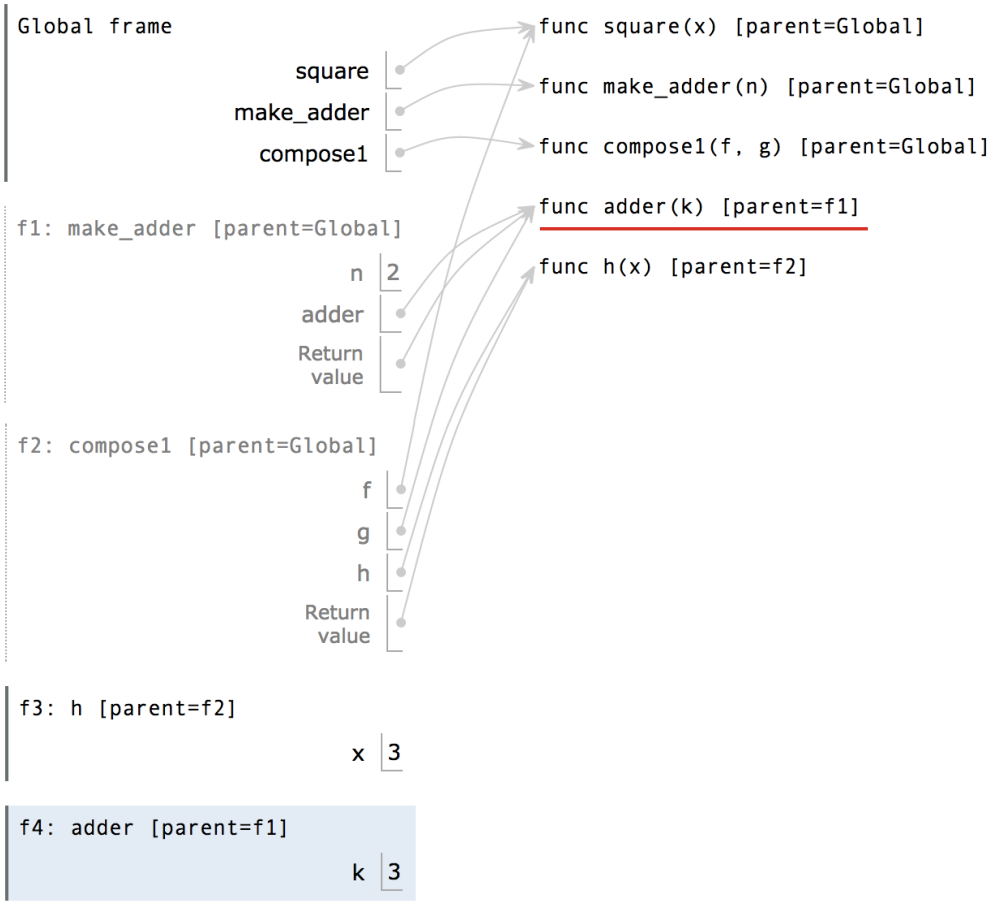

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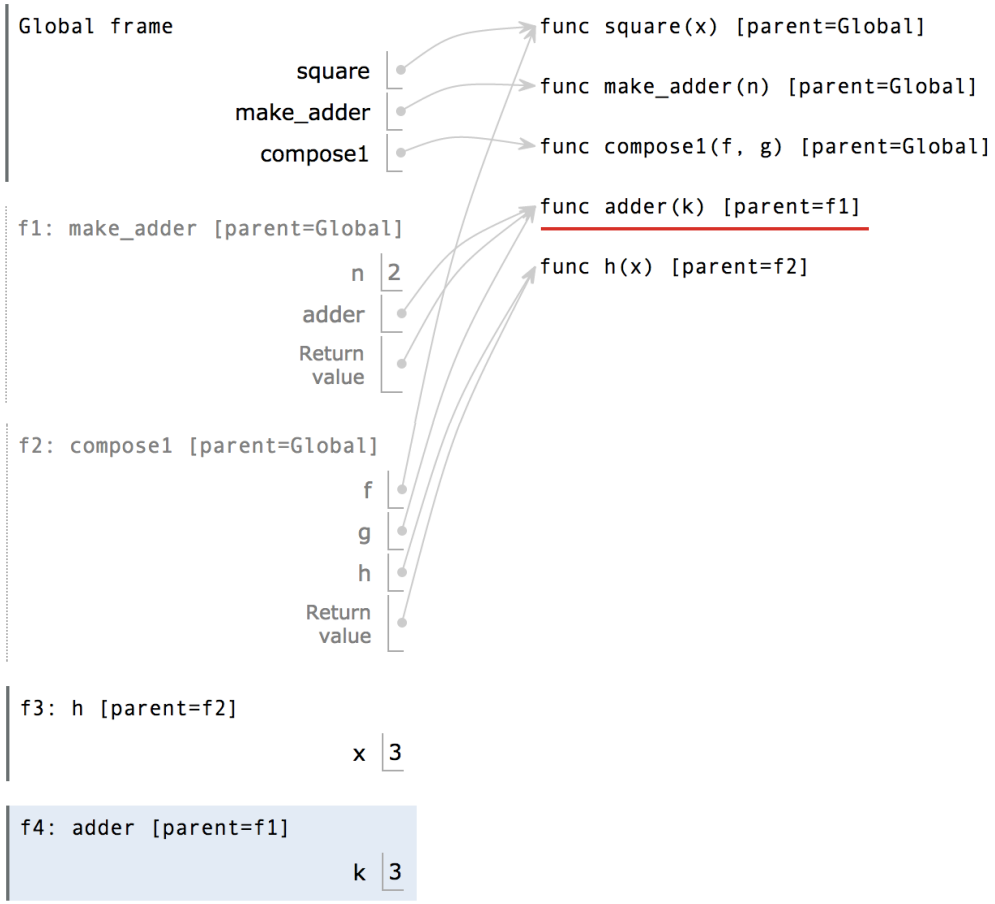
