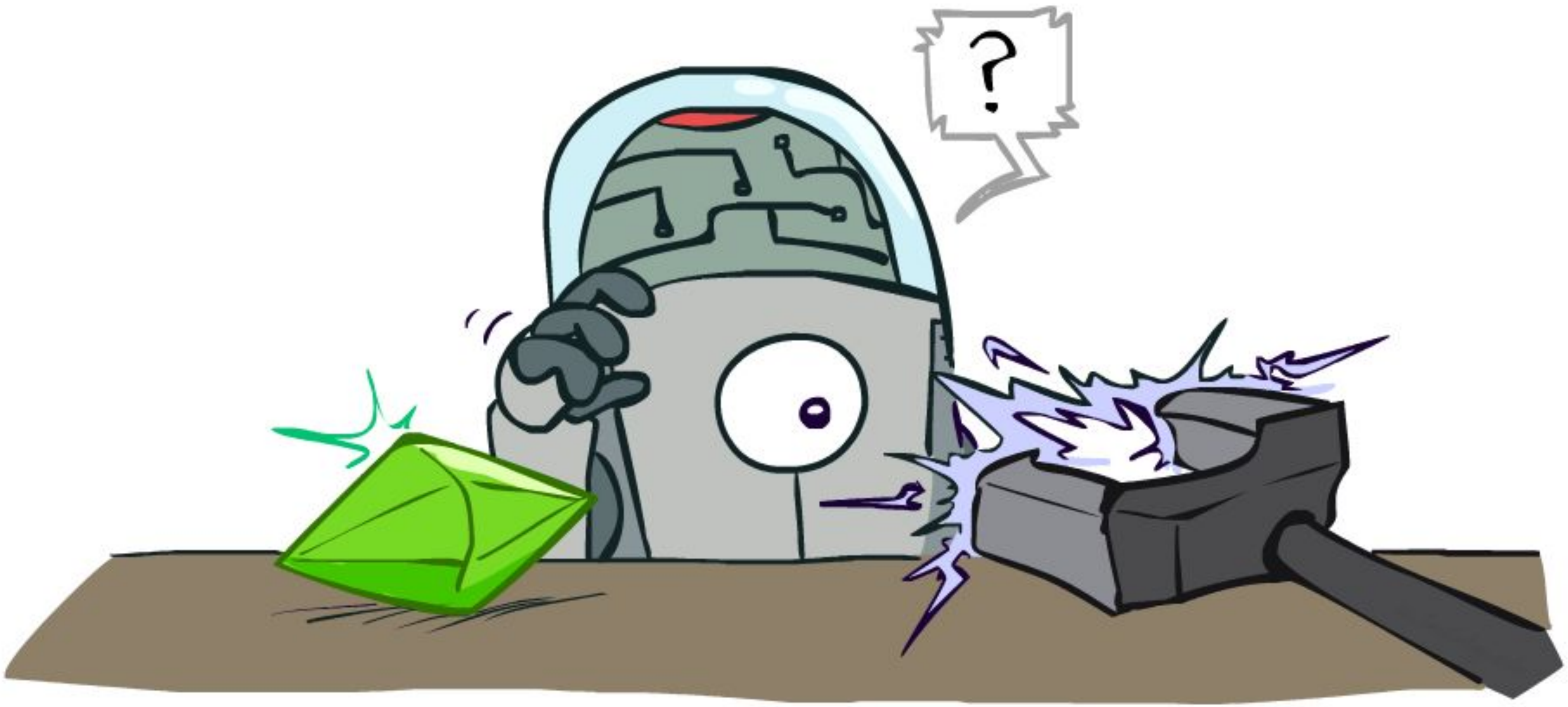
CS 188: Artificial Intelligence

Neural Networks II



Instructor: Stuart Russell and Dawn Song

University of California, Berkeley

Recap: Bayesian learning

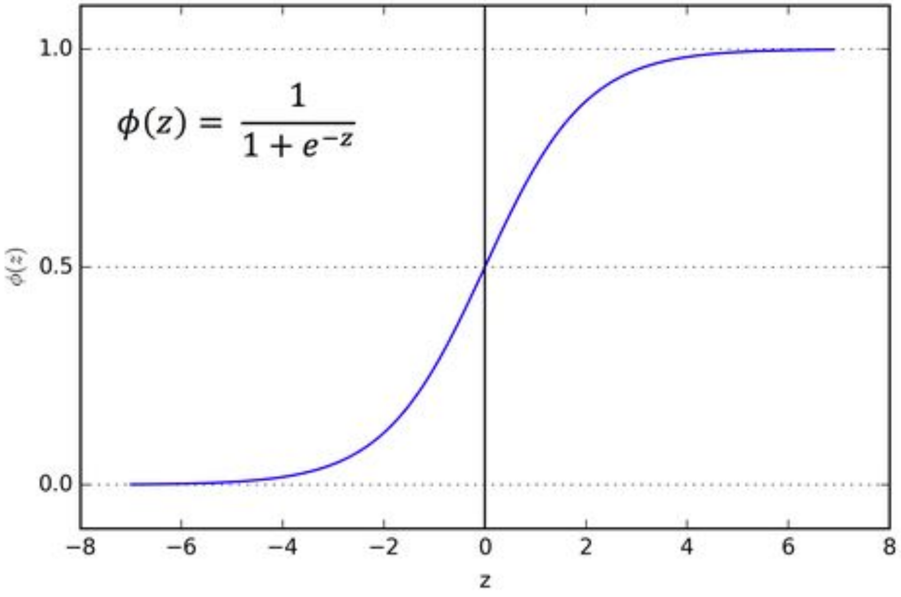
- Prior $P(H)$, training data $\mathbf{X}=\mathbf{x}_1,\ldots,\mathbf{x}_N$
- Given the data so far, each hypothesis has a posterior probability:
  - $P(h_k \mid \mathbf{X}) = \alpha\, P(\mathbf{X} \mid h_k)P(h_k)$ = α x Likelihood x Prior
- Predictions use a likelihood-weighted average over the hypotheses:
  - $P(\mathbf{x}_{N+1} \mid \mathbf{X}) = \Sigma_k\, P(\mathbf{x}_{N+1} \mid \mathbf{X},h_k)P(h_k \mid \mathbf{X}) = \Sigma_k\, P(\mathbf{x}_{N+1} \mid h_k)P(h_k \mid \mathbf{X})$
- No need to pick one best-guess hypothesis!
  - Drawback: Σ_k may be expensive/impossible for large/infinite H

Recap: Logistic Regression

- If $z = w \cdot f(x)$ very positive, then want probability going to 1
- If $z = w \cdot f(x)$ very negative, then want probability going to 0

- Sigmoid function

$$\phi(z) = \frac{1}{1 + e^{-z}}$$



Recap: Maximum Likelihood Estimation for Logistic Regression

- Maximum likelihood estimation:

$$\max_w \quad ll(w) = \max_w \quad \sum_i \log P(y^{(i)}|x^{(i)}; w)$$

with:

$$P(y^{(i)} = +1|x^{(i)}; w) = \frac{1}{1 + e^{-w \cdot f(x^{(i)})}}$$

$$P(y^{(i)} = -1|x^{(i)}; w) = 1 - \frac{1}{1 + e^{-w \cdot f(x^{(i)})}}$$

Recap: Gradient Ascent

- Perform update in uphill direction for each coordinate
- The steeper the slope (i.e. the higher the derivative) the bigger the step for that coordinate
- E.g., consider: $g(w_1, w_2)$
  - Updates:

