CS 61A Fall 2020

Final Review

Discussion 14: December 2, 2020 Solutions

1 Recursion

1.1 (Adapted from Fall 2013) Fill in the blanks in the implementation of `paths`, which takes as input two positive integers `x` and `y`. It returns a list of paths, where each path is a list containing steps to reach `y` from `x` by repeated incrementing or doubling. For instance, we can reach 9 from 3 by incrementing to 4, doubling to 8, then incrementing again to 9, so one path is `[3, 4, 8, 9]`

```
def paths(x, y):
    """Return a list of ways to reach y from x by repeated
    incrementing or doubling.
    >>> paths(3, 5)
    [[3, 4, 5]]
    >>> sorted(paths(3, 6))
    [[3, 4, 5, 6], [3, 6]]
    >>> sorted(paths(3, 9))
    [[3, 4, 5, 6, 7, 8, 9], [3, 4, 8, 9], [3, 6, 7, 8, 9]]
    >>> paths(3, 3) # No calls is a valid path
    [[3]]
    """
    if ____________________________:

        return ____________________________________________

    elif ____________________________:

        return ____________________________________________

    else:

        a = ____________________________________________

        b = ____________________________________________

        return ____________________________________________
```

```
def paths(x, y):
    if x > y:
        return []
    elif x == y:
        return [[x]]
```

```
    else:
        a = paths(x + 1, y)
        b = paths(x * 2, y)
        return [[x] + subpath for subpath in a + b]
```

1.2 We will now write one of the faster sorting algorithms commonly used, known as *merge sort*. Merge sort works like this:

1. If there is only one (or zero) item(s) in the sequence, it is already sorted!
2. If there are more than one item, then we can split the sequence in half, sort each half recursively, then merge the results, using the `merge` procedure described below. The result will be a sorted sequence.

Using the algorithm described, write a function `mergesort(seq)` that takes an unsorted sequence and sorts it.

Recall the `merge` procedure is as follows:

```
def merge(s1, s2):
    """ Merges two sorted lists """
    if len(s1) == 0:
        return s2
    elif len(s2) == 0:
        return s1
    elif s1[0] < s2[0]:
        return [s1[0]] + merge(s1[1:], s2)
    else:
        return [s2[0]] + merge(s1, s2[1:])
```

```
def mergesort(seq):


    if len(seq) <= 1:
        return seq
    else:
        mid = len(seq) // 2
        return merge(mergesort(seq[:mid]),
            mergesort(seq[mid:]))
```

2 Trees

2.1 Implement `long_paths`, which returns a list of all *paths* in a tree with length at least `n`. A path in a tree is a linked list of node values that starts with the root and ends at a leaf. Each subsequent element must be from a child of the previous value's node. The *length* of a path is the number of edges in the path (i.e. one less than the number of nodes in the path). Paths are listed in order from left to right. See the doctests for some examples.

```
def long_paths(tree, n):
    """Return a list of all paths in tree with length at least n.

    >>> t = Tree(3, [Tree(4), Tree(4), Tree(5)])
    >>> left = Tree(1, [Tree(2), t])
    >>> mid = Tree(6, [Tree(7, [Tree(8)]), Tree(9)])
    >>> right = Tree(11, [Tree(12, [Tree(13, [Tree(14)])])])
    >>> whole = Tree(0, [left, Tree(13), mid, right])
    >>> for path in long_paths(whole, 2):
    ...     print(path)
    ...
    <0 1 2>
    <0 1 3 4>
    <0 1 3 4>
    <0 1 3 5>
    <0 6 7 8>
    <0 6 9>
    <0 11 12 13 14>
    >>> for path in long_paths(whole, 3):
    ...     print(path)
    ...
    <0 1 3 4>
    <0 1 3 4>
    <0 1 3 5>
    <0 6 7 8>
    <0 11 12 13 14>
    >>> long_paths(whole, 4)
    [Link(0, Link(11, Link(12, Link(13, Link(14)))))]
    """


    paths = []
    if n <= 0 and tree.is_leaf():
        paths.append(Link(tree.label))
    for b in tree.branches:
        for path in long_paths(b, n - 1):
            paths.append(Link(tree.label, path))
```

```
return paths
```

