CS 188: Artificial Intelligence

Local search

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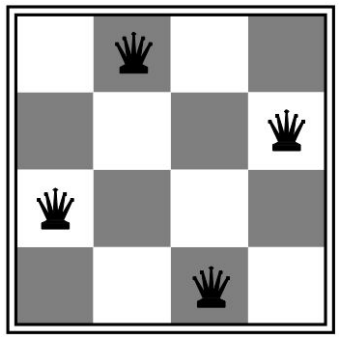
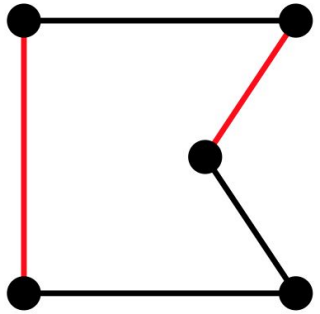
Local search algorithms

- In many optimization problems, ***path*** is irrelevant; the goal state ***is*** the solution
- Then state space = set of "complete" configurations;
  find ***configuration satisfying constraints***, e.g., n-queens problem; or, find ***optimal configuration***, e.g., travelling salesperson problem





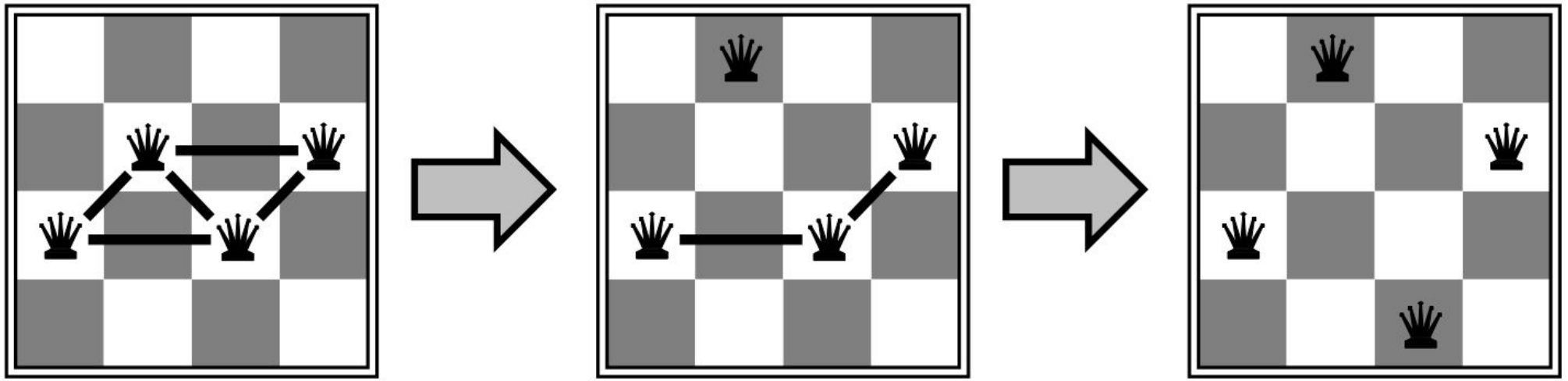
- In such cases, can use ***iterative improvement*** algorithms: keep a single "current" state, try to improve it
- Constant space, suitable for online as well as offline search
- More or less unavoidable if the "state" is yourself (i.e., learning)

Hill Climbing

- Simple, general idea:
  - Start wherever
  - Repeat: move to the best neighboring state
  - If no neighbors better than current, quit

Heuristic for *n*-queens problem

- Goal: n queens on board with no ***conflicts***, i.e., no queen attacking another
- States: n queens on board, one per column
- Heuristic value function: number of conflicts

