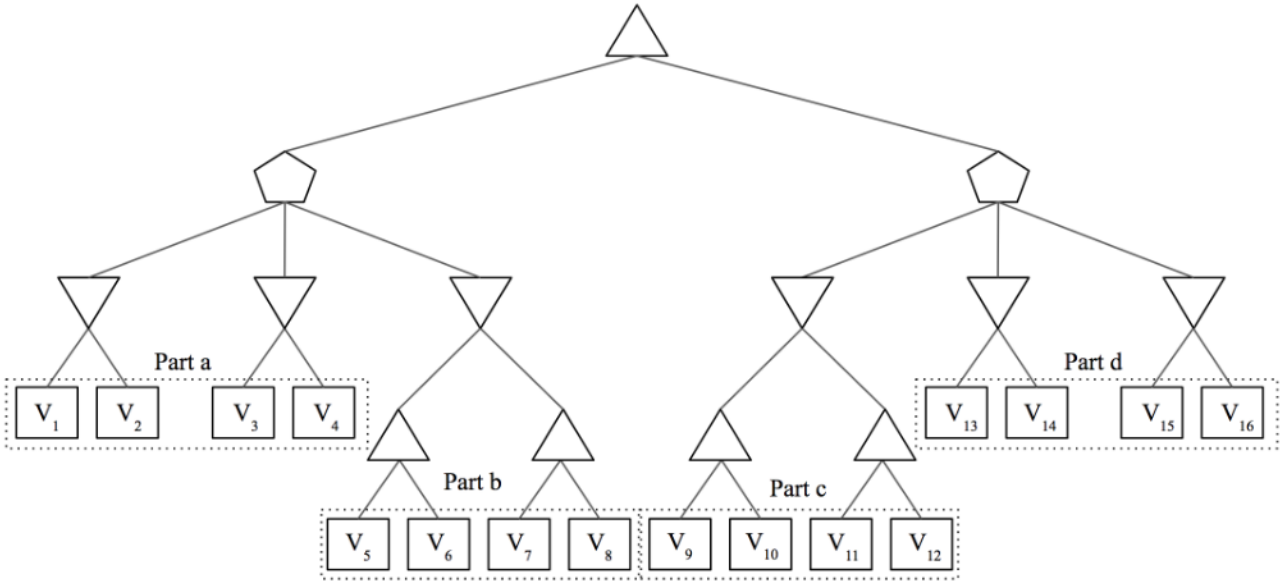
CS 188 Spring 2021 Introduction to Artificial Intelligence

Exam Prep 3 Solutions

Q1. MedianMiniMax

You're living in utopia! Despite living in utopia, you still believe that you need to maximize your utility in life, other people want to minimize your utility, and the world is a 0 sum game. But because you live in utopia, a benevolent social planner occasionally steps in and chooses an option that is a compromise. Essentially, the social planner (represented as the pentagon) is a median node that chooses the successor with median utility. Your struggle with your fellow citizens can be modelled as follows:



There are some nodes that we are sometimes able to prune. In each part, mark all of the terminal nodes such that **there exists a possible situation** for which the node **can be pruned**. In other words, you must consider **all** possible pruning situations. Assume that evaluation order is **left to right** and all V_i's are **distinct.**

Note that as long as there exists ANY pruning situation (does not have to be the same situation for every node), you should mark the node as prunable. Also, alpha-beta pruning does not apply here, simply prune a sub-tree when you can reason that its value will not affect your final utility.