2.2 Write a function that takes a `Tree` object and returns the elements at the depth with the most elements.

In this problem, you may find it helpful to use the second optional argument to `sum`, which provides a starting value. All items in the sequence to be summed will be concatenated to the starting value. By default, start will default to 0, which allows you to sum a sequence of numbers. We provide an example of sum starting with a list, which allows you to concatenate items in a list.

```
def widest_level(t):
    """
    >>> sum([[1], [2]], [])
    [1, 2]
    >>> t = Tree(3, [Tree(1, [Tree(1), Tree(5)]),
    ...              Tree(4, [Tree(9, [Tree(2)])])])
    >>> widest_level(t)
    [1, 5, 9]
    """
    levels = []
    x = [t]

    while ________________________________________________:

        ____________________________________________________

        __________ = sum(_________________________________, [])

    return max(levels, key=_________________________________)
```

```
def widest_level(t):
    levels = []
    x = [t]
    while x:
        levels.append([t.label for t in x])
        x = sum([t.branches for t in x], [])
    return max(levels, key=len)
```

Main idea: we'll traverse each level of the tree and keep track of the elements of the levels. After we're done, we return the level with the most items.

Here, `x` keeps track of the trees in the current level. To get the next level of trees, we take all the branches from all the trees in the current level. The special `sum` call is needed to make sure we get a list of trees, instead of a list of branches (since branches are a list of trees themselves).

Finally, we use `max` with a key to select the list with the longest length from our list of levels.

3 Mutation

3.1 For each row below, fill in the blanks in the output displayed by the interactive Python interpreter when the expression is evaluated. Expressions are evaluated in order, and expressions may affect later expressions.

```
>>> cats = [1, 2]
>>> dogs = [cats, cats.append(23), list(cats)]
>>> cats
```

```
[1, 2, 23]
```

```
>>> dogs[1] = list(dogs)
>>> dogs[1]
```

```
[[1, 2, 23], None, [1, 2, 23]]
```

```
>>> dogs[0].append(2)
>>> cats
```

```
[1, 2, 23, 2]
```

```
>>> cats[1::2]
```

```
[2, 2]
```

```
>>> cats[:3]
```

```
[1, 2, 23]
```

```
>>> dogs[2].extend([list(cats).pop(0), 3])
>>> dogs[3]
```

```
Index Error
```

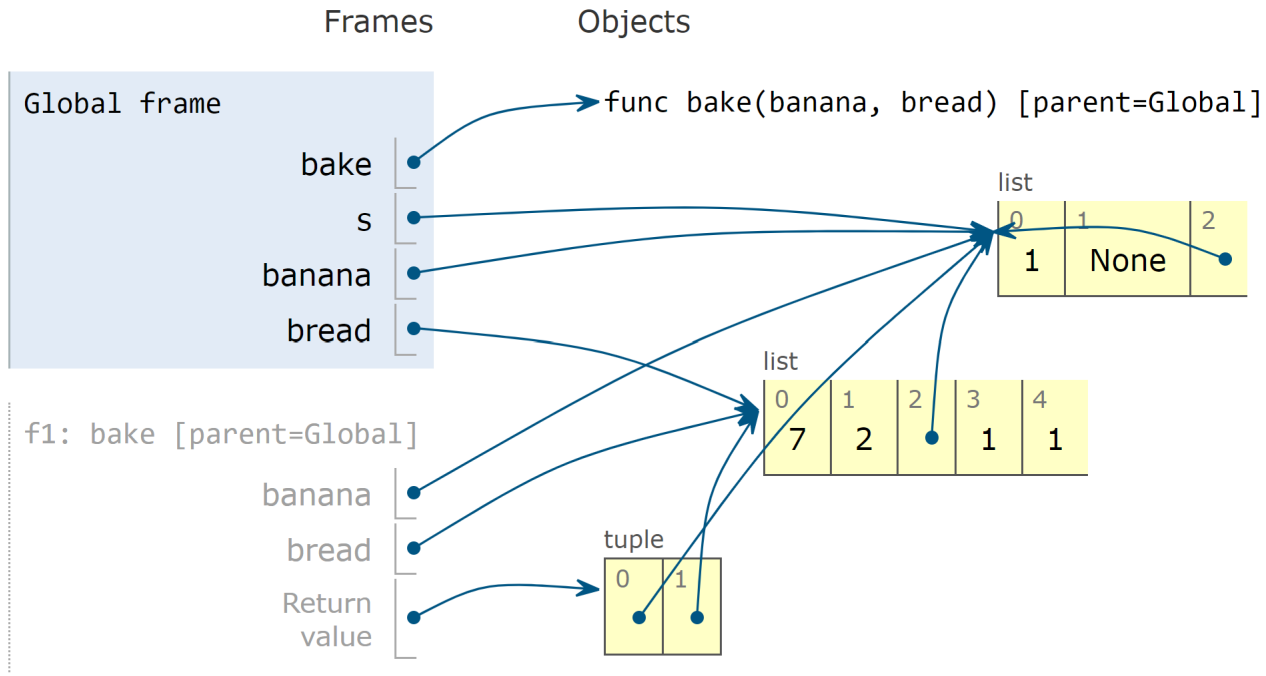
```
>>> dogs
```

```
[[1, 2, 23, 2], [[1, 2, 23, 2], None, [1, 2, 23, 1, 3]], [1, 2, 23, 1, 3]]
```