Return value of make_adder is an argument to compose1



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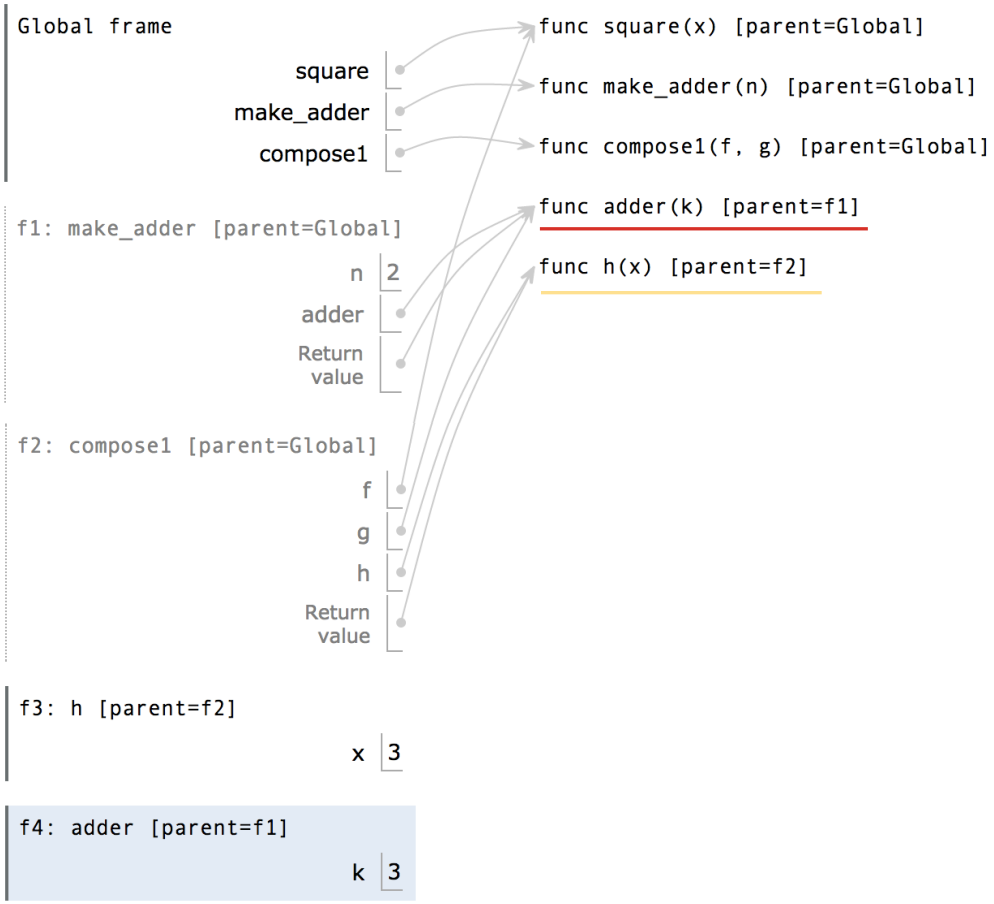