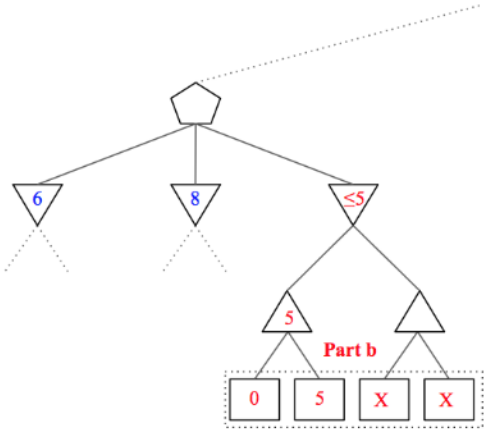
(a)
- ☐ V_1
- ☐ V_2
- ☐ V_3
- ☐ V_4
- ■ None

(b)
- ☐ V_5
- ■ V_6
- ■ V_7
- ■ V_8
- ☐ None

(c)
- ☐ V_9
- ☐ V_{10}
- ■ V_{11}
- ■ V_{12}
- ☐ None

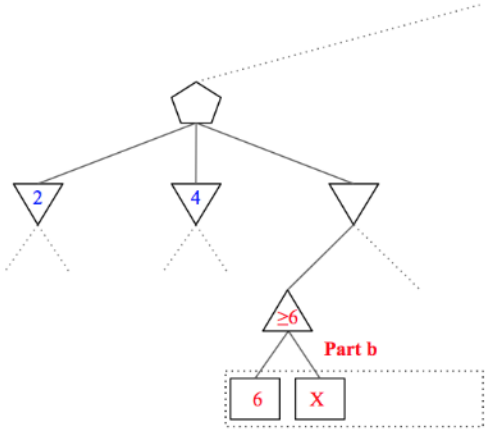
(d)
- ☐ V_{13}
- ■ V_{14}
- ■ V_{15}
- ■ V_{16}
- ☐ None

Part a:
For the left median node with three children, at least two of the childrens' values must be known since one of them will be guaranteed to be the value of the median node passed up to the final maximizer. For this reason, none of the nodes in part a can be pruned.



The value of this subtree will only get smaller.

The median node will **NOT** choose the value of this subtree. 6 is the median.



The value of this subtree will only get bigger.

*If the value of this subtree is chosen by the minimizer**, it will **NOT** be chosen by the median node.

**It is possible that the median is the value of the subtree to the right that we haven't looked at yet*

Part b (pruning V_7, V_8):
Let min_1, min_2, min_3 be the values of the three minimizer nodes in this subtree.

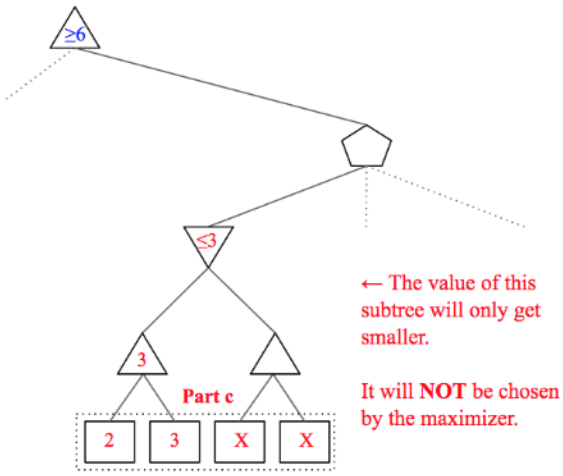
In this case, we may not need to know the final value min_3. The reason for this is that we may be able to put a bound on its value after exploring only partially, and determine the value of the median node as either min_1 or min_2 if $min_3 \leq \min(min_1, min_2)$ or $min_3 \geq \max(min_1, min_2)$.

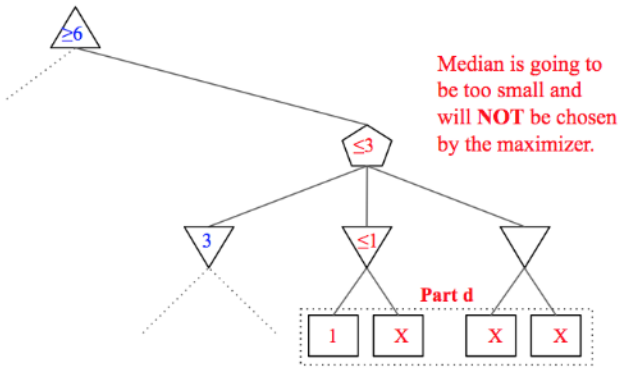
We can put an upper bound on min_3 by exploring the left subtree V_5, V_6 and if $\max(V_5, V_6)$ is lower than both min_1 and min_2, the median node's value is set as the smaller of min_1, min_2 and we don't have to explore V_7, V_8 in Figure 1.