$$w_1 \leftarrow w_1 + \alpha * \frac{\partial g}{\partial w_1}(w_1, w_2)$$

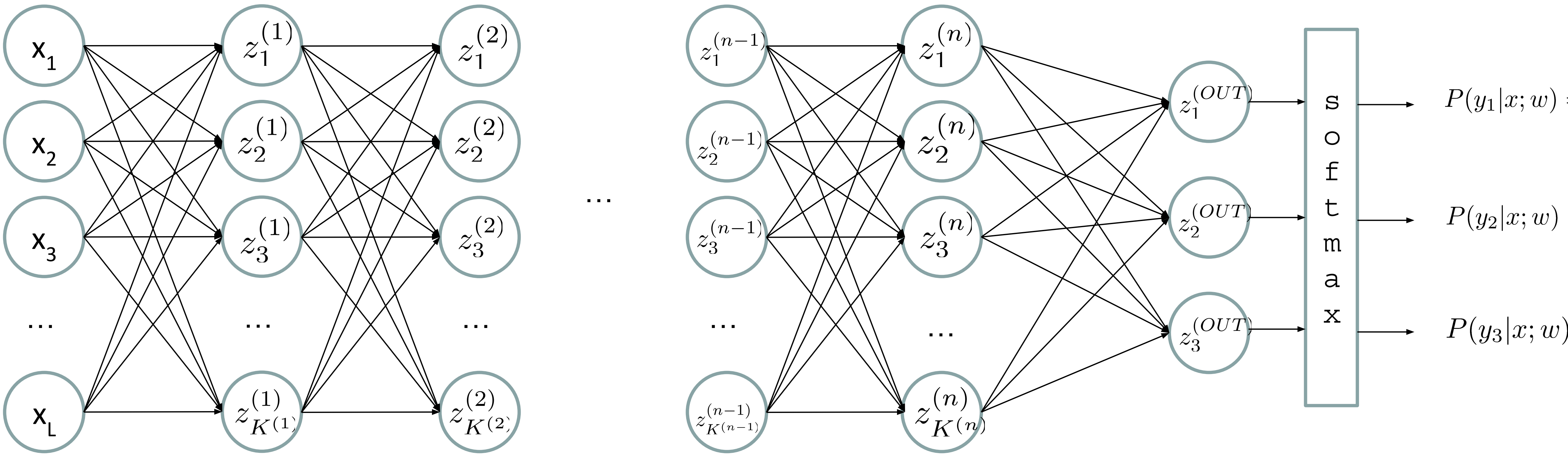
$$w_2 \leftarrow w_2 + \alpha * \frac{\partial g}{\partial w_2}(w_1, w_2)$$

- Updates in vector notation:

$$w \leftarrow w + \alpha * \nabla_w g(w)$$

with: $$\nabla_w g(w) = \begin{bmatrix} \frac{\partial g}{\partial w_1}(w) \\ \frac{\partial g}{\partial w_2}(w) \end{bmatrix}$$ **= gradient**

Recap: Neural Networks



$$z_i^{(k)} = g(\sum_j W_{i,j}^{(k-1,k)} z_j^{(k-1)})$$

g = nonlinear activation function

Neural Networks Properties

- Theorem (Universal Function Approximators). A two-layer neural network with a sufficient number of neurons can approximate any continuous function to any desired accuracy.

- Practical considerations
  - Can be seen as learning the features
  - Large number of neurons
    - Danger for overfitting

Universal Function Approximation Theorem*

Hornik theorem 1: Whenever the activation function is *bounded and nonconstant*, then, for any finite measure μ, standard multilayer feedforward networks can approximate any function in $L^p(\mu)$ (the space of all functions on R^k such that $\int_{R^k} |f(x)|^p d\mu(x) < \infty$) arbitrarily well, provided that sufficiently many hidden units are available.

Hornik theorem 2: Whenever the activation function is *continuous, bounded and nonconstant*, then, for arbitrary compact subsets $X \subseteq R^k$, standard multilayer feedforward networks can approximate any continuous function on X arbitrarily well with respect to uniform distance, provided that sufficiently many hidden units are available.

- In words: Given any continuous function f(x), if a 2-layer neural network has enough hidden units, then there is a choice of weights that allow it to closely approximate f(x).

Cybenko (1989) “Approximations by superpositions of sigmoidal functions”
Hornik (1991) “Approximation Capabilities of Multilayer Feedforward Networks”
Leshno and Schocken (1991) ”Multilayer Feedforward Networks with Non-Polynomial Activation Functions Can Approximate Any Function”

Universal Function Approximation Theorem*

Math. Control Signals Systems (1989) 2: 303–314

Mathematics of Control, Signals, and Systems

Approximation by Superpositions of a Sigmoidal Function*

G. Cybenko†

Abstract. In this paper we demonstrate that finite linear combinations of compositions of a fixed, univariate function and a set of affine functionals can uniformly approximate any continuous function of n real variables with support in the unit hypercube; only mild conditions are imposed on the univariate function. Our results settle an open question about representability in the class of single hidden layer neural networks. In particular, we show that arbitrary decision regions can be arbitrarily well approximated by continuous feedforward neural networks with only a single internal, hidden layer and any continuous sigmoidal nonlinearity. The paper discusses approximation properties of other possible types of nonlinearities that might be implemented by artificial neural networks.

Key words. Neural networks, Approximation, Completeness.

1. Introduction

A number of diverse application areas are concerned with the representation of general functions of an n-dimensional real variable, $x \in \mathbb{R}^n$, by finite linear combinations of the form

$$\sum_{j=1}^{N} \alpha_j \sigma(y_j^{\mathrm{T}} x + \theta_j), \tag{1}$$

where $y_j \in \mathbb{R}^n$ and $\alpha_j, \theta \in \mathbb{R}$ are fixed. (y^{T} is the transpose of y so that $y^{\mathrm{T}}x$ is the inner product of y and x.) Here the univariate function σ depends heavily on the context of the application. Our major concern is with so-called sigmoidal σ's:

$$\sigma(t) \to \begin{cases} 1 & \text{as } t \to +\infty, \\ 0 & \text{as } t \to -\infty. \end{cases}$$

Such functions arise naturally in neural network theory as the activation function of a neural node (or *unit* as is becoming the preferred term) [L1], [RHM]. The main result of this paper is a demonstration of the fact that sums of the form (1) are dense in the space of continuous functions on the unit cube if σ is any continuous sigmoidal

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ORIGINAL CONTRIBUTION

Approximation Capabilities of Multilayer Feedforward Networks

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Technische Universität Wien, Vienna, Austria

(*Received 30 January 1990; revised and accepted 25 October 1990*)

Abstract—*We show that standard multilayer feedforward networks with as few as a single hidden layer and arbitrary bounded and nonconstant activation function are universal approximators with respect to $L^p(\mu)$ performance criteria, for arbitrary finite input environment measures μ, provided only that sufficiently many hidden units are available. If the activation function is continuous, bounded and nonconstant, then continuous mappings can be learned uniformly over compact input sets. We also give very general conditions ensuring that networks with sufficiently smooth activation functions are capable of arbitrarily accurate approximation to a function and its derivatives.*

Keywords—Multilayer feedforward networks, Activation function, Universal approximation capabilities, Input environment measure, $L^p(\mu)$ approximation, Uniform approximation, Sobolev spaces, Smooth approximation.

1. INTRODUCTION

The approximation capabilities of neural network architectures have recently been investigated by many authors, including Carroll and Dickinson (1989), Cybenko (1989), Funahashi (1989), Gallant and White (1988), Hecht-Nielsen (1989), Hornik, Stinchcombe, and White (1989, 1990), Irie and Miyake (1988), Lapedes and Farber (1988), Stinchcombe and White (1989, 1990). (This list is by no means complete.)

If we think of the network architecture as a rule for computing values at l output units given values at k input units, hence implementing a class of mappings from R^k to R^l, we can ask how well arbitrary mappings from R^k to R^l can be approximated by the network, in particular, if as many hidden units as required for internal representation and computation may be employed.

How to measure the accuracy of approximation depends on how we measure closeness between functions, which in turn varies significantly with the specific problem to be dealt with. In many applications, it is necessary to have the network perform *simultaneously* well on all input samples taken from some compact input set X in R^k. In this case, closeness is measured by the uniform distance between functions on X, that is,

$$\rho_{\mu,X}(f, g) = \sup_{x \in X} |f(x) - g(x)|.$$

In other applications, we think of the inputs as random variables and are interested in the *average performance* where the average is taken with respect to the input environment measure μ, where $\mu(R^k) < \infty$. In this case, closeness is measured by the $L^p(\mu)$ distances

$$\rho_{p,\mu}(f, g) = \left[\int_{R^k} |f(x) - g(x)|^p \, d\mu(x) \right]^{1/p},$$

$1 \le p < \infty$, the most popular choice being $p = 2$, corresponding to mean square error.

Of course, there are many more ways of measuring closeness of functions. In particular, in many applications, it is also necessary that the *derivatives* of the approximating function implemented by the network closely resemble those of the function to be approximated, up to some order. This issue was first taken up in Hornik et al. (1990), who discuss the sources of need of smooth functional approximation in more detail. Typical examples arise in robotics (learning of smooth movements) and signal processing (analysis of chaotic time series); for a recent application to problems of nonparametric inference in statistics and econometrics, see Gallant and White (1989).

All papers establishing certain approximation ca-

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MULTILAYER FEEDFORWARD NETWORKS WITH NON-POLYNOMIAL ACTIVATION FUNCTIONS CAN APPROXIMATE ANY FUNCTION

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How about computing all the derivatives?