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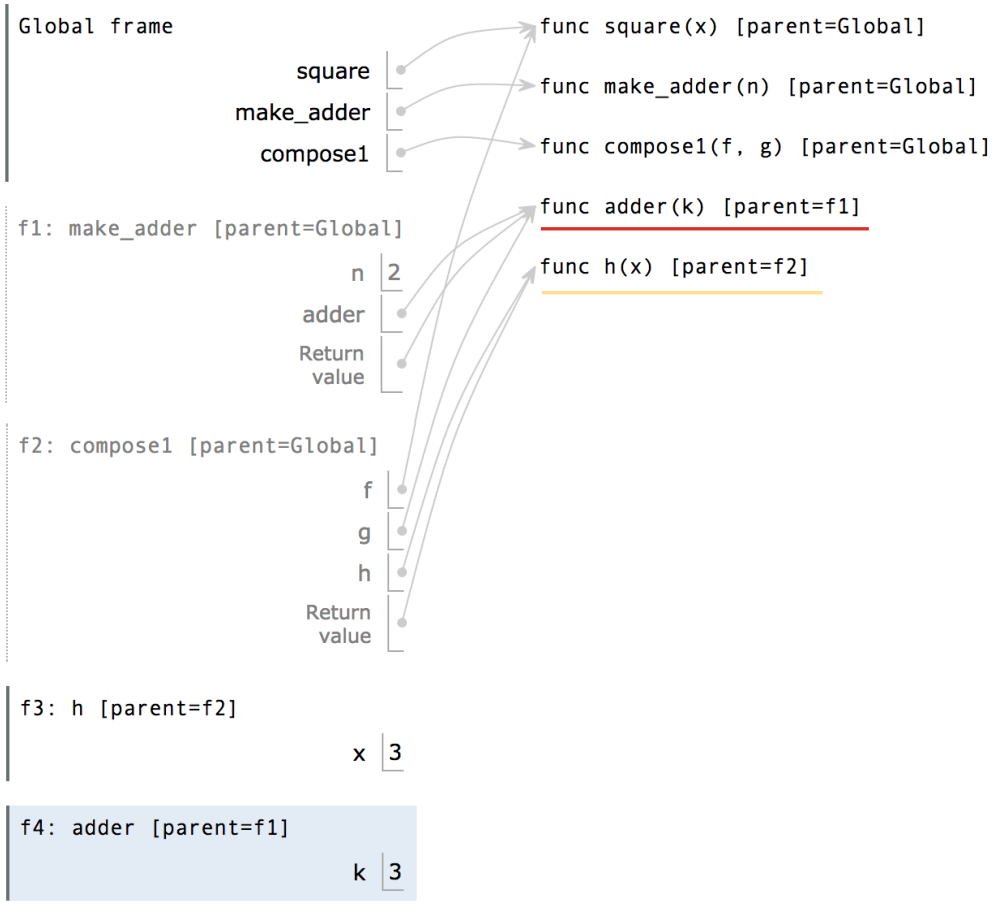