Part b (pruning V_6):
It's possible for us to put a lower bound on min_3. If V_5 is larger than both min_1 and min_2, we do not need to explore V_6.

The reason for this is subtle, but if the minimizer chooses the left subtree, we know that $min_3 \geq V_5 \geq \max(min_1, min_2)$ and we don't need V_6 to get the correct value for the median node which will be the larger of min_1, min_2.

If the minimizer chooses the value of the right subtree, the value at V_6 is unnecessary again since the minimizer never chose its subtree.





Part c (pruning V_{11}, V_{12}):
Assume the highest maximizer node has a current value $max_1 \geq Z$ set by the left subtree and the three minimizers on this right subtree have value min_1, min_2, min_3.

In this part, if $min_1 \leq \max(V_9, V_{10}) \leq Z$, we do not have to explore V_{11}, V_{12}. Once again, the reasoning is subtle, but we can now realize if either $min_2 \leq Z$ or $min_3 \leq Z$ then the value of the right median node is for sure $\leq Z$ and is useless.

Only if both $min_2, min_3 \geq Z$ will the whole right subtree have an effect on the highest maximizer, but in this case the exact value of min_1 is not needed, just the information that it is $\leq Z$. Clearly in both cases, V_{11}, V_{12} are not needed since an exact value of min_1 is not needed.

We will also take the time to note that if $V_9 \geq Z$ we do have to continue the exploring as V_{10} could be even greater and the final value of the top maximizer, so V_{10} can't really be pruned.

Part d (pruning V_{14}, V_{15}, V_{16}):
Continuing from part c, if we find that $min_1 \leq Z$ and $min_2 \leq Z$ we can stop.

We can realize this as soon we explore V_{13}. Once we figure this out, we know that our median node's value must be one of these two values, and neither will replace Z so we can stop.

Q2. Games

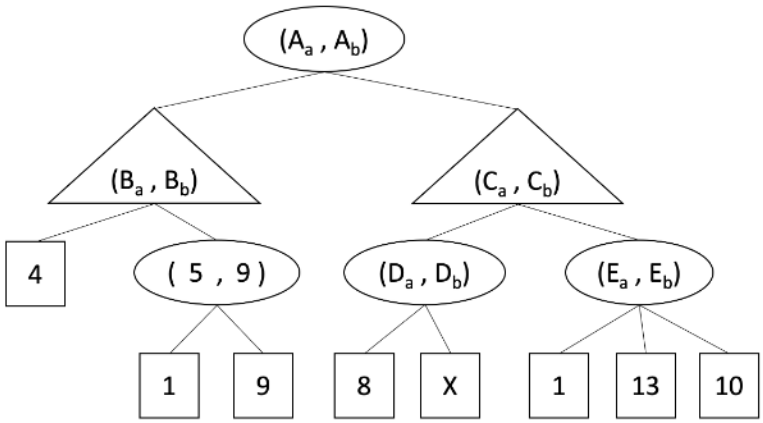
Alice is playing a two-player game with Bob, in which they move alternately. Alice is a maximizer. Although Bob is also a maximizer, Alice believes Bob is a minimizer with probability 0.5, and a maximizer with probability 0.5. Bob is aware of Alice's assumption.

In the game tree below, square nodes are the outcomes, triangular nodes are Alice's moves, and round nodes are Bob's moves. Each node for Alice/Bob contains a tuple, the left value being Alice's expectation of the outcome, and the right value being Bob's expectation of the outcome.