3.2 Fill in the following lines so that the code creates the environment diagram shown below. **You may only use** `append`, `extend`, `1`, `banana`, **and** `bread` **in your solution.**

```
def bake(banana, bread):
    _____(_____(_____)) # This line is Multiple Choice
    # Select all correct answers for the blank above
    # A. banana.append(bread.append(1))
    # B. bread.append(banana.append(1))
    # C. banana.extend([bread.extend([1])])
    # D. bread.extend([banana.extend([1])])
    bread += banana[: (len(_____) - _____)]
    banana._____(bread[_____[_____]])
    return _____, _____
```

```
s = [1]
banana, bread = bake(s, [7, 2, s])
```




```
def bake(banana, bread):
    banana.append(bread.append(1))
    # or banana.extend([bread.extend([1])])
    bread += banana[: (len(banana) - 1)]
    banana.append(bread[bread[1]])
    return banana, bread
```

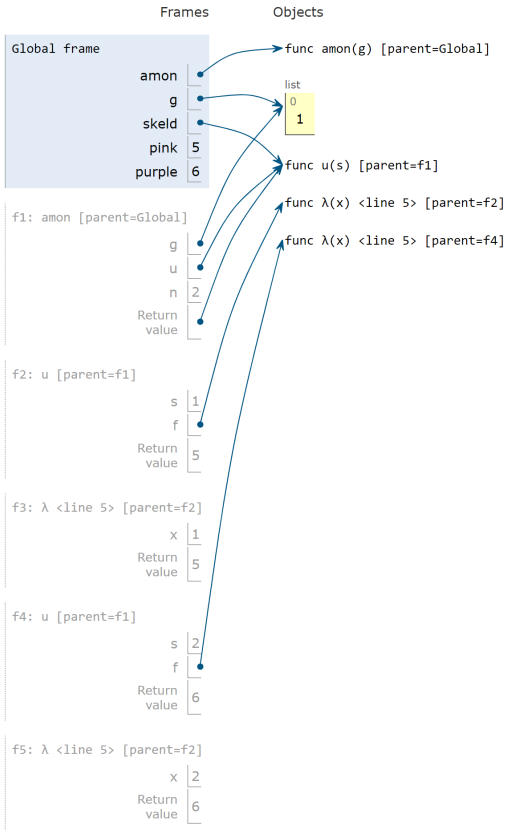
Note: This worksheet is a problem bank—most TAs will not cover all the problems in discussion section.

```
s = [1]
banana, bread = bake(s, [7, 2, s])
```

3.3 Fill in the following lines so that the code creates the environment diagram shown below.

```
def amon(g):
    ____________________
           (a)
    def u(s):
        ______________________
              (b)
        f = lambda x: x + g._________ + n
                                (c)
        ________________
               (d)
        return f(s)
    return u

g = [1, 2, 3]
skeld = amon(g)
pink = skeld(1)
purple = skeld(2)
```




```
def amon(g):
    n = 0
    def u(s):
```