- Derivatives tables:

$$\frac{d}{dx}(a) = 0$$

$$\frac{d}{dx}(x) = 1$$

$$\frac{d}{dx}(au) = a\frac{du}{dx}$$

$$\frac{d}{dx}(u + v - w) = \frac{du}{dx} + \frac{dv}{dx} - \frac{dw}{dx}$$

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{1}{v}\frac{du}{dx} - \frac{u}{v^2}\frac{dv}{dx}$$

$$\frac{d}{dx}(u^n) = nu^{n-1}\frac{du}{dx}$$

$$\frac{d}{dx}(\sqrt{u}) = \frac{1}{2\sqrt{u}}\frac{du}{dx}$$

$$\frac{d}{dx}\left(\frac{1}{u}\right) = -\frac{1}{u^2}\frac{du}{dx}$$

$$\frac{d}{dx}\left(\frac{1}{u^n}\right) = -\frac{n}{u^{n+1}}\frac{du}{dx}$$

$$\frac{d}{dx}[f(u)] = \frac{d}{du}[f(u)]\frac{du}{dx}$$

$$\frac{d}{dx}[\ln u] = \frac{d}{dx}[\log_e u] = \frac{1}{u}\frac{du}{dx}$$

$$\frac{d}{dx}[\log_a u] = \log_a e \frac{1}{u}\frac{du}{dx}$$

$$\frac{d}{dx}e^u = e^u\frac{du}{dx}$$

$$\frac{d}{dx}a^u = a^u \ln a \frac{du}{dx}$$

$$\frac{d}{dx}(u^v) = vu^{v-1}\frac{du}{dx} + \ln u \; u^v\frac{dv}{dx}$$

$$\frac{d}{dx}\sin u = \cos u\frac{du}{dx}$$

$$\frac{d}{dx}\cos u = -\sin u\frac{du}{dx}$$

$$\frac{d}{dx}\tan u = \sec^2 u\frac{du}{dx}$$

$$\frac{d}{dx}\cot u = -\csc^2 u\frac{du}{dx}$$

$$\frac{d}{dx}\sec u = \sec u \tan u\frac{du}{dx}$$

$$\frac{d}{dx}\csc u = -\csc u \cot u\frac{du}{dx}$$

[source: http://hyperphysics.phy-astr.gsu.edu/hbase/Math/derfunc.html

How about computing all the derivatives?

- But neural net f is never one of those?
  - No problem: CHAIN RULE:

If $$f(x) = g(h(x))$$

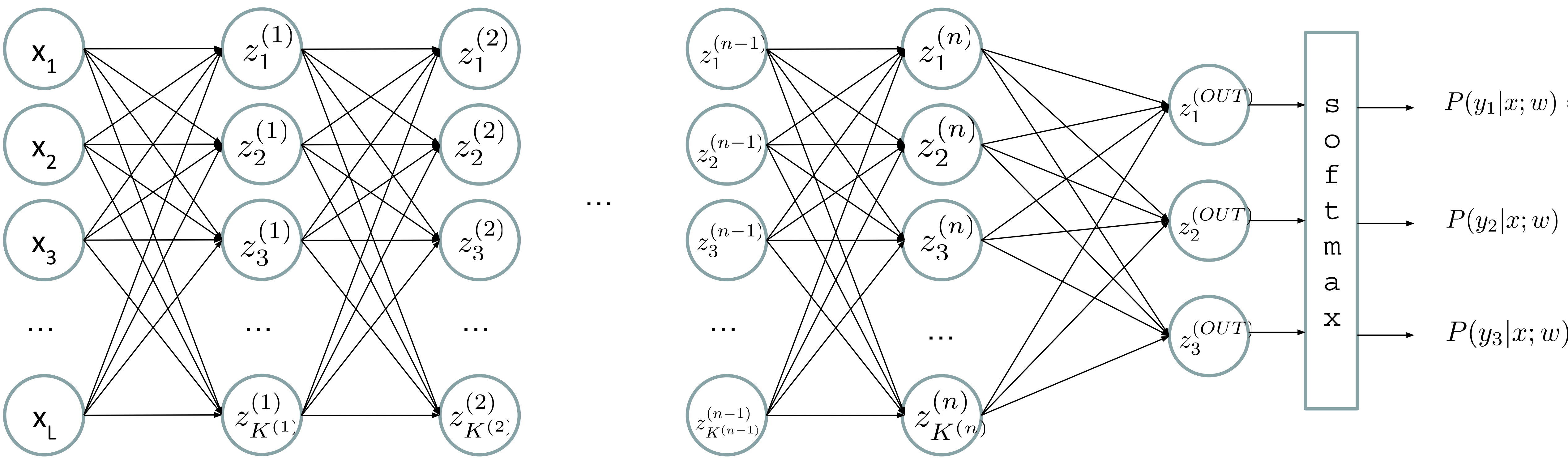
Then $$f'(x) = g'(h(x))h'(x)$$

Derivatives can be computed by following well-defined procedures

Automatic Differentiation

- Automatic differentiation software
  - e.g. Theano, TensorFlow, PyTorch, Chainer
  - Only need to program the function g(x,y,w)
  - Can automatically compute all derivatives w.r.t. all entries in w
  - This is typically done by caching info during forward computation pass of f, and then doing a backward pass = "backpropagation"
  - Autodiff / Backpropagation can often be done at computational cost comparable to the forward pass
- Need to know this exists
- How this is done? -- outside of scope of CS188

Training a Network (setting weights)



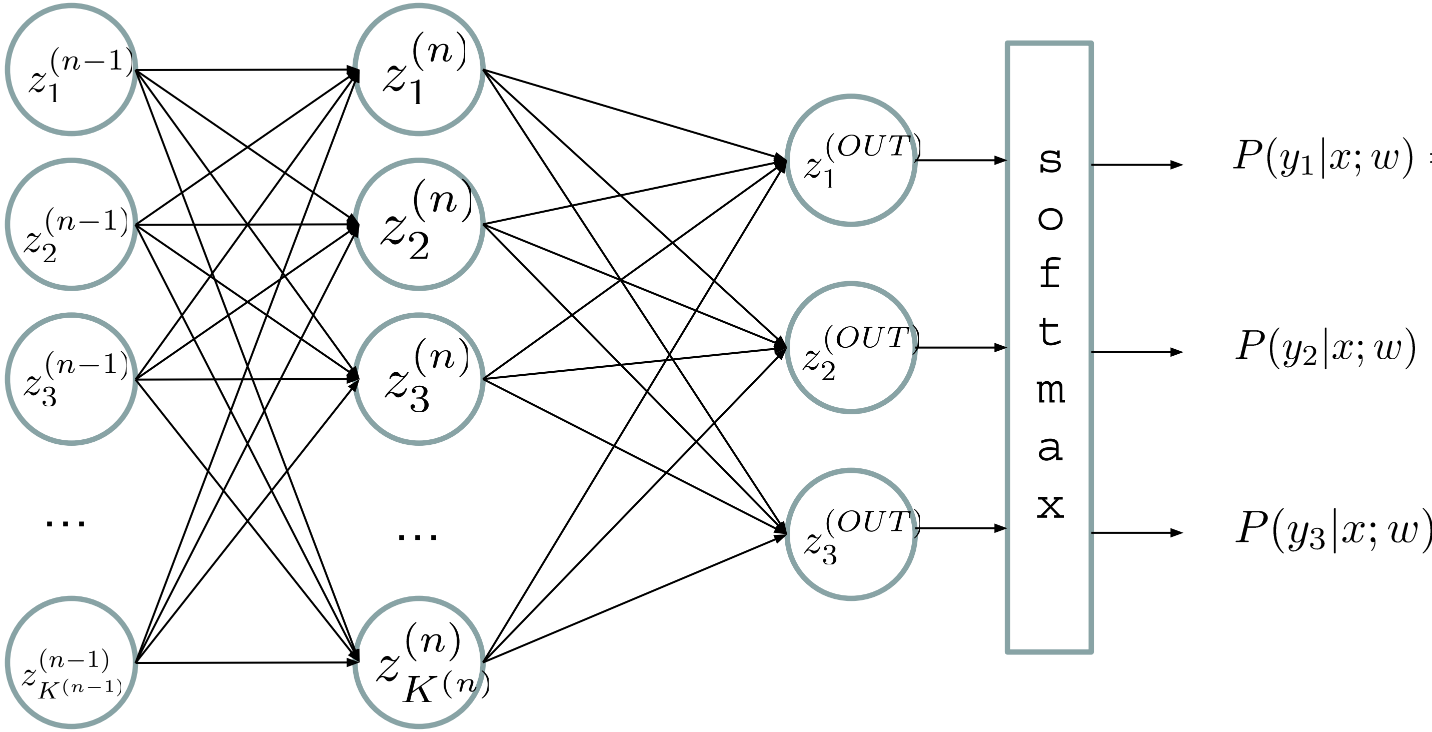
$$z_i^{(k)} = g\left(\sum_j W_{i,j}^{(k-1,k)} z_j^{(k-1)}\right)$$

g = nonlinear activation function

Training a Network

Key words:

