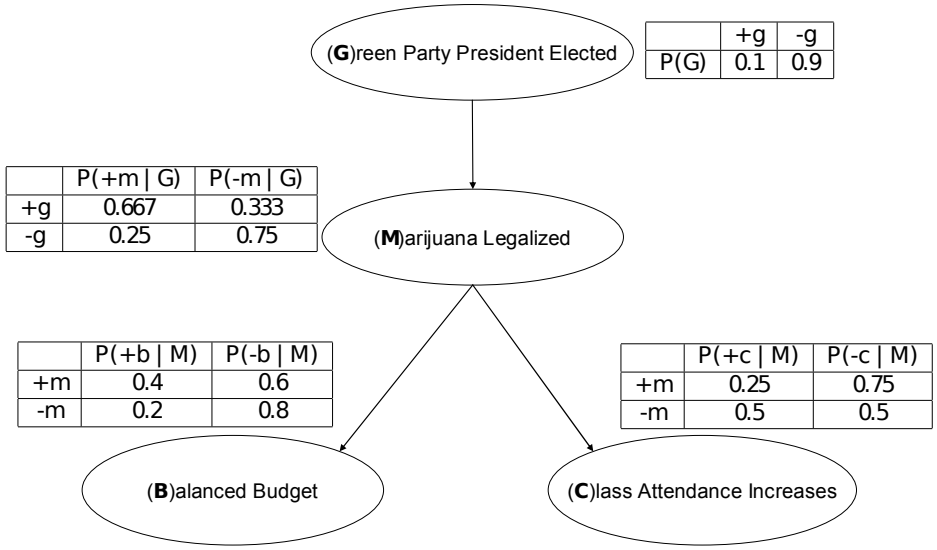
CS 188
Fall 2021

Regular Discussion 6 Solutions

1 Bayes Nets: Green Party President

In a parallel universe the Green Party is running for presidency. Whether a Green Party President is elected (G) will have an effect on whether marijuana is legalized (M), which then influences whether the budget is balanced (B), and whether class attendance increases (C). Armed with the power of probability, the analysts model the situation with the Bayes Net below.



	+g	-g
P(G)	0.1	0.9

	P(+m | G)	P(-m | G)
+g	0.667	0.333
-g	0.25	0.75

	P(+b | M)	P(-b | M)
+m	0.4	0.6
-m	0.2	0.8

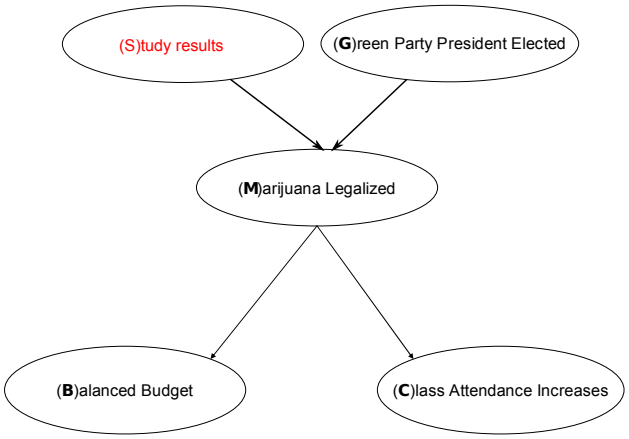
	P(+c | M)	P(-c | M)
+m	0.25	0.75
-m	0.5	0.5

1. The full joint distribution is given below. Fill in the missing values.

G	M	B	C	$P(G, M, B, C)$
+	+	+	+	0.00667
+	+	+	-	0.02
+	+	-	+	0.01
+	+	-	-	0.03
+	-	+	+	0.00333
+	-	+	-	0.00333
+	-	-	+	0.0133
+	-	-	-	0.0133

G	M	B	C	$P(G, M, B, C)$
-	+	+	+	0.0225
-	+	+	-	0.0675
-	+	-	+	0.03375
-	+	-	-	0.10125
-	-	+	+	0.0675
-	-	+	-	0.0675
-	-	-	+	0.27
-	-	-	-	0.27

2. Now, add a node S to the Bayes net that reflects the possibility that a new scientific study could influence the probability that marijuana is legalized. Assume that the study does not affect G, and that the study does not directly influence B or C. Draw the new Bayes net below. Which CPT(s) need to be modified?



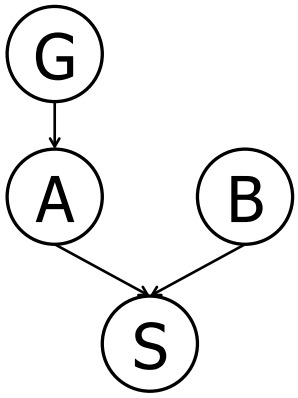
$P(M|G)$ will become $P(M|G,S)$, and will contain 8 entries instead of 4.

2 Bayes' Nets Representation and Probability

Suppose that a patient can have a symptom (S) that can be caused by two different diseases (A and B). It is known that the variation of gene G plays a big role in the manifestation of disease A. The Bayes' Net and corresponding conditional probability tables for this situation are shown below. For each part, you may leave your answer as an arithmetic expression.

$P(G)$	
$+g$	0.1
$-g$	0.9

$P(A\|G)$		
$+g$	$+a$	1.0
$+g$	$-a$	0.0
$-g$	$+a$	0.1
$-g$	$-a$	0.9



$P(B)$	
$+b$	0.4
$-b$	0.6

$P(S\|A,B)$			
$+a$	$+b$	$+s$	1.0
$+a$	$+b$	$-s$	0.0
$+a$	$-b$	$+s$	0.9
$+a$	$-b$	$-s$	0.1
$-a$	$+b$	$+s$	0.8
$-a$	$+b$	$-s$	0.2
$-a$	$-b$	$+s$	0.1
$-a$	$-b$	$-s$	0.9

(a) Compute the following entry from the joint distribution:

$P(+g,+a,+b,+s) =$ $P(+g)P(+a|+g)P(+b)P(+s|+b,+a) = (0.1)(1.0)(0.4)(1.0) = 0.04$

(b) What is the probability that a patient has disease A?

$P(+a) =$ $P(+a|+g)P(+g) + P(+a|-g)P(-g) = (1.0)(0.1) + (0.1)(0.9) = 0.19$

(c) What is the probability that a patient has disease A given that they have disease B?

$P(+a|+b) =$ $P(+a) = 0.19$ The first equality holds true since $A \perp\!\!\!\perp B$, which can be inferred from the graph of the Bayes' net.

(d) What is the probability that a patient has disease A given that they have symptom S and disease B?

$P(+a|+s,+b) = \frac{P(+a,+b,+s)}{P(+a,+b,+s)+P(-a,+b,+s)} = \frac{P(+a)P(+b)P(+s|+a,+b)}{P(+a)P(+b)P(+s|+a,+b)+P(-a)P(+b)P(+s|-a,+b)}$
$= \frac{(0.19)(0.4)(1.0)}{(0.19)(0.4)(1.0)+(0.81)(0.4)(0.8)} = \frac{0.076}{0.076+0.2592} \approx 0.2267$

(e) What is the probability that a patient has the disease carrying gene variation G given that they have disease A?

$P(+g|+a) = \frac{P(+g)P(+a|+g)}{P(+g)P(+a|+g)+P(-g)P(+a|-g)} = \frac{(0.1)(1.0)}{(0.1)(1.0)+(0.9)(0.1)} = \frac{0.1}{0.1+0.09} = 0.5263$