Tie-breaking: choose the left branch.

The left values are Alice's expectations, and are the only thing Alice can refer to when making decisions.

The right values are Bob's expectations, and they also accurately track the expected outcome of the game according to each choice of branching (regardless of it is Alice's or Bob's decision, since Bob has all the information). Hence the right values are accurate information about the game, and would be what Bob looks at when making his decisions. However, when it is Alice's turn to make decisions, Bob will think about how Alice would maximize the outcome w.r.t to what she believes, and he will update his expectations accordingly.



(a) In the blanks below, fill in the tuple values for tuples (B_a, B_b) and (E_a, E_b) from the above game tree.

$(B_a, B_b) = ($ 5 , 9 $)$

$(E_a, E_b) = ($ 7 , 13 $)$

For a square node, its value v means the same to Alice and Bob, i.e., we can think of it as a tuple (v,v).

The left value of Alice's nodes is the maximum of the left values of it's children nodes, since Alice believes that the values of the nodes are given by left values, and it's her turn of action, so she will choose the largest value.

The right value of Alice's nodes is the right value from the child node that attains the maximum left value since Bob's expectation is consistent with how Alice will act.

So for a triangular node, its tuple is the same as its child that has the maximum left value.

The left value of Bob's nodes is the average of the maximum and minimum of the left values of it's children nodes since Alice believes Bob is 50% possible to be adversarial and 50% possible to be friendly.

The right value of Bob's nodes is the maximum of the right values of the immediate children nodes since Bob would choose the branch that gives the maximum outcome during his turn.

So for a round node, left = 0.5(max(children.left) + min(children.left)), and right = max(children.right).

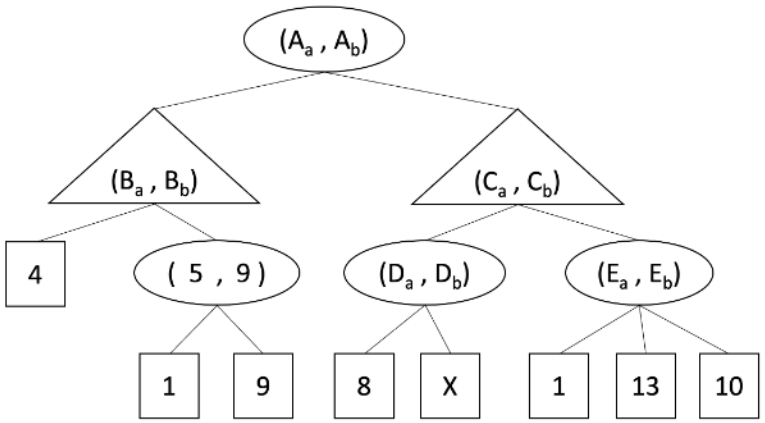
(b) In this part, we will determine the values for tuple (D_a, D_b).

(i) $D_a =$ ○ 8 ○ X ○ 8+X ● 4+0.5X ○ min(8,X) ○ max(8,X)

(ii) $D_b =$ ○ 8 ○ X ○ 8+X ○ 4+0.5X ○ min(8,X) ● max(8,X)

It's a round node, so left = 0.5(max(children.left) + min(children.left)), and right = max(children.right).

Its children: (8,8) and (X,X). So left = 0.5(8+X) = 4+0.5X, and right = max(8, X).



(The graph of the tree is copied for your convenience. You may do problem e on this graph.)

(c) Fill in the values for tuple (C_a, C_b) below. For the bounds of X, you may write scalars, ∞ or $-\infty$.
If your answer contains a fraction, please write down the corresponding **simplified decimal value** in its place. (i.e., 4 instead of $\frac{8}{2}$, and 0.5 instead of $\frac{1}{2}$).