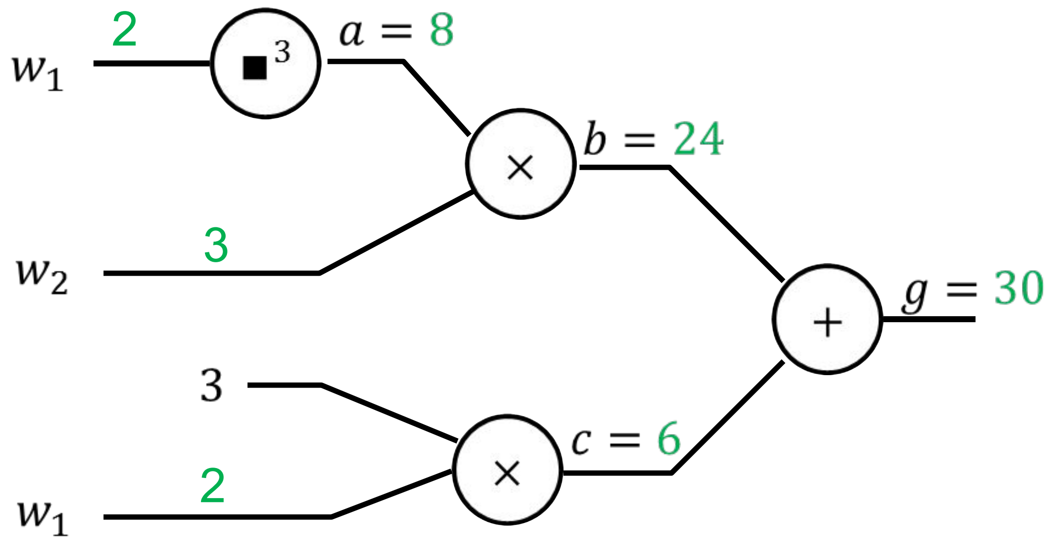
- Forward
- Backwards
- Gradient
- Backprop



g = nonlinear activation function

Back Propagation: $g(\boldsymbol{w}) = w_1^3 w_2 + 3w_1$

- Suppose we have $g(\boldsymbol{w}) = w_1^3 w_2 + 3w_1$ and want the gradient at $\boldsymbol{w} = [2, 3]$
- Think of the function as a composition of many functions.
  - Can use derivative chain rule to compute $\partial g / \partial w_1$ and $\frac{\partial g}{\partial w_2}$
  - $\frac{\partial g}{\partial w_1} =$ ________________________
  - $\frac{\partial g}{\partial w_2} =$ ________________________



Computation Graph

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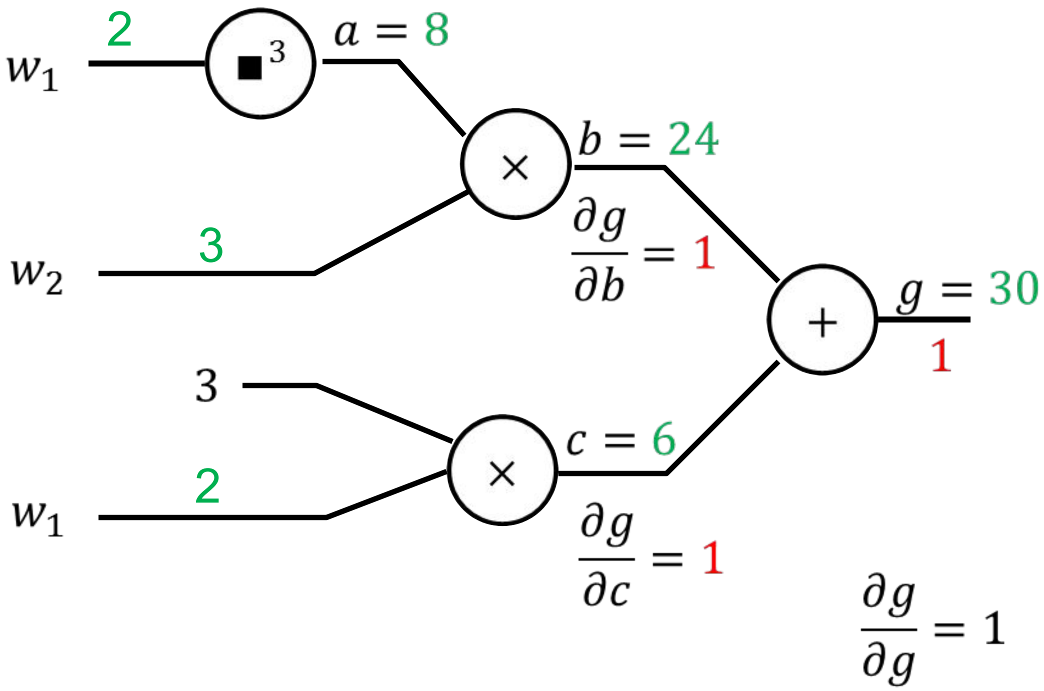

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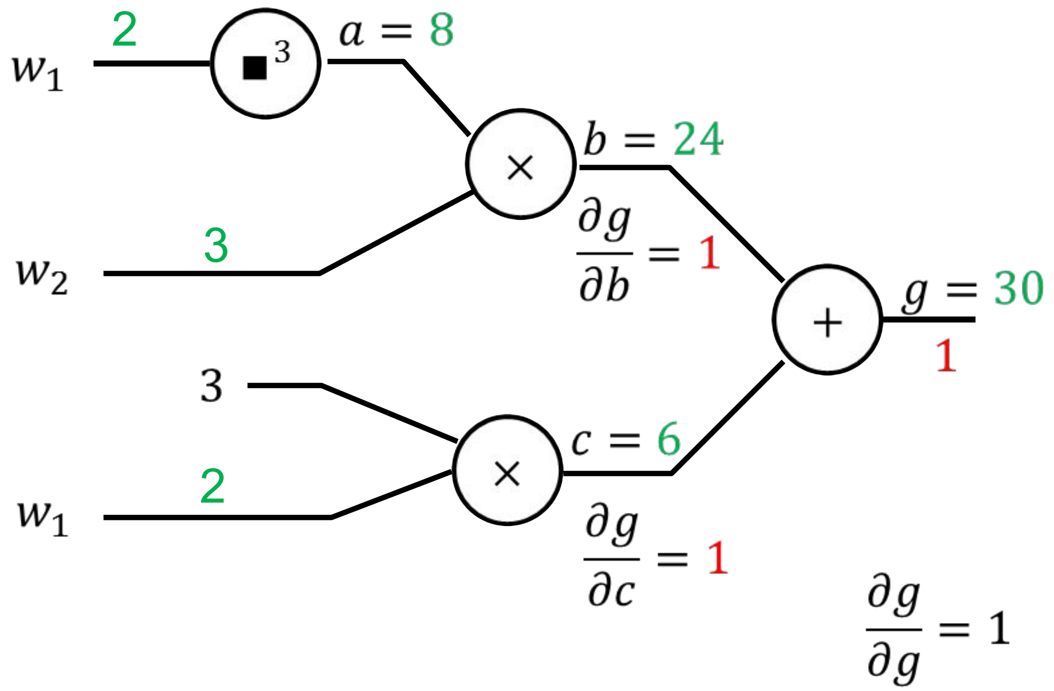
  - $\frac{\partial g}{\partial a} = \frac{\partial g}{\partial b} \frac{\partial b}{\partial a}$



