CS 188 Fall 2022 | Introduction to Artificial Intelligence | Challenge Q8 HW6

- **Due:** Friday 10/21 at 11:59pm.
- **Policy:** Can be solved in groups (acknowledge collaborators) but must be submitted individually.
- **Make sure to show all your work and justify your answers**.
- **Note:** This is a typical exam-level question. On the exam, you would be under time pressure, and have to complete this question on your own. We strongly encourage you to first try this on your own to help you understand where you currently stand. Then feel free to have some discussion about the question with other students and/or staff, before independently writing up your solution.
- Your submission on Gradescope should be a PDF that matches this template. Each page of the PDF should align with the corresponding page of the template (page 1 has name/collaborators, question begins on page 2.). **Do not reorder, split, combine, or add extra pages**. The intention is that you print out the template, write on the page in pen/pencil, and then scan or take pictures of the pages to make your submission. You may also fill out this template digitally (e.g. using a tablet.)

First name	
Last name	
SID	
Collaborators	

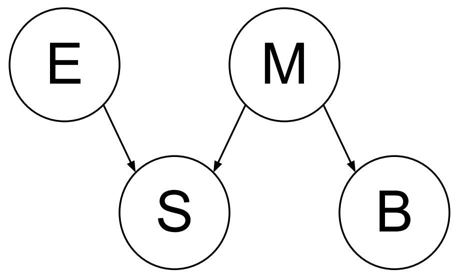
For staff use only:

Q8. [23 pts] Bayes Nets

Consider the following Bayes Net that has four variables E, S, M and B. The associated probabilities are also given to you in the following tables.

$P(E)$	
$+e$	0.4
$-e$	0.6

$P(S\|E,M)$			
$+e$	$+m$	$+s$	1.0
$+e$	$+m$	$-s$	0.0
$+e$	$-m$	$+s$	0.8
$+e$	$-m$	$-s$	0.2
$-e$	$+m$	$+s$	0.3
$-e$	$+m$	$-s$	0.7
$-e$	$-m$	$+s$	0.1
$-e$	$-m$	$-s$	0.9



$P(M)$	
$+m$	0.1
$-m$	0.9

$P(B\|M)$		
$+m$	$+b$	1.0
$+m$	$-b$	0.0
$-m$	$+b$	0.1
$-m$	$-b$	0.9

Figure 1: Bayes Net and probability tables.

In the following five questions, do not just answer with a numerical value. Write explicitly what probability expression you are calculating. For instance you should write P(A)P(B) = 0.2 instead of just 0.2.

10.1) (2 pts) Write an expression to compute the value of the joint distribution $P(-e,-s,-m,-b)$. What is its value?

$P(-e,-s,-m,-b) = P(-e)P(-m)P(-s|-e,-m)P(-b|-m) = (0.6)(0.9)(0.9)(0.9) = 0.4374$ by expanding the joint according to the chain rule of conditional probability.

10.2) (2 pts) Write an expression to compute the value of $P(+b)$. What is its value?

$P(+b) = P(+b|+m)P(+m) + P(+b|-m)P(-m) = (1.0)(0.1) + (0.1)(0.9) = 0.19$ by marginalizing out m according to the law of total probability.

10.3) (2 pts) Write an expression to compute the value of $P(+m|+b)$. What is its value?

$P(+m|+b) = \frac{P(+b|+m)P(+m)}{P(+b)} = \frac{(1)(0.1)}{0.19} \approx 0.5263$ by marginalizing out m according to the law of total probability.

10.4) (2 pts) Write an expression to compute the value of $P(+m|+s,+b,+e)$. What is its value?

$P(+m|+s,+b,+e) = \frac{P(+m,+s,+b,+e)}{\sum_m P(m,+s,+b,+e)} = \frac{P(+e)P(+m)P(+s|+e,+m)P(+b|+m)}{\sum_m P(+e)P(m)P(+s|+e,m)P(+b|m)} \frac{(0.4)(0.1)(1.0)(1.0)}{(0.4)(0.1)(1.0)(1.0)+(0.4)(0.9)(0.8)(0.1)} \approx 0.5814.$

10.5) (2 pts) Write an expression to compute the value of $P(+e|+m)$. What is its value?