http://pythontutor.com/composingprograms.html#code=def%20square%28x%29%3A%0A%20%20%20%20return%20x%20*%20x%0A%0Adef%20make_adder%28n%29%3A%0A%20%20%20%20def%20adder%28k%29%3A%0A%20%20%20%20%20%20%20%20return%20k%20%2B%20n%0A%20%20%20%20return%20adder%0A%20%20%20%20%0Adef%20compose1%28f,%20g%29%3A%0A%20%20%20%20def%20h%28x%29%3A%0A%20%20%20%20%20%20%20%20return%20f%28g%28x%29%29%0A%20%20%20%20return%20h%0A%20%20%20%20%0Acompose1%28square,%20make_adder%282%29%29%283%29&cumulative=true&curInstr=0&mode=display&origin=composingprograms.js&py=3&rawInputLstJSON=%5B%5D

The Environment Diagram for Function Composition

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12     return h
13
14 compose1(square, make_adder(2))(3)
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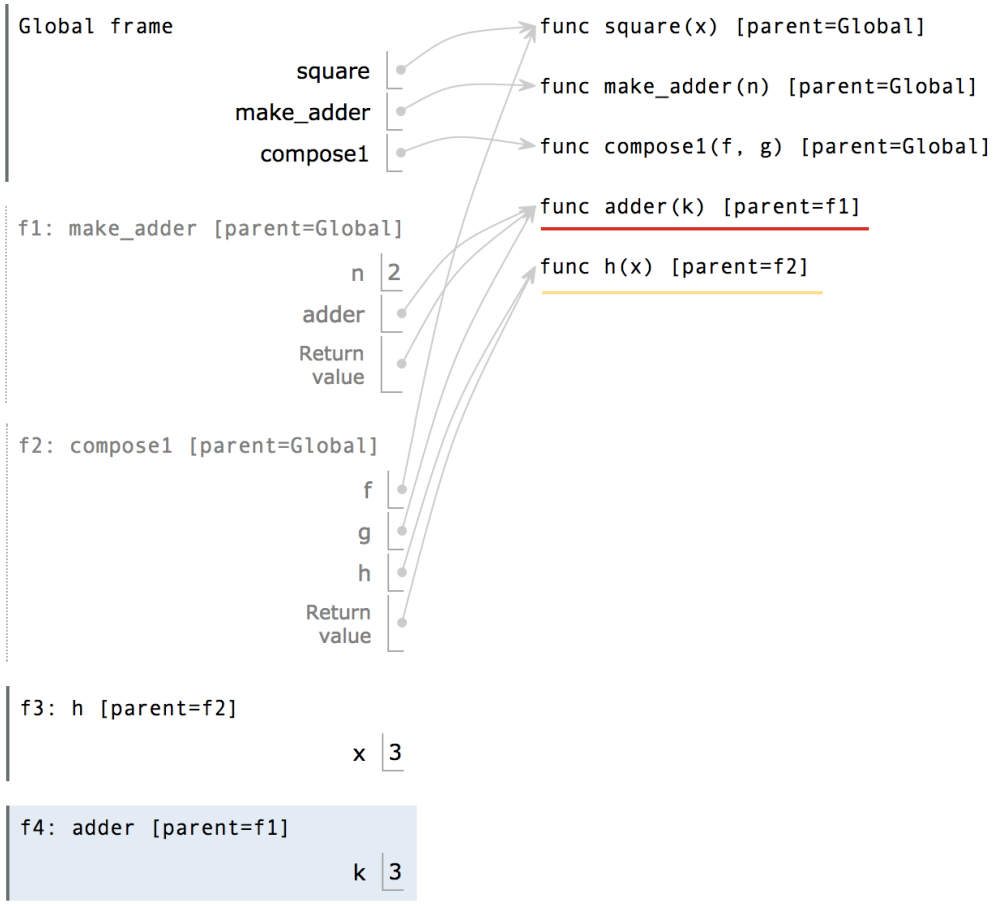

