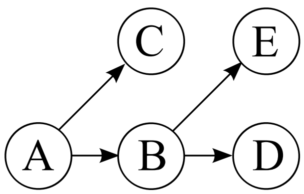
CS 188 Fall 2021 | Introduction to Artificial Intelligence | Exam Prep 7 Solutions

Q1. Bayes Nets: Variable Elimination



	$P(A)$
$+a$	0.25
$-a$	0.75

$P(B\|A)$	$+b$	$-b$
$+a$	0.5	0.5
$-a$	0.25	0.75

$P(C\|A)$	$+c$	$-c$
$+a$	0.2	0.8
$-a$	0.6	0.4

$P(D\|B)$	$+d$	$-d$
$+b$	0.6	0.4
$-b$	0.8	0.2

$P(E\|B)$	$+e$	$-e$
$+b$	0.25	0.75
$-b$	0.1	0.9

(a) Using the Bayes' Net and conditional probability tables above, calculate the following quantities:

(i) $P(+b|+a) =$ 0.5

(ii) $P(+a,+b) =$

$0.25 * 0.5 = 0.125 = \frac{1}{8}$

(iii) $P(+a|+b) =$

$\frac{0.25*0.5}{0.25*0.5+0.25*0.75} = 0.4 = \frac{2}{5}$

(b) Now we are going to consider variable elimination in the Bayes' Net above.

(i) Assume we have the evidence $+c$ and wish to calculate $P(E \mid +\text{c})$. What factors do we have initially?

$P(A)$, $P(B \mid A)$, $P(+\text{c} \mid A)$, $P(D \mid B)$, $P(E \mid B)$

(ii) If we eliminate variable B, we create a new factor. What probability does that factor correspond to?

$P(D, E \mid A)$

(iii) What is the equation to calculate the factor we create when eliminating variable B?

$f(A, D, E) = \sum_B P(B \mid A) * P(D \mid B) * P(E \mid B)$

(iv) After eliminating variable B, what are the new set of factors? As in (ii), write the probabilities that the factors represent. For each factor, also provide its size.

Factor	Size after elimination
$P(A)$	2
$P(+\text{c} \mid A)$	2
$P(D, E \mid A)$	2^3

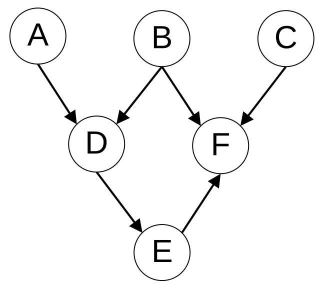
(v) Now assume we have the evidence $-c$ and are trying to calculate $P(A|-c)$. What is the most efficient elimination ordering? If more than one ordering is most efficient, provide any one of them. E, D, B or D, E, B

(vi) Once we have run variable elimination and have $f(A, -c)$ how do we calculate $P(+a \mid -c)$? $\frac{f(+a,-c)}{f(+a,-c)+f(-a,-c)}$ or note that elimination is unnecessary - just use Bayes' rule

Q2. Bayes Nets: Independence

(a) For the following graphs, explicitly state the minimum size set of edges that must be removed such that the corresponding independence relations are guaranteed to be true.

Marked the removed edges with an 'X' on the graphs.

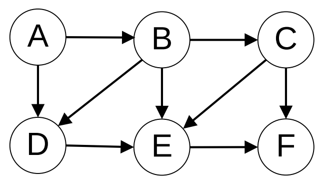


$A \perp\!\!\!\perp B|F$

$A \perp\!\!\!\perp F|D$

$B \perp\!\!\!\perp C$

Solution: AD



$A \perp\!\!\!\perp D|B$

$A \perp\!\!\!\perp F|C$

$C \perp\!\!\!\perp D|B$

Solution: AD, (EF OR AB)

(b) You're performing variable elimination over a Bayes Net with variables A, B, C, D, E. So far, you've finished joining over (but not summing out) C, when you realize you've lost the original Bayes Net!

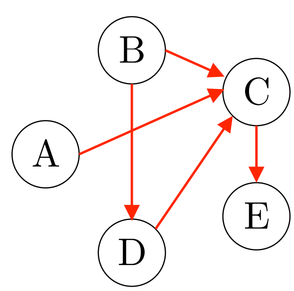
Your current factors are $f(A)$, $f(B)$, $f(B, D)$, $f(A, B, C, D, E)$. Note: these are factors, NOT joint distributions. You don't know which variables are conditioned or unconditioned.

(i) What's the smallest number of edges that could have been in the original Bayes Net? Draw out one such Bayes Net below.

Number of edges = 5

The original Bayes net must have had 5 factors, 1 for each node. f(A) and f(B) must have corresponded to nodes A and B, and indicate that neither A nor B have any parents. f(B, D), then, must correspond to node D, and indicates that D has only B as a parent. Since there is only one factor left, f(A, B, C, D, E), for the nodes C and E, those two nodes must have been joined while you were joining C. This implies two things: 1) E must have had C as a parent, and 2) every other node must have been a parent of either C or E.

The below figure is one possible solution that uses the fewest possible edges to satisfy the above.



(ii) What's the largest number of edges that could have been in the original Bayes Net? Draw out one such Bayes Net below.

Number of edges = 8

The constraints are the same as outlined in part i). To maximize the number of edges, we make each of A, B, and D a parent of both C and E, as opposed to a parent of one of them.

The below figure is the only possible solution.