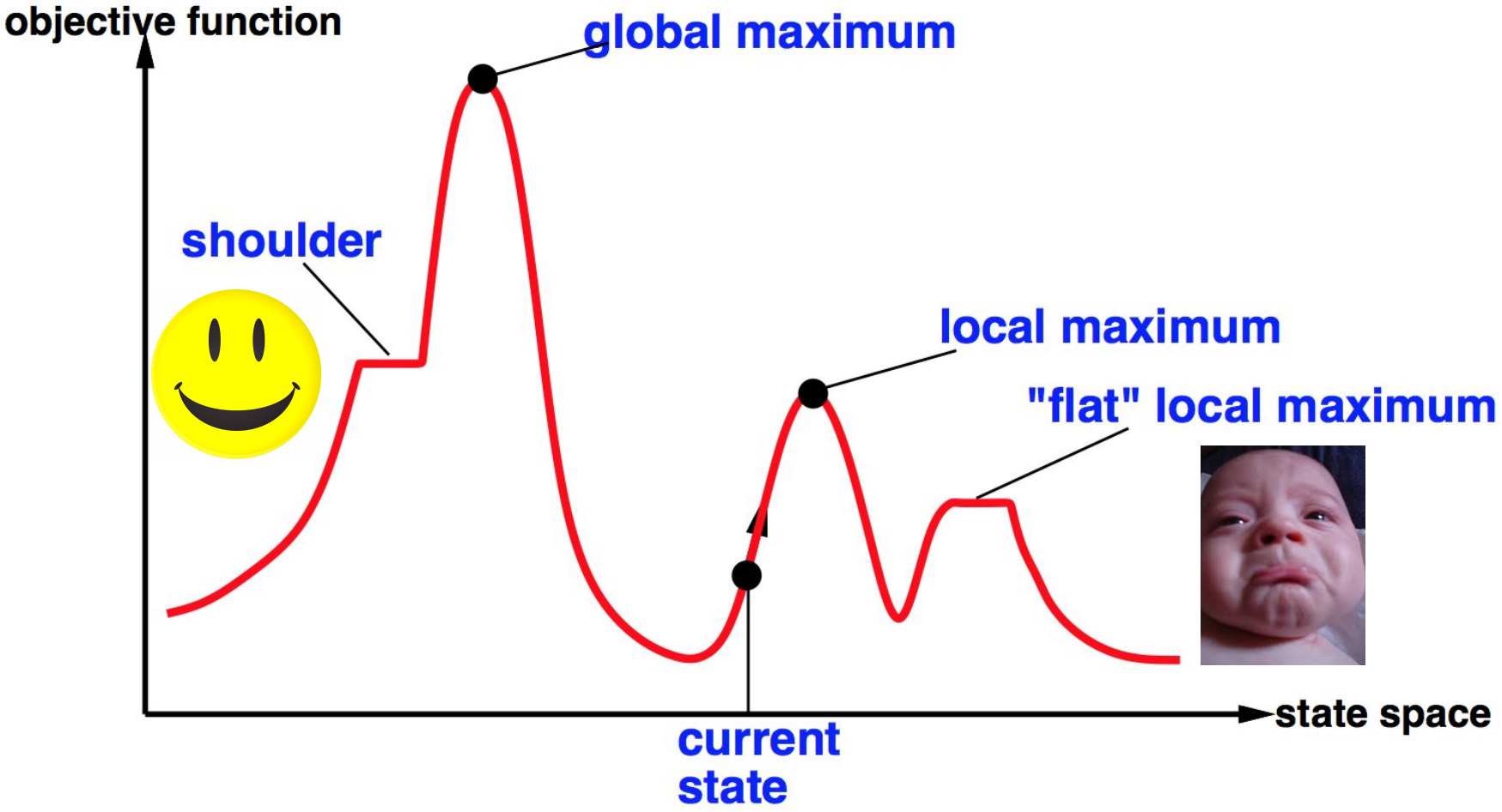


Hill-climbing algorithm

```
function HILL-CLIMBING(problem) returns a state
    current ← make-node(problem.initial-state)
    loop do
        neighbor ← a highest-valued successor of current
        if neighbor.value ≤ current.value then
            return current.state
        current ← neighbor
```

"Like climbing Everest in thick fog with amnesia"