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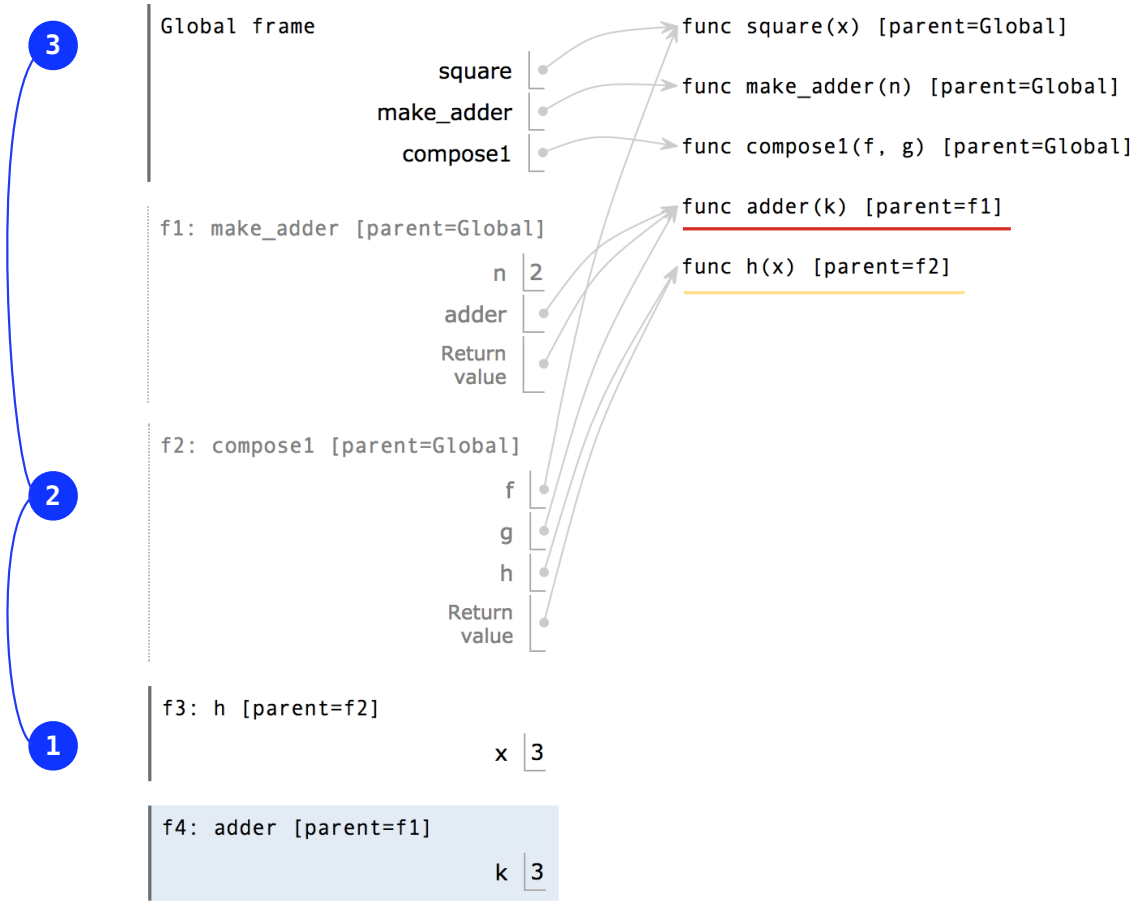