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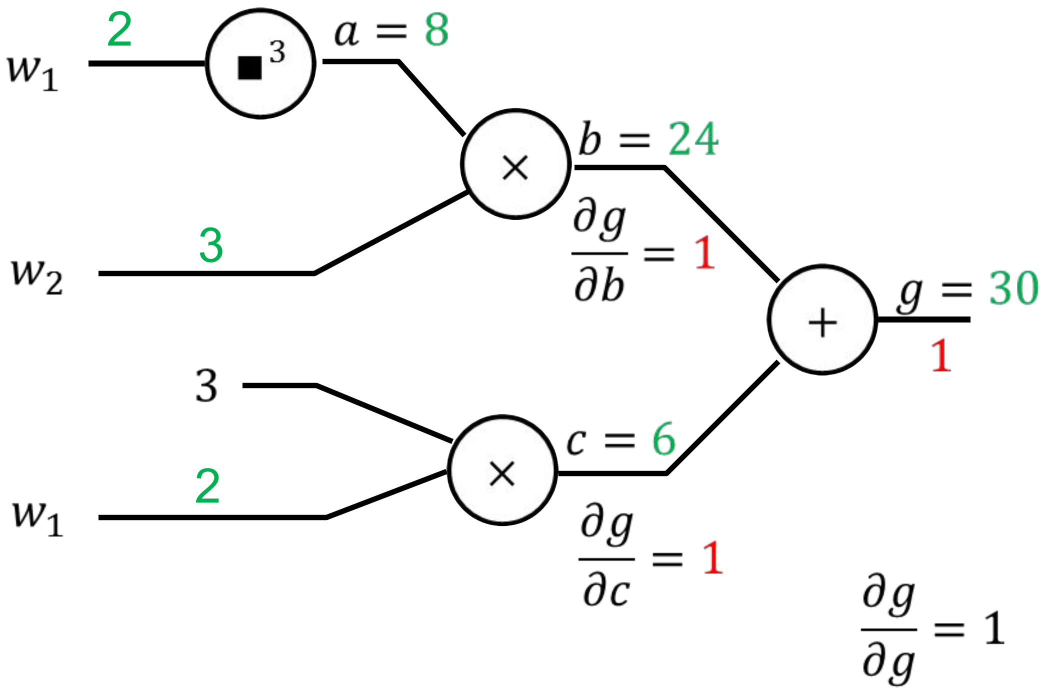

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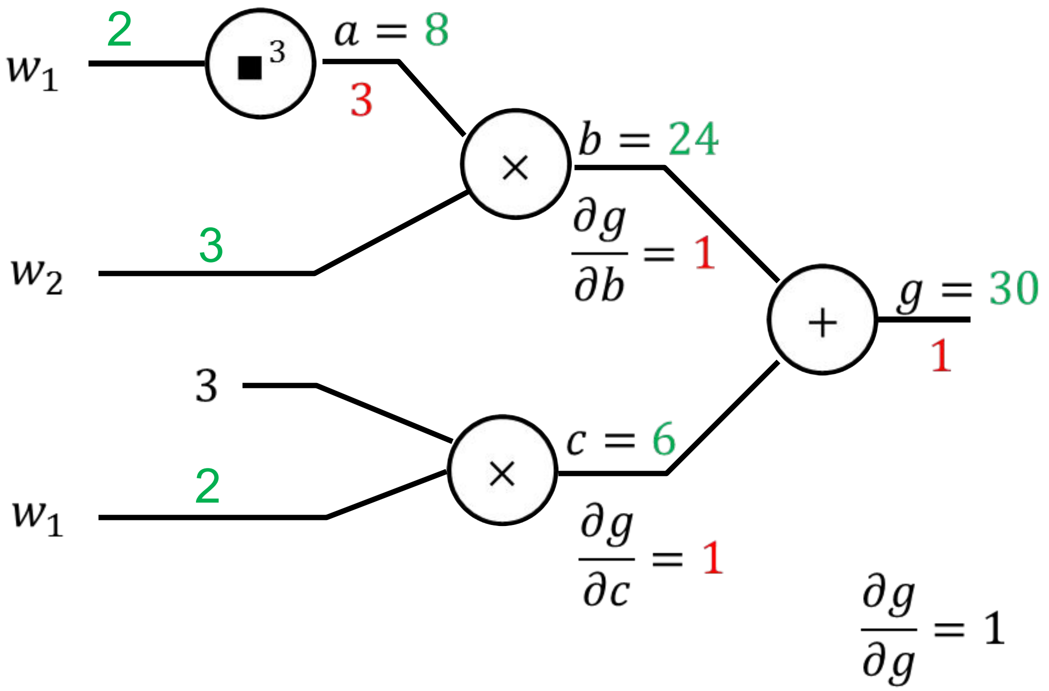

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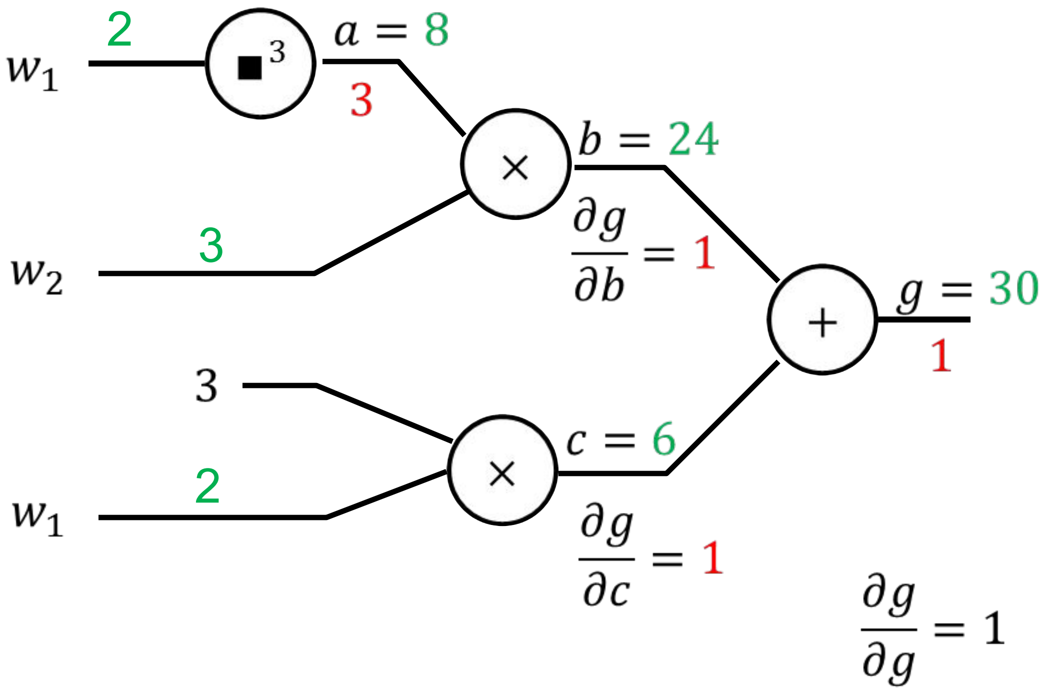
- $a = w_1^3$
  - $\frac{\partial g}{\partial w_1} =?????$



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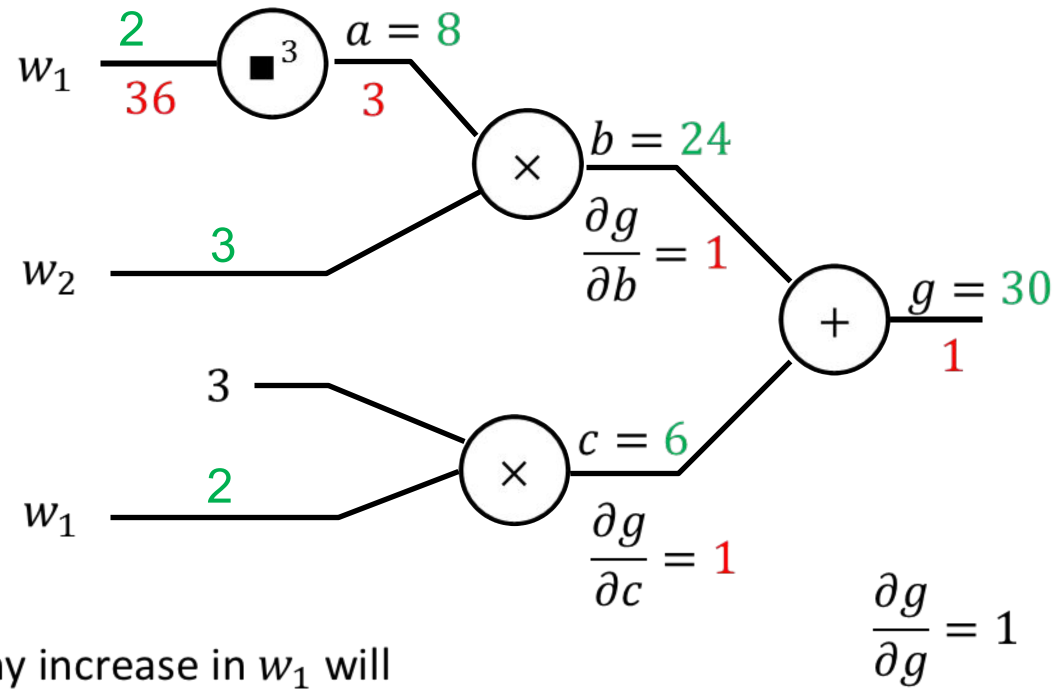
- $a = w_1^3$
  - $\frac{\partial g}{\partial w_1} = \frac{\partial g}{\partial a}\frac{\partial a}{\partial w_1} = 3 \cdot 3w_1^2 = 36$



Interpretation: A tiny increase in w_1 will result in an approximately 36 times increase in g due to this computation path.

