Note: Your TA probably will not cover all the problems. This is totally fine, the discussion worksheets are not designed to be finished in an hour. They are deliberately made long so they can serve as a resource you can use to practice, reinforce, and build upon concepts discussed in lecture, readings, and the homework.

1 Secret Santa

Imagine you are throwing a party and you want to play Secret Santa. Thus you would like to assign to every person at the party another partier to whom they must anonymously give a gift. However, there are some restrictions on who can give gifts to who. For instance, nobody should be assigned to give a gift to themselves or to their spouse. Since you are the host, you know all of these restrictions. Give an efficient algorithm that determines if you and your guests can play Secret Santa.

Zero Sum Games: In this game, there are two players: a maximizer and a minimizer. We generally write the payoff matrix M in perspective of the maximizer, so every row corresponds to an action that the maximizer can take, every column corresponds to an action that the minimizer can take, and a positive entry corresponds to the maximizer winning. M is a n by m matrix, where n is the number of choices the maximizer has, and m is the number of choices the minimizer has.

A linear program that represents fixing the maximizer's choices to a probabilistic distribution where the maximizer has n choices, and the probability that the maximizer chooses choice i is p_i is the following:

$$\begin{gathered}
\max(z) \\
M_{1,1}(p_1) + \cdots + M_{n,1}(p_n) \geq z \\
M_{1,2}(p_1) + \cdots + M_{n,2}(p_n) \geq z \\
\vdots \\
M_{1,n}(p_1) + \cdots + M_{n,n}(p_n) \geq z \\
p_1 + p_2 + \cdots + p_n = 1 \\
p_1, p_2, \cdots, p_n \geq 0
\end{gathered}$$

The dual represents fixing the minimizers choices to a probabilistic distribution.

By strong duality, the optimal value of the game is the same if you fix the minimizer's distribution first or the maximizer's distribution first.

2 Weighted Rock-Paper-Scissors

You and your friend used to play rock-paper-scissors, and have the loser pay the winner 1 dollar. However, you then learned in CS170 that the best strategy is to pick each move uniformly at random, which took all the fun out of the game.

Your friend, trying to make the game interesting again, suggests playing the following variant: If you win by beating rock with paper, you get 5 dollars from your opponent. If you win by beating scissors with rock, you get 3 dollars. If you win by beating paper with scissors, you get 1 dollar.

(a) Draw the payoff matrix for this game.

(b) Write a linear program to find the optimal strategy.

3 Domination

In this problem, we explore a concept called *dominated strategies*. Consider a zero-sum game with the following payoff matrix for the row player:

Column:

		A	B	C
	D	1	2	-3
Row:	E	3	2	-2
	F	-1	-2	2

(a) If the row player plays optimally, can you find the probability that they pick D without directly solving for the optimal strategy? (Hint: Notice that the payoff for E is always greater than the payoff for D. When this happens, we say that E *dominates* D, i.e. D is a *dominated strategy*).

(b) Given the answer to part a, if the both players play optimally, what is the probability that the column player picks A?

(c) Given the answers to part a and b, what are both players' optimal strategies? (You might be able to figure this out without writing or solving any LP).