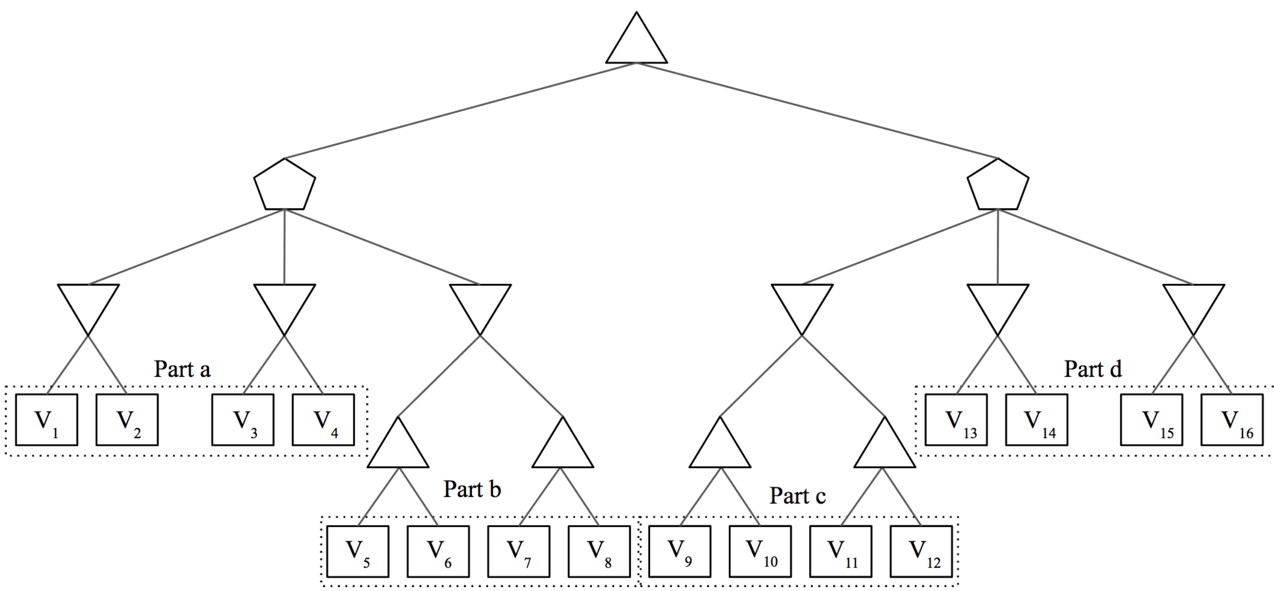
# Q1. MedianMiniMax

You're living in utopia! Despite living in utopia, you still believe that you need to maximize your utility in life, other people want to minimize your utility, and the world is a 0 sum game. But because you live in utopia, a benevolent social planner occasionally steps in and chooses an option that is a compromise. Essentially, the social planner (represented as the pentagon) is a median node that chooses the successor with median utility. Your struggle with your fellow citizens can be modelled as follows:




There are some nodes that we are sometimes able to prune. In each part, mark all of the terminal nodes such that **there exists a possible situation** for which the node **can be pruned**. In other words, you must consider **all** possible pruning situations. Assume that evaluation order is **left to right** and all $V_i$'s are **distinct.**

Note that as long as there exists ANY pruning situation (does not have to be the same situation for every node), you should mark the node as prunable. Also, alpha-beta pruning does not apply here, simply prune a sub-tree when you can reason that its value will not affect your final utility.