Global and local maxima



Random restarts

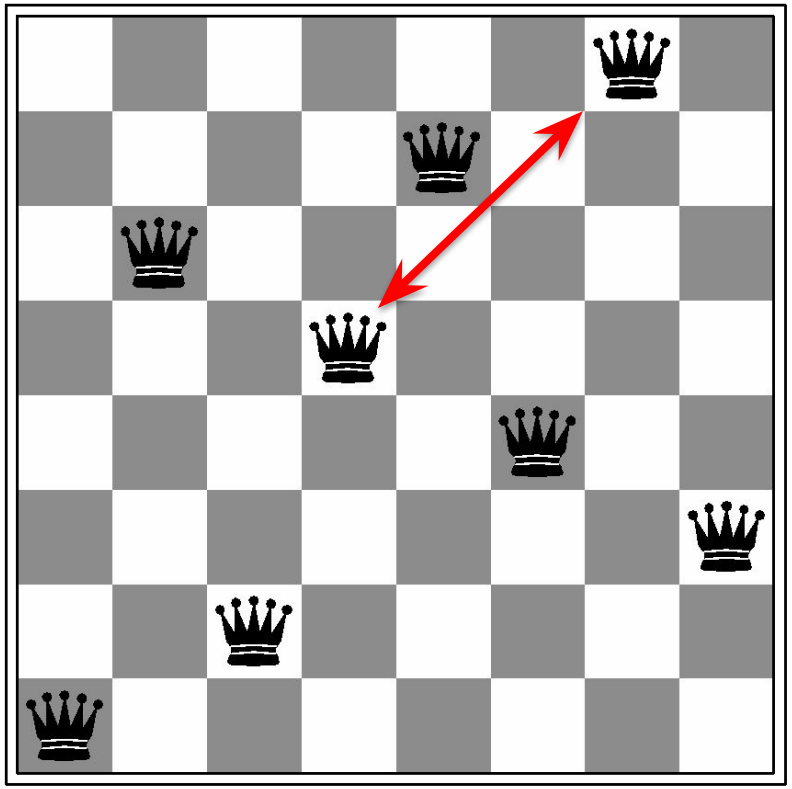
- find global optimum
- duh

Random sideways moves

- Escape from shoulders
- Loop forever on flat local maxima

Hill-climbing on the 8-queens problem

- No sideways moves:
  - Succeeds w/ prob. 0.14
  - Average number of moves per trial:
    - 4 when succeeding, 3 when getting stuck
  - Expected total number of moves needed:
    - 3(1-p)/p + 4 =~ 22 moves
- Allowing 100 sideways moves:
  - Succeeds w/ prob. 0.94
  - Average number of moves per trial:
    - 21 when succeeding, 65 when getting stuck
  - Expected total number of moves needed:
    - 65(1-p)/p + 21 =~ 25 moves



Moral: algorithms with knobs to twiddle are irritating

Simulated annealing

- Resembles the annealing process used to cool metals slowly to reach an ordered (low-energy) state
- Basic idea:
  - Allow “bad” moves occasionally, depending on “temperature”
  - High temperature => more bad moves allowed, shake the system out of its local minimum
  - Gradually reduce temperature according to some schedule
  - Sounds pretty flaky, doesn’t it?

Simulated annealing algorithm

function SIMULATED-ANNEALING(problem,schedule) **returns** a state

current ← problem.initial-state

for t = 1 **to** ∞ **do**

T ←schedule(t)