Back Propagation: $g(\boldsymbol{w}) = w_1^3 w_2 + 3w_1$

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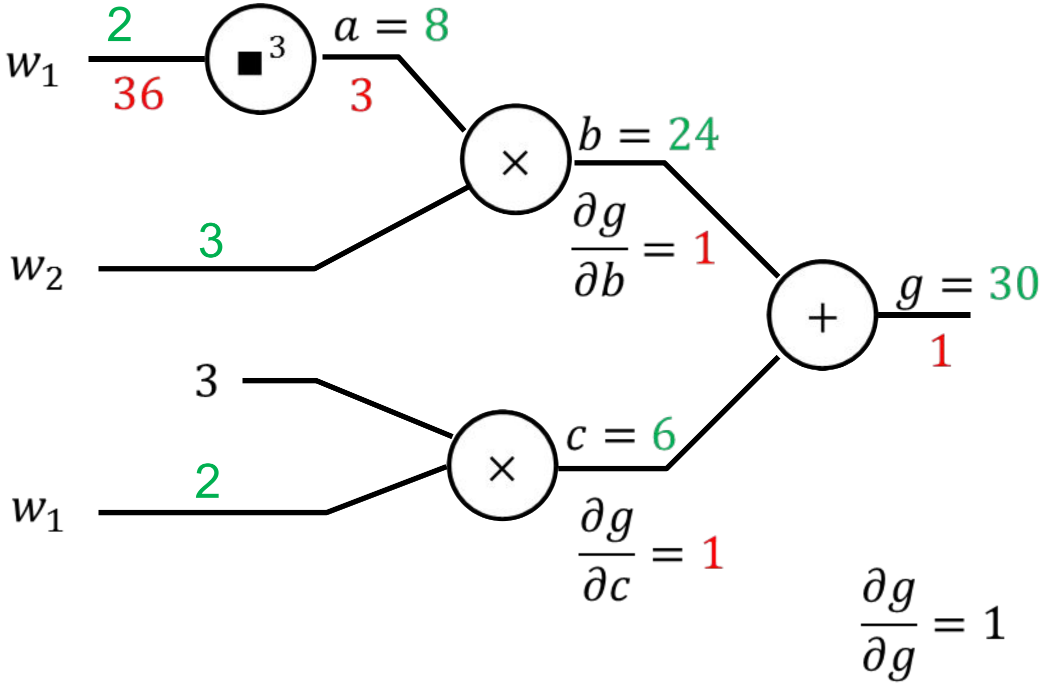
- $\frac{\partial g}{\partial w_2} = ???$ Hint: $b = a \times 3$ may be useful.



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  - $\frac{\partial g}{\partial w_2} = \frac{\partial g}{\partial b}\frac{\partial b}{\partial w_2} = 1 \frac{\partial b}{\partial w_2} = 1 \cdot 8 = 8$
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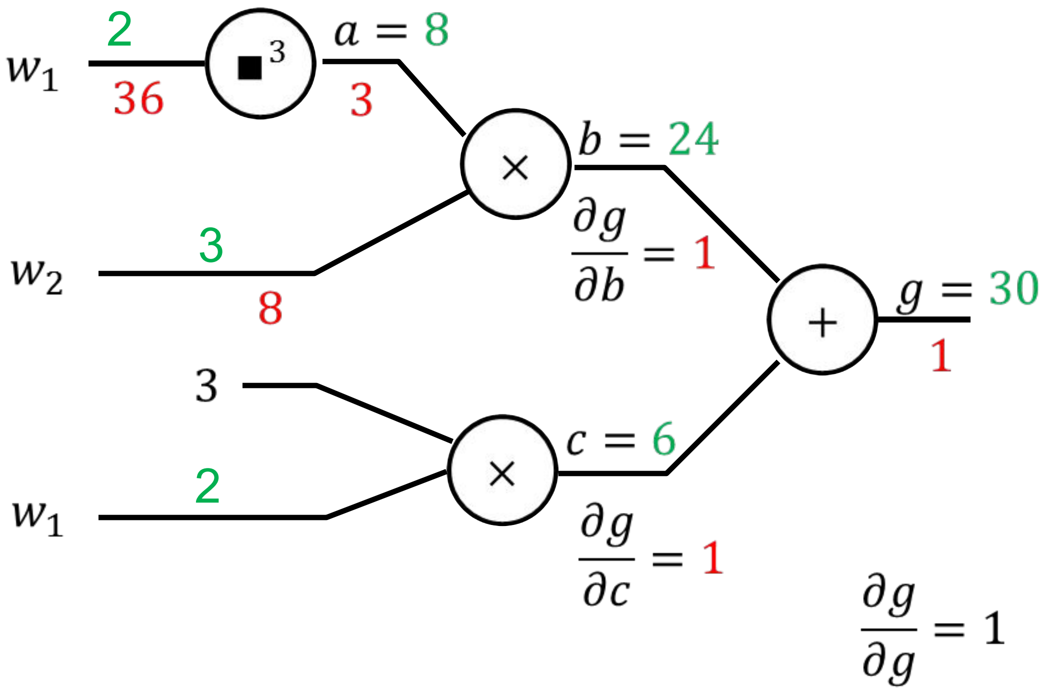

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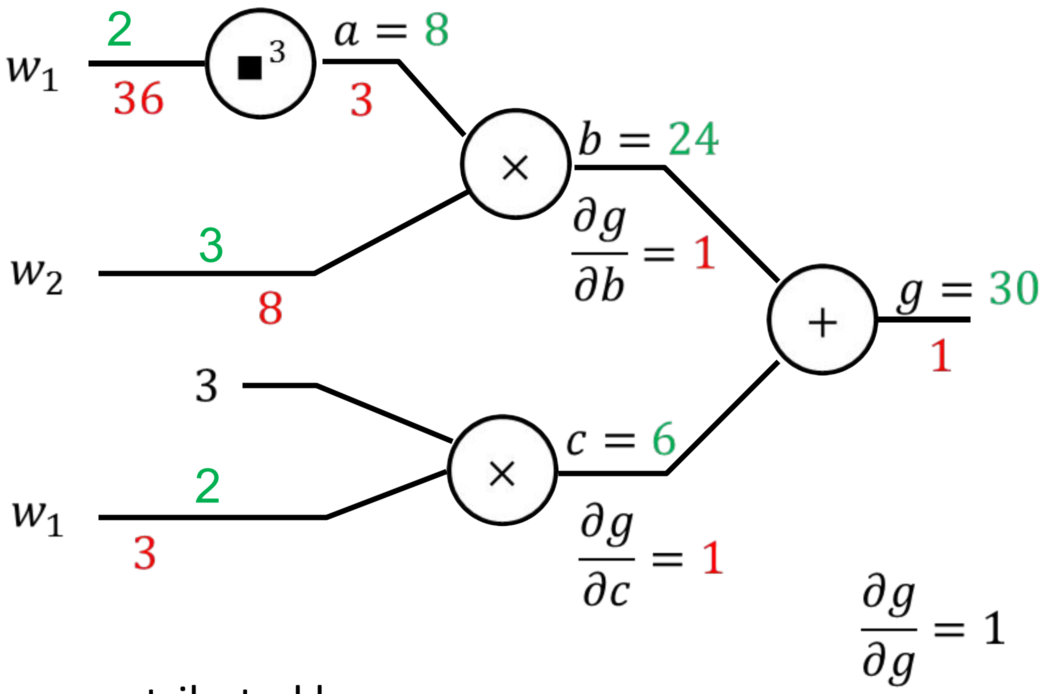
- $a = w_1^3$
  - $\frac{\partial g}{\partial w_1} = \frac{\partial g}{\partial a}\frac{\partial a}{\partial w_1} = 3 \cdot 3w_1^2 = 36$
- $c = 3w_1$
  - $\frac{\partial g}{\partial w_1} = \frac{\partial g}{\partial c}\frac{\partial c}{\partial w_1} = 1 \cdot 3 = 3$