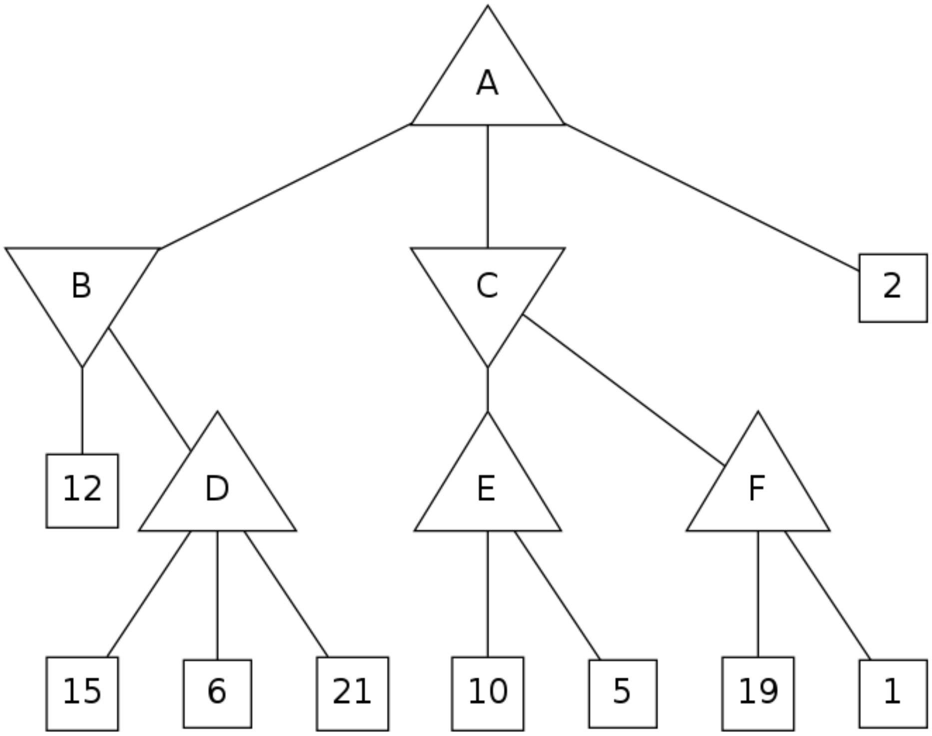
**(a)**
- ☐ $V_1$
- ☐ $V_2$
- ☐ $V_3$
- ☐ $V_4$
- ☐ None

**(b)**
- ☐ $V_5$
- ☐ $V_6$
- ☐ $V_7$
- ☐ $V_8$
- ☐ None

**(c)**
- ☐ $V_9$
- ☐ $V_{10}$
- ☐ $V_{11}$
- ☐ $V_{12}$
- ☐ None

**(d)**
- ☐ $V_{13}$
- ☐ $V_{14}$
- ☐ $V_{15}$
- ☐ $V_{16}$
- ☐ None

# Q2. Games




**(a)** What is the minimax value of node A in the tree above?

**(b)** Cross off the nodes that are pruned by alpha-beta pruning. Assume the standard left-to-right traversal of the tree. If a non-terminal state (A, B, C, D, E, or F) is pruned, cross off the entire subtree.

**(c)** If a function $F$ is strictly increasing, then $F(a) < F(b)$ for all $a < b$ for $a, b \in \mathbb{R}$. Consider applying a strictly increasing function $F$ to the leaves of a game tree and comparing the old tree and the new tree.

Are the claims below true or false? For true cases, justify your reasoning in a single sentence. For false cases, provide a counterexample (specifically, a game tree, including terminal values).

In a *Minimax* two player zero-sum game, applying $F$ will not change the optimal *action*.

True False

In a *Minimax* two player zero-sum game, applying $F$ will not affect which nodes are pruned by alpha-beta pruning.

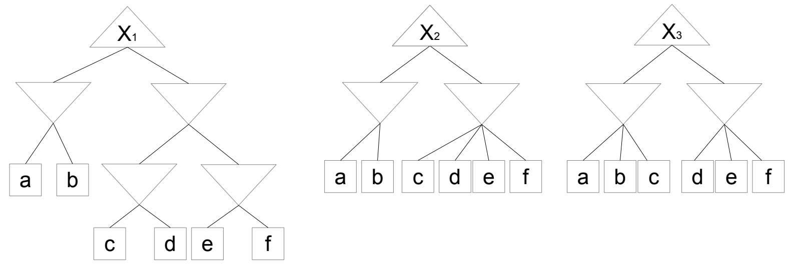
True False

In a *Minimax* two player non-zero-sum game (where the utilities of players do not necessarily add up to zero), applying $F$ will not change the optimal *action*.

True False

In an *Expectimax* two player zero-sum game, applying $F$ will not change the optimal *action*.

True False

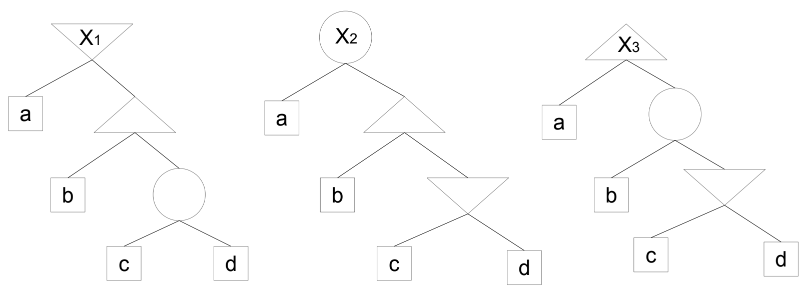



**(d)** Let $X_1$, $X_2$, and $X_3$ be the values at each root in the above minimax game trees. In these trees $a$, $b$, $c$, $d$, $e$, and $f$ are constants (they are the same across all three trees). Determine which of the following statements are true for all possible assignments to constants $a$, $b$, $c$, $d$, $d$, and $f$.