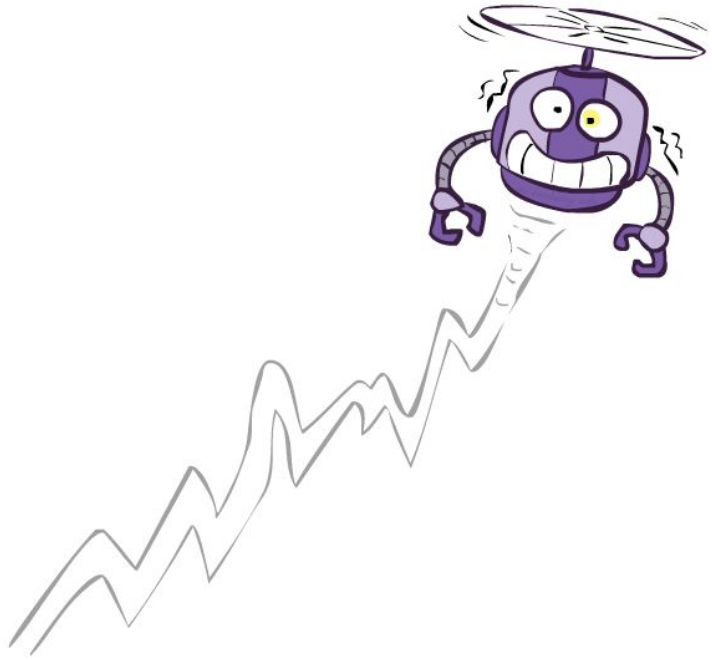
if T = 0 **then return** current

next ← a randomly selected successor of current

ΔE ← next.value – current.value

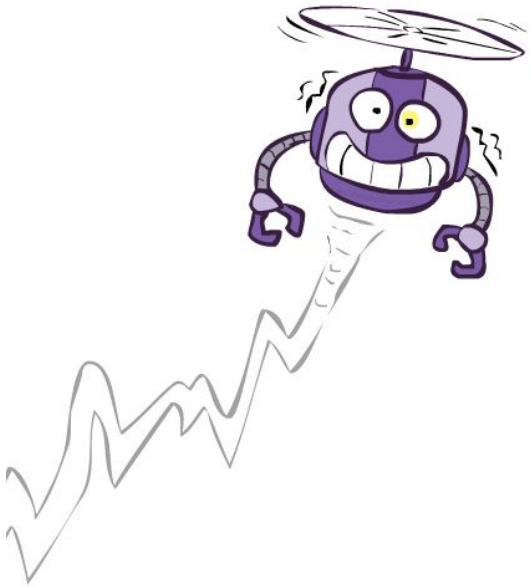
if ΔE > 0 **then** current ← next

else current ← next only with probability $e^{\Delta E/T}$



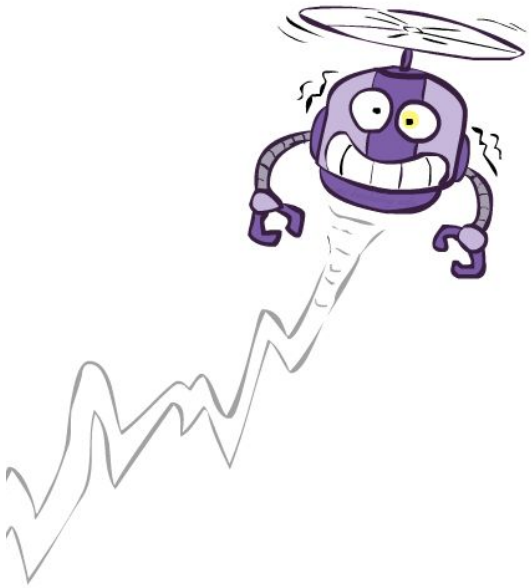
Simulated Annealing

- Theoretical guarantee:
  - Stationary distribution (Boltzmann):
  - If T decreased slowly enough, will converge to optimal state!
- Proof sketch (for reversible case: $x \rightarrow y$ iff $y \rightarrow x$):
  - Let $P(x)$, $P(y)$ be the equilibrium occupancy probabilities at T
  - Let $P(x \rightarrow y)$ be the probability that state x transitions to state y
  - Assume $E(y) > E(x)$ [and the algorithm seeks *high* values of E]



Simulated Annealing

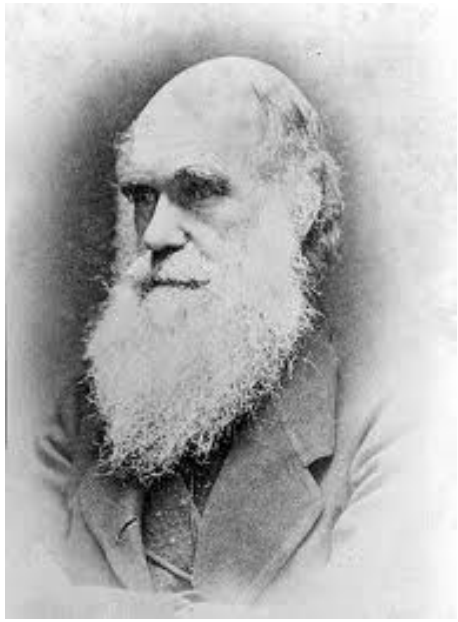
- Is this convergence an interesting guarantee?
- Sounds like magic, but reality is reality:
  - The more downhill steps you need to escape a local optimum, the less likely you are to ever make them all in a row
  - "Slowly enough" may mean exponentially slowly
  - Random restart hillclimbing also converges to optimal state…
- Simulated annealing and its relatives are a key workhorse in VLSI layout and other optimal configuration problems



