CS 188 Fall 2022 — Introduction to Artificial Intelligence — Written HW1

- **Due:** Friday 9/9 at 11:59pm.
- **Policy:** Can be solved in groups (acknowledge collaborators) but must be submitted individually.
- **Make sure to show all your work and justify your answers**.
- **Note:** This is a typical exam-level question. On the exam, you would be under time pressure, and have to complete this question on your own. We strongly encourage you to first try this on your own to help you understand where you currently stand. Then feel free to have some discussion about the question with other students and/or staff, before independently writing up your solution.
- Your submission on Gradescope should be a PDF that matches this template. Each page of the PDF should align with the corresponding page of the template (page 1 has name/collaborators, question begins on page 2.). **Do not reorder, split, combine, or add extra pages**. The intention is that you print out the template, write on the page in pen/pencil, and then scan or take pictures of the pages to make your submission. You may also fill out this template digitally (e.g. using a tablet.)

First name	
Last name	
SID	
Collaborators	

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Q1. [18 pts] Search

It is training day for Pacbabies, also known as Hungry Running Maze Games day. Each of k Pacbabies starts in its own assigned start location s_i in a large maze of size $M \times N$ and must return to its own Pacdad who is waiting patiently but proudly at g_i along the way, the Pacbabies must, between them, eat all the dots in the maze.

At each step, all k Pacbabies move one unit to any open adjacent square. The only legal actions are Up, Down, Left, or Right. It is illegal for a Pacbaby to wait in a square, attempt to move into a wall, or attempt to occupy the same square as another Pacbaby. To set a record, the Pacbabies must find an optimal collective solution.

1.1) (5 pts) Define a minimal state space representation for this problem.

The minimal state space is defined by the current locations of k Pacbabies and, for each square of the grid, a Boolean variable that indicates whether there is food there or not.

1.2) (2 pts) How large is the state space?

Given the minimal state representation defined above an upper bound on the size of the state space is $(MN)^k \cdot 2^{MN}$. The first part is $(MN)^k$ as the pacbabies can move to any state in the state-space. The second term 2^{MN} accounts for all the possible food configurations on the grid. You could also point out that given that two pacbabies cannot be on the same place at the same time the first term is $(MN) * (MN-1) * \cdots (MN-(k-1))$. Both approaches are considered correct.

1.3) (2 pts) What is the maximum branching factor for this problem?

A) 4^k

B) 8^k

C) $4^k 2^{MN}$

D) $4^k 2^4$

For each distinct action of a pacbaby we will end up in a possibly different child node. Given that we have k pacbabies then the answer is 4^k as each of the k Pacbabies has a choice of 4 actions.

1.4) (5 pts) Let $MH(p,q)$ be the Manhattan distance between positions p and q and F be the set of all positions of remaining food pellets and p_i be the current position of Pacbaby i. Which of the following are admissible heuristics?

A) $\frac{\sum_{i=1}^{k} MH(p_i, g_i)}{k}$

B) $\max_{1 \le i \le k} MH(p_i, g_i)$

C) $\max_{1 \le i \le k}[\max_{f \in F} MH(p_i, f)]$

D) $\max_{1 \le i \le k}[\min_{f \in F} MH(p_i, f)]$

E) $\min_{1 \le i \le k}[\min_{f \in F} MH(p_i, f)]$

F) $\min_{f \in F}[\max_{1 \le i \le k} MH(p_i, f)]$

A), B), E)

- A) is admissible because the total Pacbaby–Pacdad distance can be reduced by at most k at each time step.
- B) is admissible because it will take at least this many steps for the furthest Pacbaby to reach its Pacdad.
- C) is inadmissible because it looks at the distance from each Pacbaby to its most distant food square and in the optimal solution we might have another Pacbaby, that is closer, going to that square so this heuristic is inadmissible.

- D) same logic as C).
- E) is admissible because some Pacbaby will have to travel at least this far to eat one piece of food.
- F) is inadmissible because it connects each food square to the most distant Pacbaby, which may not be the one who eats it.

1.5) (2 pts) Give one pair of heuristics h_i, h_j from part (1.4) such that their maximum, $h(n) = \max(h_i(n), h_j(n))$, is an admissible heuristic.

Any pair between A), B), and E) would make $h(n)$ admissible as the max of two admissible heuristics is still admissible.

1.6) (2 pts) Is there a pair of heuristics h_i, h_j from part (1.4) such that their convex combination, defined as

$$h(n) = \alpha h_i(n) + (1 - \alpha)h_j(n)), \alpha \in [0, 1],$$

is an admissible heuristic for any value of α between 0 and 1?

Again, any pair between A), B), and E) would make $h(n)$ admissible as the convex combination of two functions is dominated by the max of those functions, $h(n) = \alpha h_i(n) + (1 - \alpha)h_j(n)) \leq \max(h_i(n), h_j(n))$ for any $\alpha \in [0, 1]$, and since from the previous part the max is admissible the same holds for the convex combination.