Note: This worksheet is a problem bank—most TAs will not cover all the problems in discussion section.

```
        nonlocal n
        f = lambda x: x + g.pop() + n
        n += 1
        return f(s)
    return u

g = [1, 2, 3]
skeld = amon(g)
pink = skeld(1)
purple = skeld(2)
```

4 OOP

4.1 Fill in the classes `Emotion`, `Joy`, and `Sadness` below so that you get the following output from the Python interpreter.

```
>>> Emotion.num
0
>>> joy = Joy()
>>> sadness = Sadness()
>>> Emotion.num # number of Emotion instances created
2
>>> joy.power
5
>>> joy.catchphrase() # Print Joy's catchphrase
Think positive thoughts
>>> sadness.catchphrase() #Print Sad's catchphrase
I'm positive you will get lost
>>> sadness.power
5
>>> joy.feeling(sadness) # When both Emotions have same power value, print "Together"
Together
>>> sadness.feeling(joy)
Together
>>> joy.power = 7
>>> joy.feeling(sadness) # Print the catchphrase of the more powerful feeling before the less
    powerful feeling
Think positive thoughts
I'm positive you will get lost
>>> sadness.feeling(joy)
Think positive thoughts
I'm positive you will get lost
```

```
class Emotion(_______):
```

```
class Emotion(object):
    num = 0

    def __init__(self):

        self.power = 5
        Emotion.num += 1


    def feeling(self, other):

        if self.power > other.power:
            self.catchphrase()
            other.catchphrase()
        elif other.power > self.power:
            other.catchphrase()
            self.catchphrase()
        else:
            print("Together")
```

```
class Joy(_______):
```

```
class Joy(Emotion):

    def catchphrase(self):

        print("Think positive thoughts!")
```

```
class Sadness(_______):
```

```
class Sadness(Emotion):

    def catchphrase(self):

        print("I'm positive you will get lost.")
```

5 Mutable Linked Lists and Trees

5.1 Write a function that takes a sorted linked list of integers and mutates it so that all duplicates are removed.

```
def remove_duplicates(lnk):
    """
    >>> lnk = Link(1, Link(1, Link(1, Link(1, Link(5)))))
    >>> remove_duplicates(lnk)
    >>> lnk
    Link(1, Link(5))
    """
```

Recursive solution:

```
    if lnk is Link.empty or lnk.rest is Link.empty:
        return
    if lnk.first == lnk.rest.first:
        lnk.rest = lnk.rest.rest
        remove_duplicates(lnk)
    else:
        remove_duplicates(lnk.rest)
```

For a list of one or no items, there are no duplicates to remove.

Now consider two possible cases:

- If there is a duplicate of the first item, we will find that the first and second items in the list will have the same values (that is, `lnk.first == lnk.rest.first`). We can confidently state this because we were told that the input linked list is in sorted order, so duplicates are adjacent to each other. We'll remove the second item from the list.

  Finally, it's tempting to recurse on the remainder of the list (`lnk.rest`), but remember that there could still be more duplicates of the first item in the rest of the list! So we have to recurse on `lnk` instead. Remember that we have removed an item from the list, so the list is one element smaller than before. Normally, recursing on the same list wouldn't be a valid subproblem.

- Otherwise, there is no duplicate of the first item. We can safely recurse on the remainder of the list.

Iterative solution:

```
    while lnk is not Link.empty and lnk.rest is not Link.empty:
        if lnk.first == lnk.rest.first:
            lnk.rest = lnk.rest.rest
        else:
            lnk = lnk.rest
```

The loop condition guarantees that we have at least one item left to consider with `lnk`.

For each item in the linked list, we pause and remove all adjacent items that have the same value. Once we see that `lnk.first != lnk.rest.first`, we can safely advance to the next item. Once again, this takes advantage of the property that our input linked list is sorted.