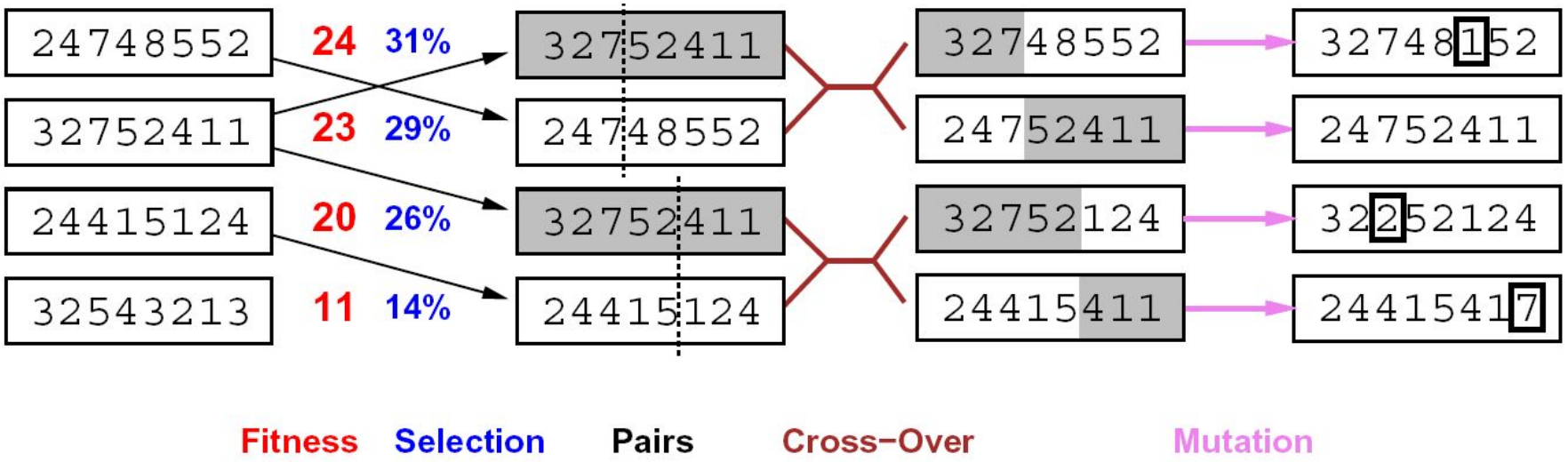
Local beam search

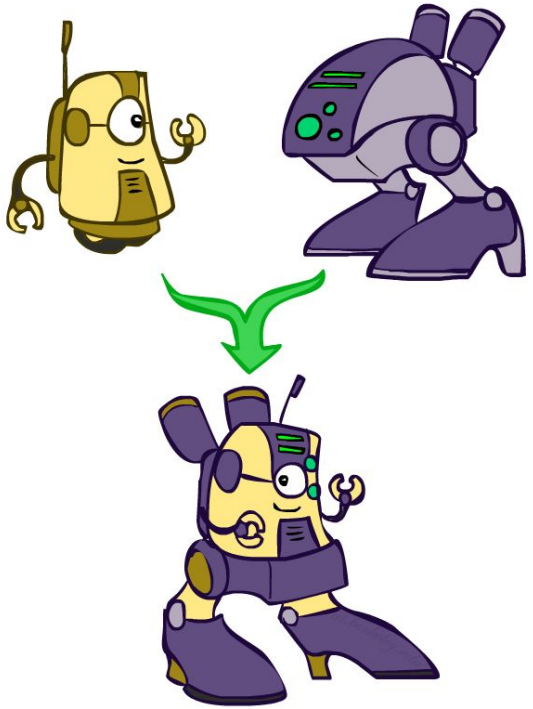
- Basic idea:
  - *K* copies of a local search algorithm, initialized randomly
  - For each iteration
    - Generate ALL successors from *K* current states
    - Choose best *K* of these to be the new current states
- Why is this different from *K* local searches in parallel?
  - The searches ***communicate***! "Come over here, the grass is greener!"
- What other well-known algorithm does this remind you of?
  - Evolution!