Adding the changes to g contributed by change in w_1 together

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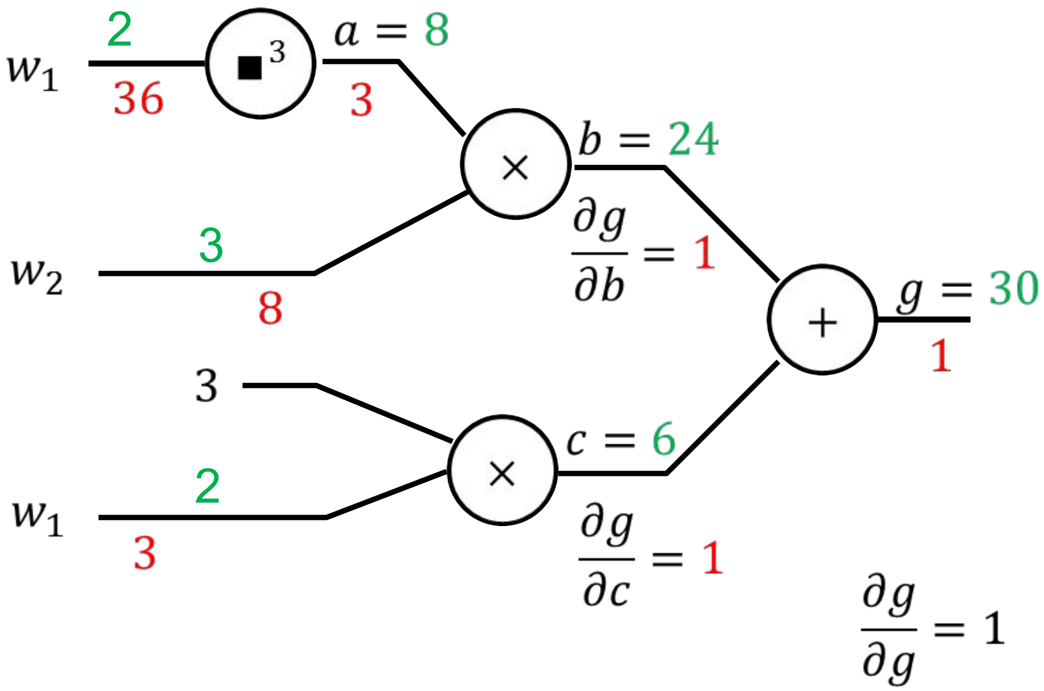
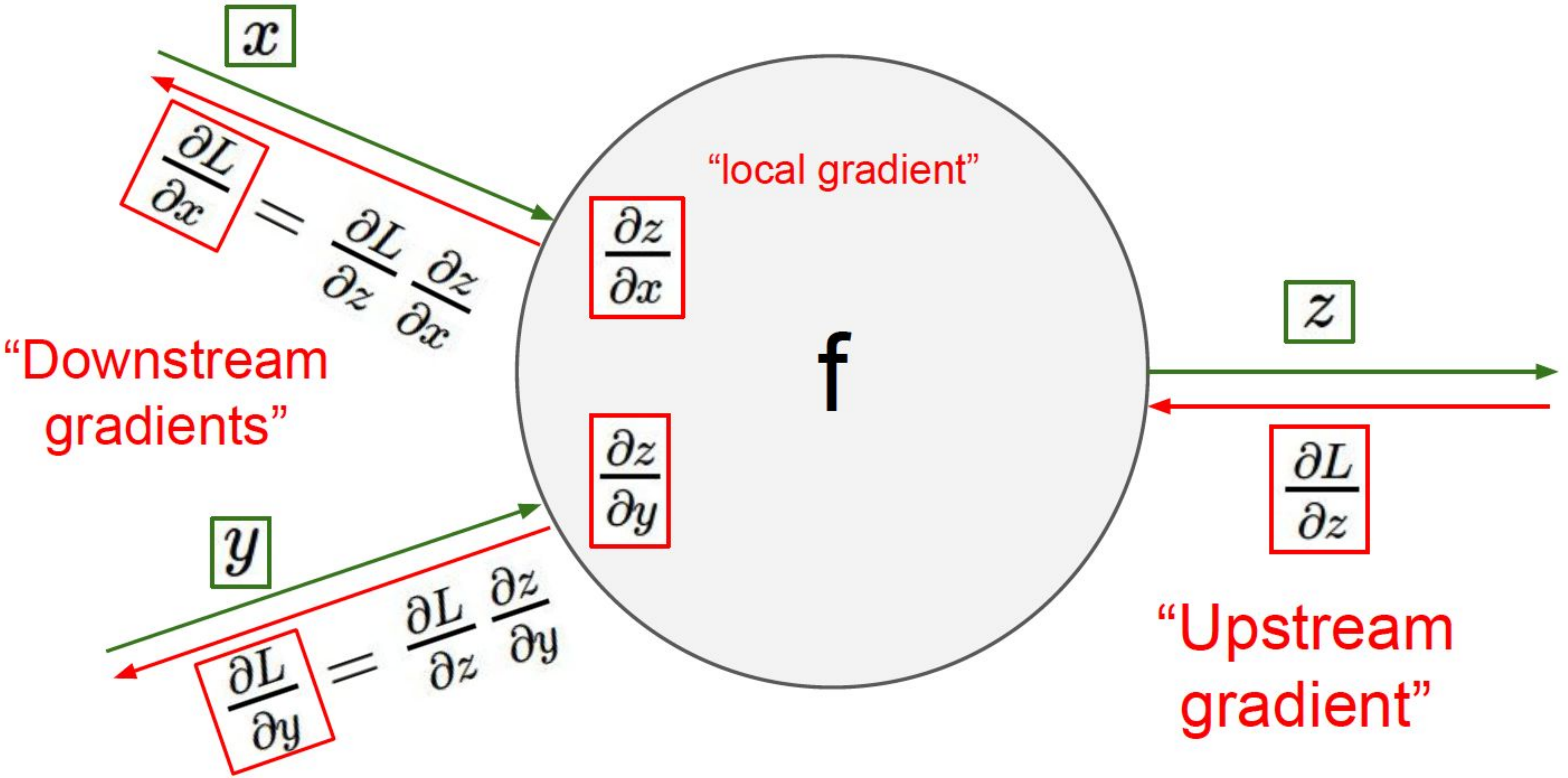
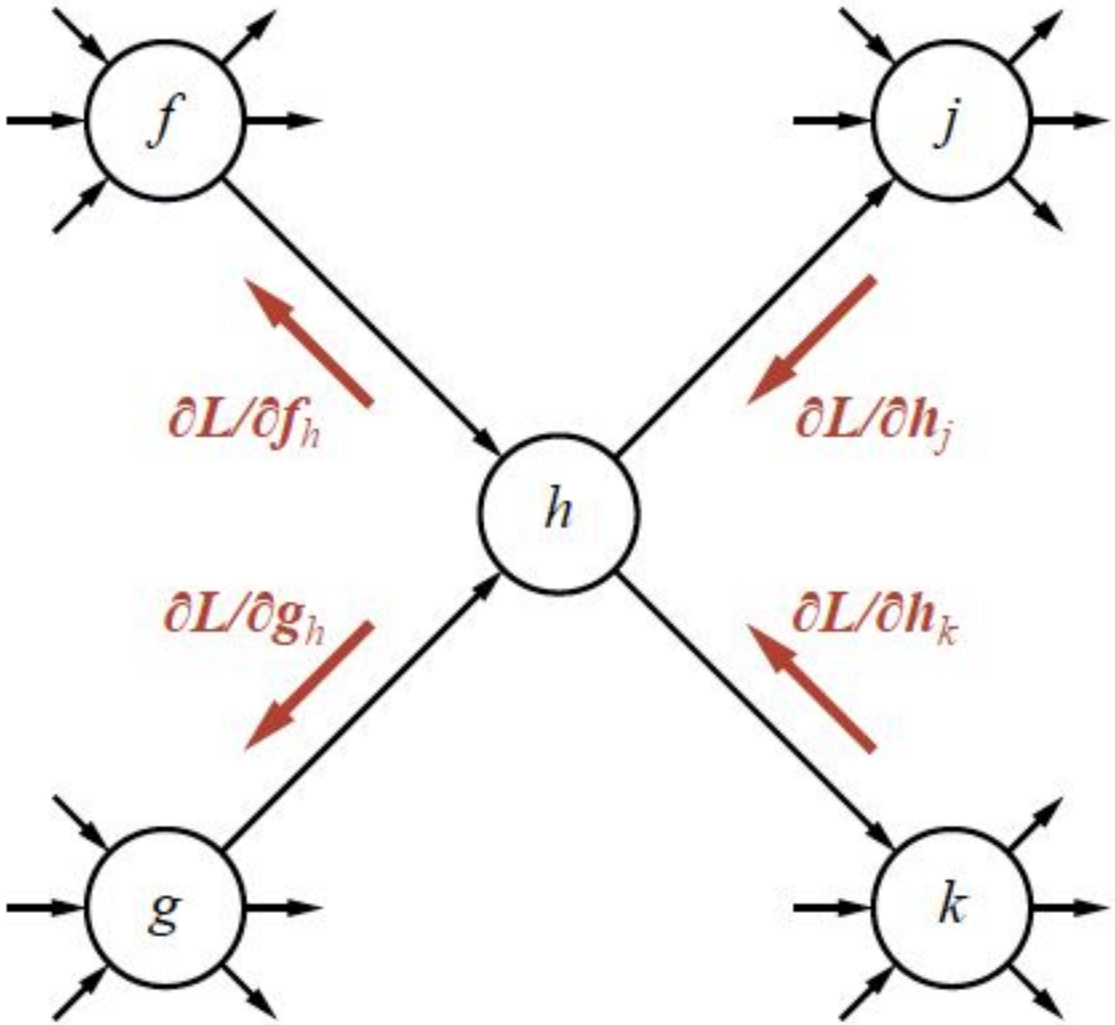
- $b = a \times w_2$
  - $\frac{\partial g}{\partial a} = \frac{\partial g}{\partial b}\frac{\partial b}{\partial a} = 1\frac{\partial b}{\partial a} = 1 \cdot 3 = 3$
  - $\frac{\partial g}{\partial w_2} = \frac{\partial g}{\partial b}\frac{\partial b}{\partial w_2} = 1\frac{\partial b}{\partial w_2} = 1 \cdot 8 = 8$
- $a = w_1^3$
  - $\frac{\partial g}{\partial w_1} = \frac{\partial g}{\partial a}\frac{\partial a}{\partial w_1} = 3 \cdot 3w_1^2 = 36$
- $c = 3w_1$
  - $\frac{\partial g}{\partial w_1} = \frac{\partial g}{\partial c}\frac{\partial c}{\partial w_1} = 1 \cdot 3 = 3$



