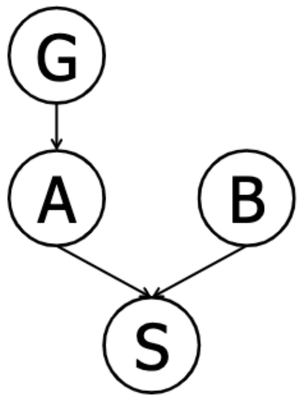
CS 188 Spring 2021 — Regular Discussion 5

1 Probability

Suppose that a patient can have a symptom (S) that can be caused by two different, independent diseases (A and B). It is known that the variation of gene G plays a big role in the manifestation of disease A. A model and some conditional probability tables for this situation are shown below. For each part, you may leave your answer as an arithmetic expression.

$P(G)$	
$+g$	0.1
$-g$	0.9

$P(A\|G)$		
$+g$	$+a$	1.0
$+g$	$-a$	0.0
$-g$	$+a$	0.1
$-g$	$-a$	0.9



$P(B)$	
$+b$	0.4
$-b$	0.6

$P(S\|A,B)$			
$+a$	$+b$	$+s$	1.0
$+a$	$+b$	$-s$	0.0
$+a$	$-b$	$+s$	0.9
$+a$	$-b$	$-s$	0.1
$-a$	$+b$	$+s$	0.8
$-a$	$+b$	$-s$	0.2
$-a$	$-b$	$+s$	0.1
$-a$	$-b$	$-s$	0.9

(a) Compute the following entry from the joint distribution:

$P(+g, +a, +b, +s) =$

(b) What is the probability that a patient has disease A?

$P(+a) =$

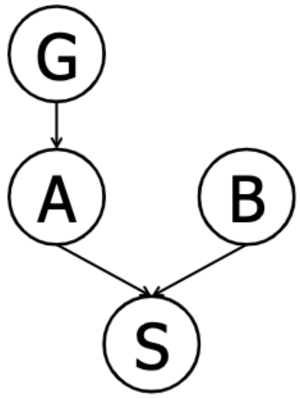
(c) What is the probability that a patient has disease A given that they have disease B?

$P(+a| + b) =$

The figures and table below are identical to the ones on the previous page and are repeated here for your convenience.

$P(G)$	
$+g$	0.1
$-g$	0.9

$P(A\|G)$		
$+g$	$+a$	1.0
$+g$	$-a$	0.0
$-g$	$+a$	0.1
$-g$	$-a$	0.9



$P(B)$	
$+b$	0.4
$-b$	0.6

$P(S\|A,B)$			
$+a$	$+b$	$+s$	1.0
$+a$	$+b$	$-s$	0.0
$+a$	$-b$	$+s$	0.9
$+a$	$-b$	$-s$	0.1
$-a$	$+b$	$+s$	0.8
$-a$	$+b$	$-s$	0.2
$-a$	$-b$	$+s$	0.1
$-a$	$-b$	$-s$	0.9

(d) What is the probability that a patient has disease A given that they have symptom S and disease B?

$P(+a| + s, +b) =$

(e) What is the probability that a patient has the disease carrying gene variation G given that they have disease A?

$P(+g| + a) =$

2 Independence

1. Suppose you have two random variables, C and N. C is the result of flipping a biased coin that lands on heads (h) with probability 0.8 and tails (t) with probability 0.2. N is the number of heads that result from two independent coin flips of a fair coin.

   (a) Fill in the probability tables for $P(C)$, $P(N)$, and $P(C, N)$.

C	$P(C)$
h	
t	

N	$P(N)$
0	
1	
2	

C	N	$P(C, N)$
h	0	
h	1	
h	2	
t	0	
t	1	
t	2	

   (b) Using the probability tables above, what is $P(N = 1|C = t)$?

2. Simplify each of the following into a single probability expression using the given independence assumption.

   (a) Given that $A \perp\!\!\!\perp B$, simplify $\sum_a P(a|B)P(C|a)$.

   (b) Given that $B \perp\!\!\!\perp C|A$, simplify $\frac{P(A)P(B|A)P(C|A)}{P(B|C)P(C)}$.

   (c) Given that $A \perp\!\!\!\perp B|C$, simplify $\frac{P(C,A|B)P(B)}{P(C)}$.

3. Mark **all** expressions that are equal to $P(R, S, T)$, given no independence assumptions:

   - ☐ $P(R \mid S, T)\ P(S \mid T)\ P(T)$
   - ☐ $P(T \mid R, S)\ P(R)\ P(S)$
   - ☐ $P(R \mid S)\ P(S \mid T)\ P(T)$
   - ☐ $P(R \mid S, T)\ P(S \mid R, T)\ P(T \mid R, S)$
   - ☐ $P(T, S \mid R)\ P(R)$
   - ☐ $P(T \mid R, S)\ P(R, S)$
   - ☐ None of the above