Or, K chosen randomly with a bias towards good ones



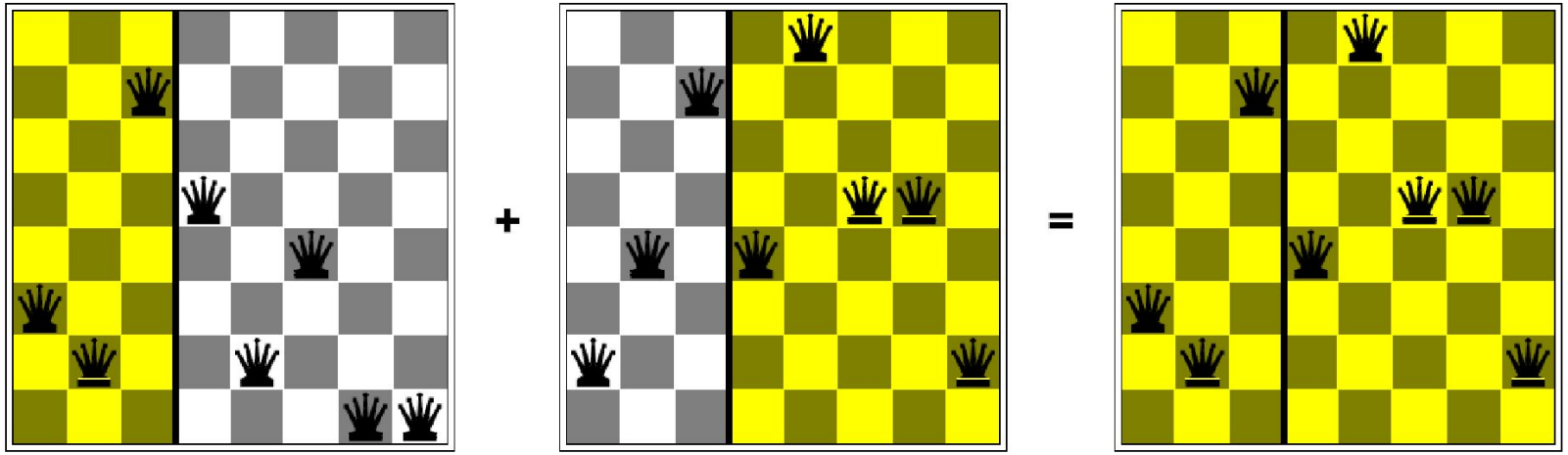
Genetic algorithms





- Genetic algorithms use a natural selection metaphor
  - Resample *K* individuals at each step (selection) weighted by fitness function
  - Combine by pairwise crossover operators, plus mutation to give variety