6 Generators

6.1 Write a generator function that yields functions that are repeated applications of a one-argument function `f`. The first function yielded should apply `f` 0 times (the identity function), the second function yielded should apply `f` once, etc.

```
def repeated(f):
    """
    >>> double = lambda x: 2 * x
    >>> funcs = repeated(double)
    >>> identity = next(funcs)
    >>> double = next(funcs)
    >>> quad = next(funcs)
    >>> oct = next(funcs)
    >>> quad(1)
    4
    >>> oct(1)
    8
    >>> [g(1) for _, g in
    ...  zip(range(5), repeated(lambda x: 2 * x))]
    [1, 2, 4, 8, 16]
    """

    g = ____________________________________________________________________

    while True:

        ____________________________________________________________________

        ____________________________________________________________________
```

```
def repeated(f):
    g = lambda x: x
    while True:
        yield g
        g = (lambda g: lambda x: f(g(x)))(g)
```

Video walkthrough

6.2 Ben Bitdiddle proposes the following alternate solution. Does it work?

```
def ben_repeated(f):
    g = lambda x: x
```

```
while True:
    yield g
    g = lambda x: f(g(x))
```

This solution does not work. The value g changes with each iteration so the bodies of the lambdas yielded change as well.

6.3 Implement `accumulate`, which takes in an `iterable` and a function `f` and yields each accumulated value from applying `f` to the running total and the next element.

```
from operator import add, mul

def accumulate(iterable, f):
    """
    >>> list(accumulate([1, 2, 3, 4, 5], add))
    [1, 3, 6, 10, 15]
    >>> list(accumulate([1, 2, 3, 4, 5], mul))
    [1, 2, 6, 24, 120]
    """
    it = iter(iterable)

    ____________________________________________________________________________

    ____________________________________________________________________________

    for ____________________________________________________________________________:

        ____________________________________________________________________________

        ____________________________________________________________________________
```

```
    total = next(it)
    yield total
    for element in it:
        total = f(total, element)
        yield total
```

7 Scheme

7.1 Write a function that takes a procedure and applies to every element in a given nested list.

The result should be a nested list with the same structure as the input list, but with each element replaced by the result of applying the procedure to that element.

Use the built-in `list?` procedure to detect whether a value is a list.

```
(define (deep-map fn lst)


  (cond ((null? lst) lst)
        ((list? (car lst)) (cons (deep-map fn (car lst)) (deep-map fn (cdr lst))))
        (else (cons (fn (car lst)) (deep-map fn (cdr lst))))
        )
   )
```

```
scm> (deep-map (lambda (x) (* x x)) '(1 2 3))
(1 4 9)
scm> (deep-map (lambda (x) (* x x)) '(1 ((4) 5) 9))
(1 ((16) 25) 81)
```

8 SQL

(Adapted from Fall 2019) The scoring table has three columns, a player column of strings, a points column of integers, and a quarter column of integers. The players table has two columns, a name column of strings and a team column of strings. Complete the SQL statements below so that they would compute the correct result even if the rows in these tables were different than those shown. Important: You may write anything in the blanks including keywords such as WHERE or ORDER BY. Use the following tables for the questions below:

```
CREATE TABLE scoring AS
    SELECT "Donald Stewart" AS player, 7 AS points, 1 AS quarter UNION
    SELECT "Christopher Brown Jr.", 7, 1 UNION
    SELECT "Ryan Sanborn", 3, 2 UNION
    SELECT "Greg Thomas", 3, 2 UNION
    SELECT "Cameron Scarlett", 7, 3 UNION
    SELECT "Nikko Remigio", 7, 4 UNION
    SELECT "Ryan Sanborn", 3, 4 UNION
    SELECT "Chase Garbers", 7, 4;

CREATE TABLE players AS
    SELECT "Ryan Sanborn" AS name, "Stanford" AS team UNION
    SELECT "Donald Stewart", "Stanford" UNION
    SELECT "Cameron Scarlett", "Stanford" UNION
    SELECT "Christopher Brown Jr.", "Cal" UNION
    SELECT "Greg Thomas", "Cal" UNION
    SELECT "Nikko Remigio", "Cal" UNION
    SELECT "Chase Garbers", "Cal";
```

8.1 Write a SQL statement to select a one-column table of quarters in which more than 10 total points were scored.

```
SELECT quarter FROM scoring GROUP BY quarter HAVING SUM(points) > 10;
```

8.2 Write a SQL statement to select a two-column table of the points scored by each team. Assume that no two players have the same name.

```
SELECT team, SUM(points) FROM scoring, players WHERE player=name GROUP BY team;
```