**(i)** $X_1 = X_2$ True False

**(ii)** $X_1 = X_3$ True False

**(iii)** $X_2 = X_3$ True False




**(e)** In this question we want to determine relations between the values at the root of the new game trees above (that is, between $X_1$, $X_2$, and $X_3$).

All three game trees use the same values at the leaves, represented by $a$, $b$, $c$, and $d$. The chance nodes can have any distribution over actions, that is, they can choose right or left with any probability. The chance node distributions can also vary between the trees.

For each case below, write the relationship between the values using $<$, $\leq$, $>$, $\geq$, $=$, or $NR$. Write $NR$ if no relation can be confirmed given the current information. Briefly justify each answer (one sentence at most). (Hint: try combinations of $\{-\infty, -1, 0, 1, +\infty\}$ for $a$, $b$, $c$, and $d$.)

**(i)** $X_1$ blank $X_3$

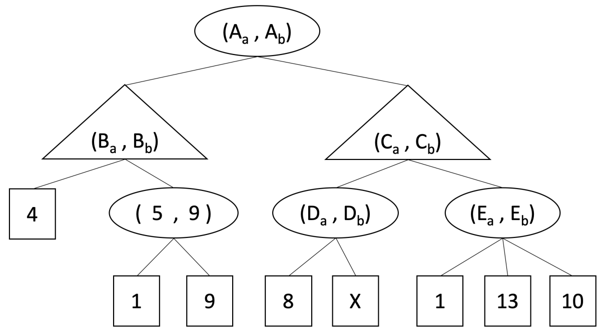
**(ii)** $X_2$ blank $X_3$

# Q3. Games

Alice is playing a two-player game with Bob, in which they move alternately. Alice is a maximizer. Although Bob is also a maximizer, Alice believes Bob is a minimizer with probability 0.5, and a maximizer with probability 0.5. Bob is aware of Alice's assumption.

In the game tree below, square nodes are the outcomes, triangular nodes are Alice's moves, and round nodes are Bob's moves. Each node for Alice/Bob contains a tuple, the left value being Alice's expectation of the outcome, and the right value being Bob's expectation of the outcome.

Tie-breaking: choose the left branch.




**(a)** In the blanks below, fill in the tuple values for tuples $(B_a, B_b)$ and $(E_a, E_b)$ from the above game tree.

$(B_a, B_b) = ($ , $)$

$(E_a, E_b) = ($ , $)$

**(b)** In this part, we will determine the values for tuple $(D_a, D_b)$.

**(i)** $D_a =$ ○ 8 ○ X ○ 8+X ○ 4+0.5X ○ min(8,X) ○ max(8,X)

**(ii)** $D_b =$ ○ 8 ○ X ○ 8+X ○ 4+0.5X ○ min(8,X) ○ max(8,X)

**(c)** Fill in the values for tuple $(C_a, C_b)$ below. For the bounds of X, you may write scalars, $\infty$ or $-\infty$.
If your answer contains a fraction, please write down the corresponding **simplified decimal value** in its place. (i.e., 4 instead of $\frac{8}{2}$, and 0.5 instead of $\frac{1}{2}$).

1. If $-\infty <$ X $<$ , $(C_a, C_b) = ($ , $)$
2. Else, $(C_a, C_b) = ($ , max( , $))$

**(d)** Fill in the values for tuple $(A_a, A_b)$ below. For the bounds of X, you may write scalars, $\infty$ or $-\infty$.
If your answer contains a fraction, please write down the corresponding **simplified decimal value** in its place. (i.e., 4 instead of $\frac{8}{2}$, and 0.5 instead of $\frac{1}{2}$).

1. If $-\infty <$ X $<$ , $(A_a, A_b) = ($ , $)$
2. Else, $(A_a, A_b) = ($ , max( , $))$

**(e)** When Alice computes the left values in the tree, some branches can be pruned and do not need to be explored. In the game tree graph above, put an 'X' on these branches. If no branches can be pruned, write "Not Possible" below. Assume that the children of a node are visited in left-to-right order and that you should not prune on equality.