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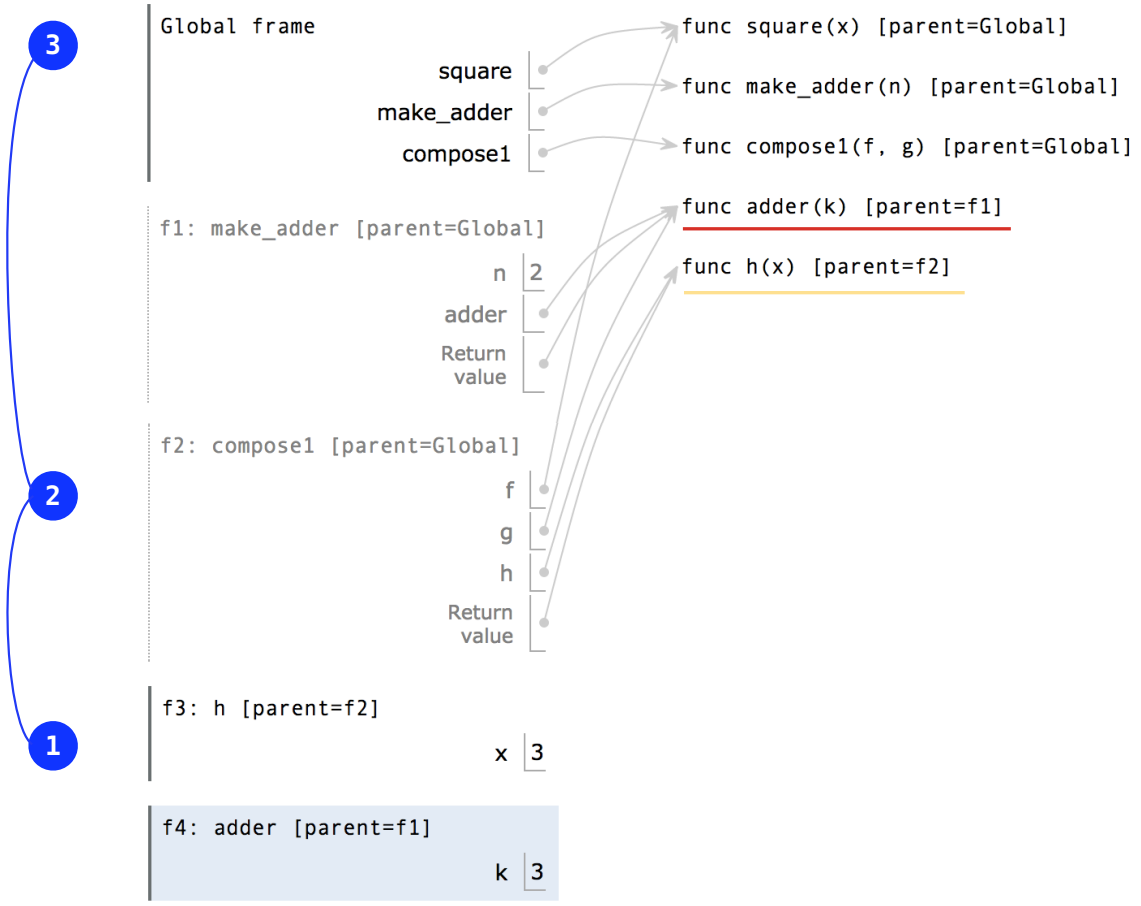