Example: N-Queens

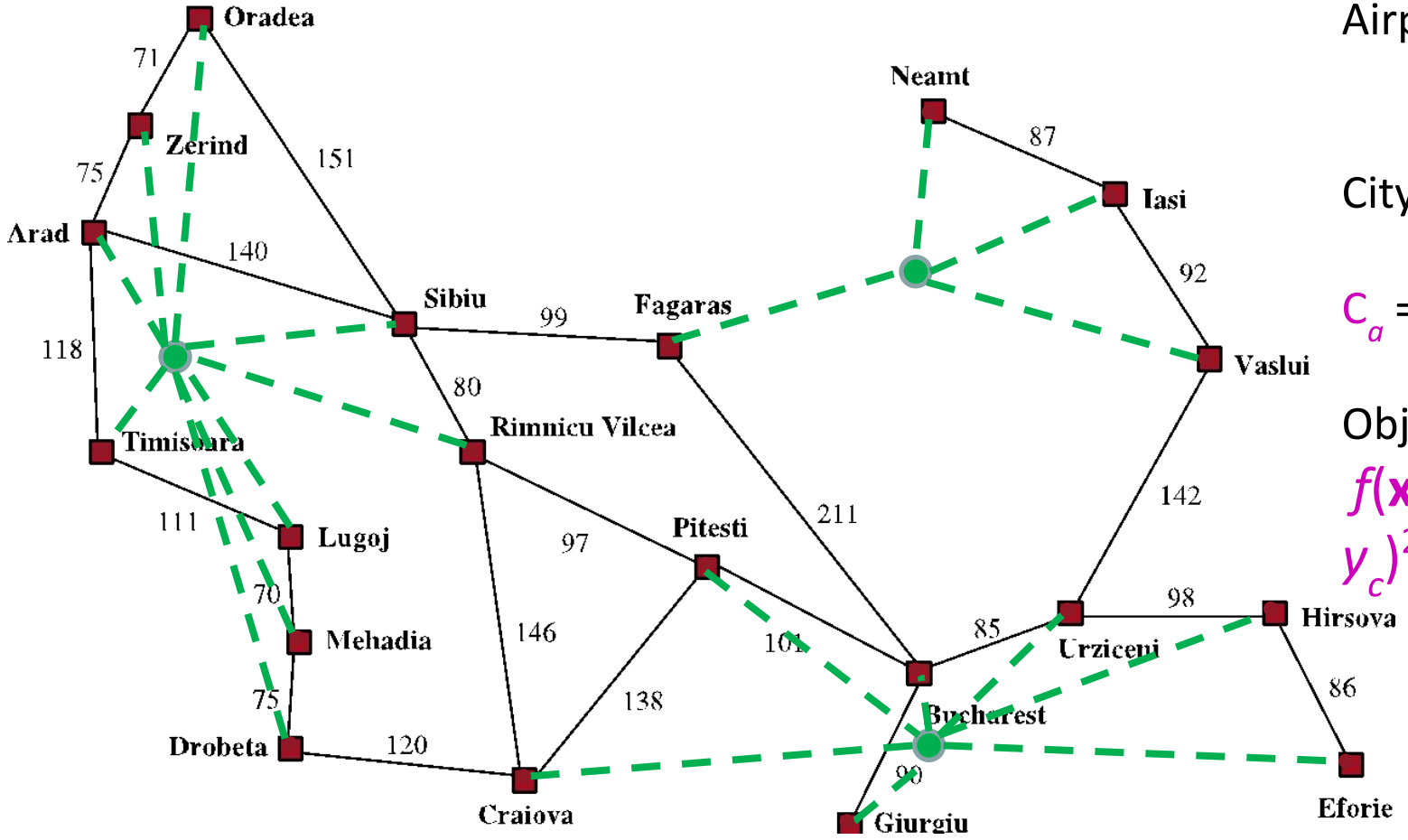


- Does crossover make sense here?
- What would mutation be?
- What would a good fitness function be?

Local search in continuous spaces

Example: Siting airports in Romania

Place 3 airports to minimize the sum of squared distances from each city to its nearest airport



Airport locations

$\mathbf{x} = (x_1,y_1), (x_2,y_2), (x_3,y_3)$

City locations (x_c,y_c)

C_a = cities closest to airport a

Objective: minimize

$f(\mathbf{x}) = \sum_a \sum_{c \in Ca} (x_a - x_c)^2 + (y_a - y_c)^2$

Handling a continuous state/action space

1. Discretize it!
   - Define a grid with increment δ , use any of the discrete algorithms
2. Choose random perturbations to the state
   a. First-choice hill-climbing: keep trying until something improves the state
   b. Simulated annealing
4. Compute gradient of $f(\mathbf{x})$ analytically

Finding extrema in continuous space

- Gradient vector $\nabla f(\mathbf{x}) = (\partial f/\partial x_1, \partial f/\partial y_1, \partial f/\partial x_2, \ldots)^T$
- For the airports, $f(\mathbf{x}) = \Sigma_a \Sigma_{c \in Ca} (x_a - x_c)^2 + (y_a - y_c)^2$
- $\partial f/\partial x_1 = \Sigma_{c \in C1} 2(x_1 - x_c)$
- At an extremum, $\nabla f(\mathbf{x}) = 0$
- Can sometimes solve in closed form: $x_1 = (\Sigma_{c \in C1} x_c)/|C_1|$
- Is this a local or global minimum of f?
- Gradient descent: $\mathbf{x} \leftarrow \mathbf{x} - \alpha \nabla f(\mathbf{x})$
  - Huge range of algorithms for finding extrema using gradients

Summary

- Many configuration and optimization problems can be formulated as local search
- General families of algorithms:
  - Hill-climbing, continuous optimization
  - Simulated annealing (and other stochastic methods)
  - Local beam search: multiple interaction searches
  - Genetic algorithms: break and recombine states

Many machine learning algorithms are local searches