$$\nabla g = \left[\frac{\partial g}{\partial w_1}, \frac{\partial g}{\partial w_2}\right] = [39, 8]$$



x
$\frac{\partial L}{\partial x} = \frac{\partial L}{\partial z}\frac{\partial z}{\partial x}$
"local gradient"
$\frac{\partial z}{\partial x}$
f
z
"Downstream gradients"
$\frac{\partial z}{\partial y}$
$\frac{\partial L}{\partial z}$
y
$\frac{\partial L}{\partial y} = \frac{\partial L}{\partial z}\frac{\partial z}{\partial y}$
"Upstream gradient"


f
j
∂L/∂f_h
∂L/∂h_j
h
∂L/∂g_h
∂L/∂h_k
g
k

Summary of Key Ideas

- Optimize probability of label given input $\max_w \quad ll(w) = \max_w \quad \sum_i \log P(y^{(i)}|x^{(i)}; w)$
- Continuous optimization
  - Gradient ascent:
    - Compute steepest uphill direction = gradient (= just vector of partial derivatives)
    - Take step in the gradient direction
    - Repeat
- Deep neural nets
  - Layered computation graph
    - Last layer = often logistic regression
    - Now also many more layers before this last layer
      - = computing the features
      - the features are learned rather than hand-designed
    - Different neural network architectures: CNN, RNN, LSTM, Transformer
  - Universal function approximation theorem
    - `If` neural net is large enough
    - `Then` neural net can represent any continuous mapping from input to output with arbitrary accuracy
    - But remember: need to avoid overfitting / memorizing the training data; early stopping!
  - Automatic differentiation gives the derivatives efficiently

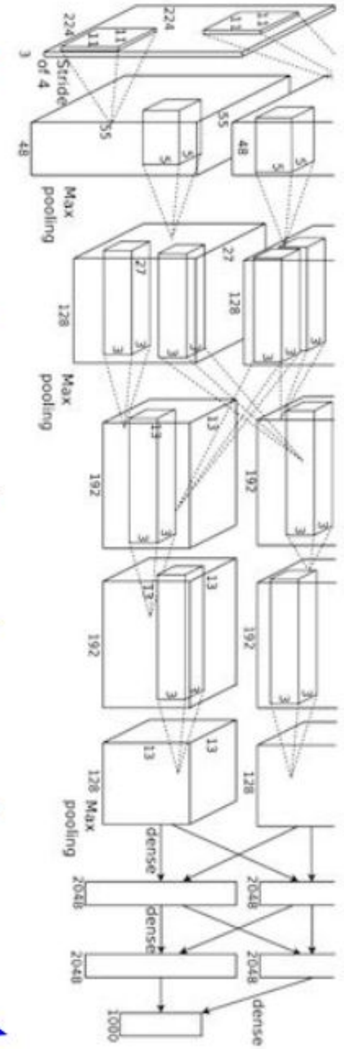
Different Neural Network Architectures

Convolutional network (AlexNet)

input image

weights

loss



Neural network as General computation graph

Krizhevsky, Suskever, Hinton, 2012

Different Neural Network Architectures

- Exploration of different neural network architectures
  - ResNet: residual networks
  - Networks with attention
  - Transformer networks
- Neural network architecture search
- Really large models
  - GPT2, GPT3
  - CLIP

In a shocking finding, scientist discovered a herd of unicorns living in a remote, previously unexplored valley, in the Andes Mountains. Even more surprising to the researchers was the fact that the unicorns spoke perfect English.

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The scientist named the population, after their distinctive horn, Ovid's
Unicorn. These four-horned, silver-white unicorns were previously unknown
to science.

Now, after almost two centuries, the mystery of what sparked this odd
phenomenon is finally solved.

Dr. Jorge Pérez, an evolutionary biologist from the University of La Paz, and
several companions, were exploring the Andes Mountains when they found a small
valley, with no other animals or humans. Pérez noticed that the valley had
what appeared to be a natural fountain, surrounded by two peaks of rock and
silver snow.

Pérez and the others then ventured further into the valley. "By the time we
reached the top of one peak, the water looked blue, with some crystals on
top," said Pérez.

Pérez and his friends were astonished to see the unicorn herd. These creatures
could be seen from the air without having to move too much to see them – they
were so close they could touch their horns.

While examining these bizarre creatures the scientists discovered that the
creatures also spoke some fairly regular English. Pérez stated, "We can see,
for example, that they have a common 'language,' something like a dialect
or dialectic."

Dr. Pérez believes that the unicorns may have originated in Argentina, where
the animals were believed to be descendants of a lost race of people who lived
there before the arrival of humans in those parts of South America.

While their origins are still unclear, some believe that perhaps the creatures
were created when a human and a unicorn met each other in a time before human
civilization. According to Pérez, "In South America, such incidents seem to be
quite common."

However, Pérez also pointed out that it is likely that the only way of knowing
for sure if unicorns are indeed the descendants of a lost alien race is
through DNA. "But they seem to be able to communicate in English quite well,
which I believe is a sign of evolution, or at least a change in social
organization," said the scientist.
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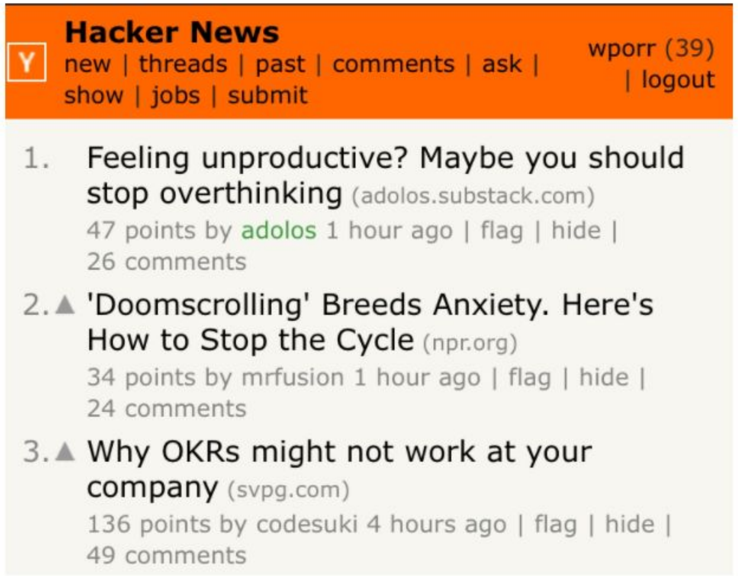
A college student used GPT-3 to write fake blog posts and ended up at the top of Hacker News

He says he wanted to prove the AI could pass as a human writer

By Kim Lyons | Aug 16, 2020, 1:55pm EDT



Porr's fake blog post, written under the fake name "adolos," reaches #1 on Hacker News. Porr says he used three separate accounts to submit and upvote his posts on Hacker News in an attempt to push them higher. The admin said this strategy doesn't work, but his click-baity headlines did.

SCREENSHOT / LIAM PORR