1. If $-\infty < X <$ [6] , $(C_a, C_b) =$ ([7] , [13])
2. Else, $(C_a, C_b) =$ ([4+0.5X] , max([8] , [X]))

It's a triangular node, so its tuple is the same as its child that has the maximum left value.
Its children: (4+0.5X, max(8,X)) and (7, 13).
So if 4+0.5X < 7, i.e. $-\infty < X < 6$, it's the same as child node (7, 13), and otherwise it's (4+0.5X, max(8,X)).

(d) Fill in the values for tuple (A_a, A_b) below. For the bounds of X, you may write scalars, ∞ or $-\infty$.
If your answer contains a fraction, please write down the corresponding **simplified decimal value** in its place. (i.e., 4 instead of $\frac{8}{2}$, and 0.5 instead of $\frac{1}{2}$).

1. If $-\infty < X <$ [6] , $(A_a, A_b) =$ ([6] , [13])
2. Else, $(A_a, A_b) =$ ([4.5+0.25X] , max([9] , [X]))

It's a round node, so left = 0.5(max(children.left) + min(children.left)), and right = max(children.right).
Its children: (5,9) and node "Part (c)".
If $-\infty < X < 6$, these children are (5,9) and (7, 13).
left = 0.5(max(children.left) + min(children.left)) = 0.5(5+7) = 6
right = max(children.right) = max(9, 13) = 13.
Otherwise ($6 < X < +\infty$), these children are (5,9) and (4+0.5X, max(8,X)).
left = 0.5(max(children.left) + min(children.left)) = 0.5(5+4+0.5X) = 4.5 + 0.25X
right = max(children.right) = max(9, max(8,X)) = max(9,X).

(e) When Alice computes the left values in the tree, some branches can be pruned and do not need to be explored. In the game tree graph on this page, put an 'X' on these branches. If no branches can be pruned, mark the "Not possible" choice below.
Assume that the children of a node are visited in left-to-right order and that you should not prune on equality.

● Not possible

It's impossible to determine the average of *min* and *max* until all children nodes are seen, so no pruning can be done for Alice. Leaving "Not possible" unmarked and no 'X' found in the graph is interpreted as 'no conclusion' and will not be given credit.

Q3. Local Search

In this problem, we will look at the classic Traveling Salesman Problem (TSP) and how it relates to local search algorithms. The goal of the Traveling Salesman Problem is "Given a list of cities and the distances between each pair of cities, what is the shortest possible route that visits each city exactly once and returns to the origin city?" In other words, you're trying to find the shortest path that starts and end at a city and goes through every other city.

(a) Why might we choose to use local search (i.e.: hill-climbing) over an algorithm like A* search for solving the Traveling Salesman Problem? In general, when would using local search be better than using A* search?

For computationally difficult problems like TSP (NP-Hard) that are unfeasible to find a global optimum for, local search can converge to a local optimum instead of a global optimum. Additionally, if something changes in your problem (e.g., a bridge is closed, a flight is cancelled, etc.), using local search we can simply resume the search and get a good solution that is close to the current one. An algorithm like A* has to be restarted, and might generate a quite different path altogether.

Essentially, local search can be used well when the path is irrelevant and the goal state itself is the solution. Problems that try to find a configuration that satisfies certain constraints (n-queens problem) or try to find an optimal configuration (TSP) are great examples of this.

(b) Suppose that you connect all the cities into a single path (not a cycle). Devise a hill-climbing algorithm to solve TSPs. (Hint: Think about breaking the path)

Here is one simple hill-climbing algorithm:

- Connect all the cities into an arbitrary path.
- Pick two points along the path at random.
- Split the path at those points, producing three pieces.
- Try all six possible ways to connect the three pieces.
- Keep the best one, and reconnect the path accordingly.
- Iterate the steps above until no improvement is observed for a while.

For other reference, look up 2-opt, 3-opt. and Lin-Kernighan.

(c) Is it possible to reach a local optimum (but not the global optimum) using your algorithm?

If the set of action(s) is a small random set, and every random set still fails to improve the tour, then it will converge to a local optimum.