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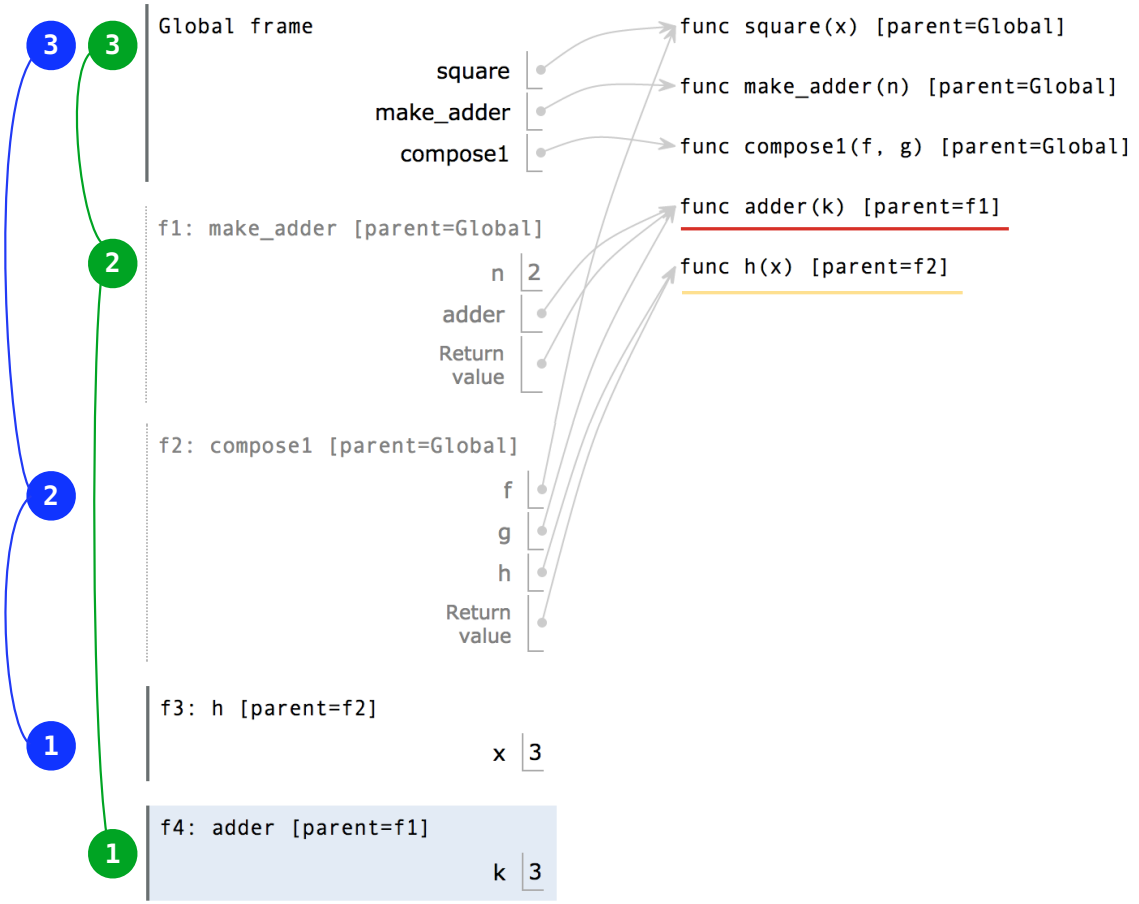

Lambda Expressions

(Demo)

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A function

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Lambda expressions in Python cannot contain statements at all!

Lambda Expressions Versus Def Statements

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VS

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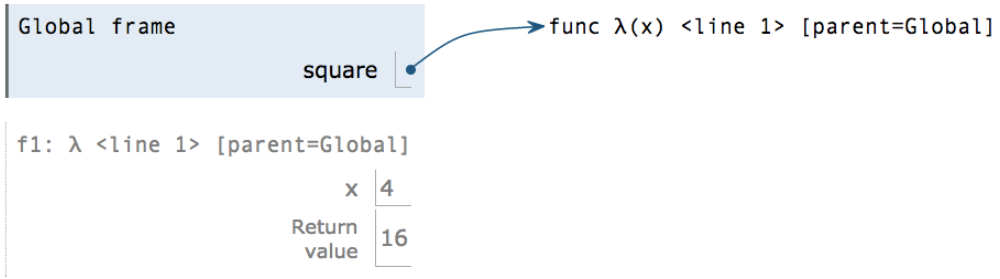

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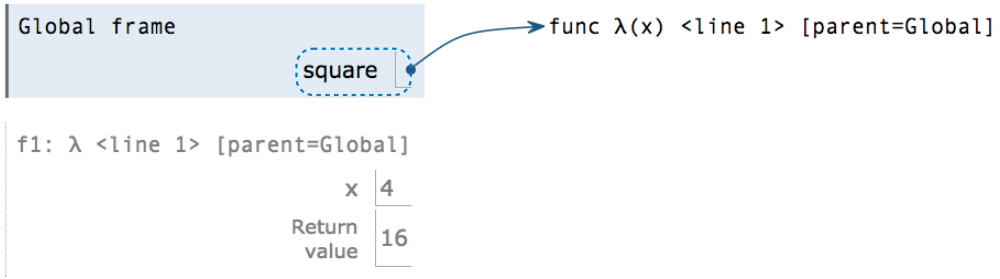