$P(+e|+m) = P(+e) = 0.4$ The first equality holds true as E is independent of M, which can be inferred from the graph of the Bayes net.

Now consider the following Bayes Net in which all the variables are **binary**. We want to compute the query $P(B,D|+f)$, so we run variable elimination with the variable elimination ordering being A, C, E, G.

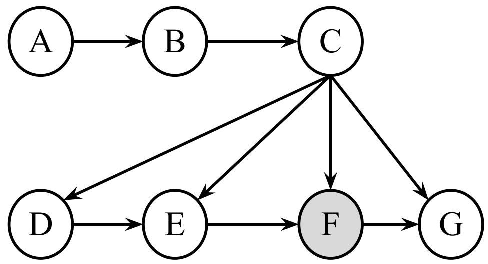


Figure 2: Bayes Net.

After observing evidence on F, we have the following factors

$$P(A),\ P(B|A),\ P(C|B),\ P(D|C),\ P(E|C,D),\ P(+f|C,E),\ P(G|C,+f)$$

When we eliminate the variable A, we then create a new factor $f_1(B) = \sum_a P(B|a)P(a)$ and the remaining factors are

$$f_1(B),\ P(C|B),\ P(D|C),\ P(E|C,D),\ P(+f|C,E),\ P(G|C,+f)$$

10.6) (2 pts) When we eliminate C next, what is the new factor f_2 we obtain? Furthermore, list the leftover factors.

$f_2(B,D,E,+f,G) = \sum_c P(c|B)P(D|c)P(E|c,D)P(+f|c,E)P(G|c,+f)$.
The remaining factors are: $f_1(B), f_2(B,D,E,+f,G)$.

10.7) (2 pts) When we eliminate E next, what is the new factor f_3 we obtain? Furthermore, list the leftover factors.

$f_3(B,D,+f,G) = \sum_e f_2(B,D,E,+f,G)$.
The remaining factors are: $f_1(B), f_3(B,D,+f,G)$.

10.8) (2 pts) When we eliminate G next, what is the new factor f_4 we obtain? Furthermore, list the leftover factors.

$f_4(B,D,+f) = \sum_g f_3(B,D,+f,G)$.
The remaining factors are: $f_1(B), f_4(B,D,+f)$.

10.9) (2 pts) How can we compute $P(B,D|+f)$ using the factors form **10.8)**? Explain in a couple of sentences.

Join $f_1 f_4$ to obtain $P(B,D,+f)$ and normalize it to get $P(B,D|+f)$. More concretely, $P(b,d|+f) = \frac{f_1(b)f_4(b,d,+f)}{\sum_{b',d'} f_1(b')f_4(b',d',+f)}$.

10.10) (1 pts) Between $f_1,\ f_2,\ f_3$ and f_4 which is the largest factor (or equivalently whose table has the most rows)?

$f_2(B,D,E,+f,G)$ is the largest factor generated. It has 4 variables, hence $2^4 = 16$ entries.

10.11) (4 pts) For the same query $P(B,\ D|+f)$, find a new variable elimination ordering that minimizes the size of the largest factor created during the process. Fill in your ordering and the generated factors on the table below. For instance, if we were to follow the original ordering you would write on the left and right columns of the first row of the table A and $f_1(B)$ respectively, and so on. *Hint: Your largest factor should be of size* 4, *i.e. have only two variables.*

Variable Eliminated	Factor Generated
A	$f_1(B)$
G	$f_2(C,+f)$
E	$f_3(C,D,+f)$
C	$f_4(B,D,+f)$

Note: multiple orderings are possible. In particular in this case all orderings with E and G before C are correct.