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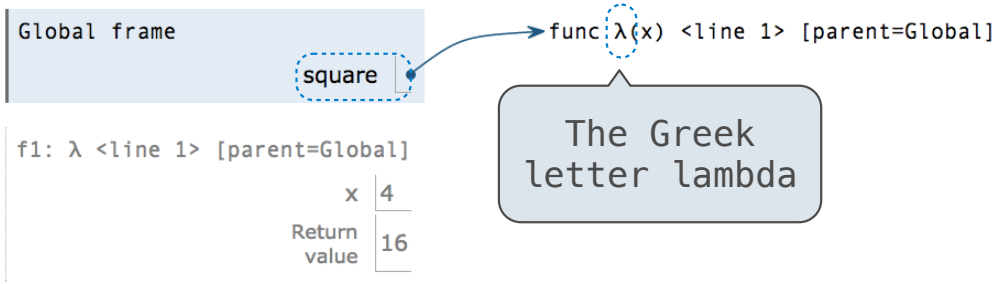

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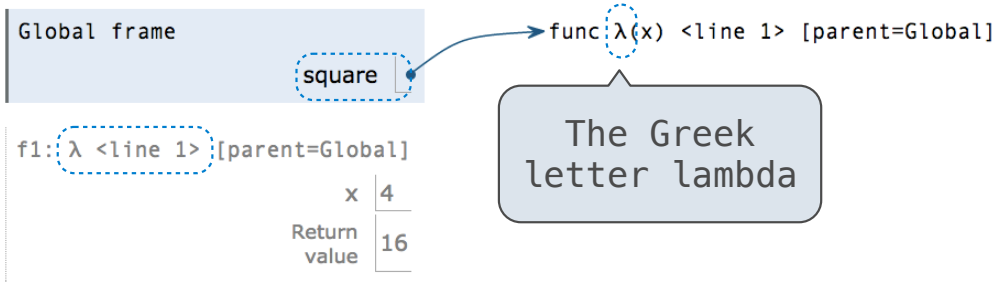

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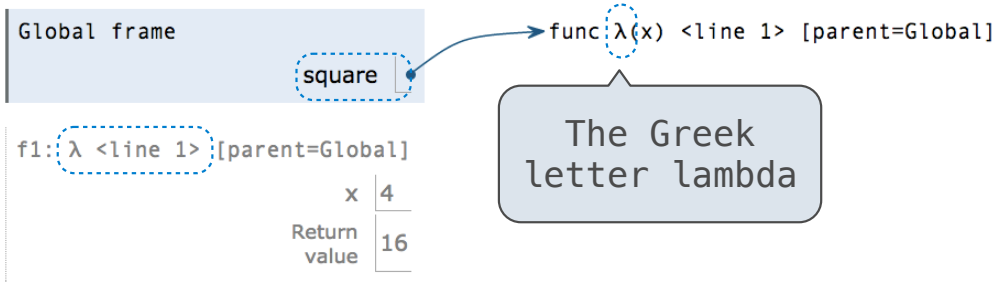

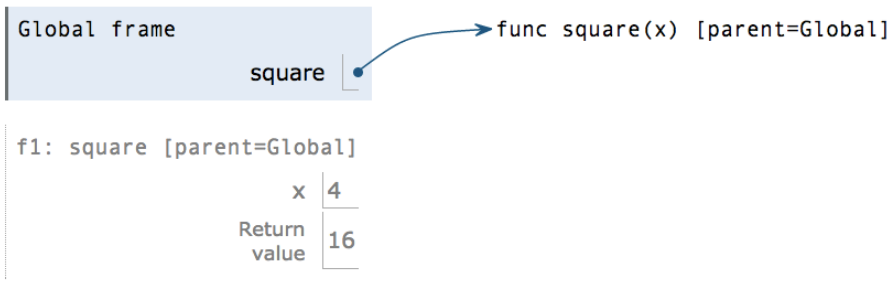

Currying

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def make_adder(n):
    return lambda k: n + k
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(Demo)

Curry: Transform a multi-argument function into a single-argument, higher-order function