1. Tying MWU + Zero-sum games $l_i^{(t)} = l_i(t)$

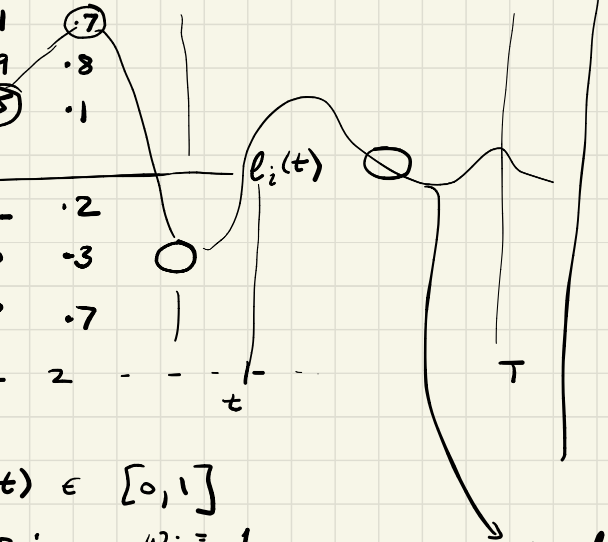
2. Review

MWU

Experts

1	.1	.7
2	.9	.8
3	.5	.1
⋮		
i		
⋮	1	.2
⋮	0	.3
n	.3	.7

Days → 1 2 - - - t - - T



Total Loss

$T/3$

$T-5$

$10 = L^*$

100

MWU:

$w_i \leftarrow w_i (1-\varepsilon)^{l_i(t)}$

Pick i w.p. $\frac{w_i}{\sum_{j=1}^{n} w_j}$

$l_i(t) \in [0,1]$

Day 0: $w_i = 1$

MWU loss $= L(T)$

Theorem: $L(T) \leq \frac{\ln n}{\varepsilon} + (1+\varepsilon) L^*$

zero sum games

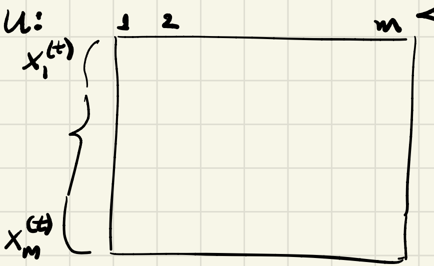
$$\begin{array}{cc|ccc} & B & y_1 & y_2 & y_3 \\ A & & 1 & 2 & 3 \\ \hline x_1 & 1 & .2 & 0 & 1 \\ x_2 & 2 & .5 & .6 & .3 \\ x_3 & 3 & .9 & .5 & .2 \end{array} = M$$

payoff $= x^T M y$

$$\min_y \max_x x^T M y \underset{\uparrow}{=} \max_x \min_y x^T M y$$

min-max theorem.

MWU:



$M =$ m×m matrix

$$\frac{L^*}{T} \quad \underset{\text{B's best response to } y}{R(y)} \le \overset{\text{A's best response to } x}{C(x)} \le \underset{\text{MWU bound.}}{\frac{L(T)}{T}}$$

:

$X(1) = x_1^{(1)} = \cdots = x_m^{(1)} = \frac{1}{m}$ = Alice play. Bob plays best response. $y^{(1)}$

$X(2) =$ ________ Alice plays Bob plays best response $y(2)$

|

$X(T)$

$\frac{1}{T}\sum_{t=1}^{T} X(t) = X$

$$T = \frac{\ln m}{\varepsilon^2}$$

(X, y) are within 2ε of optimal.

$y(T)$

$y = \frac{1}{T}\sum_{t=1}^{T} y(t)$

LP

$$\max \quad 5r + 1.5h + s$$

$$\begin{array}{ll} x: & r \le 2 \\ y: & h \le 6 \\ z: & r + h + s \le 25 \\ & r, h, s \ge 0 \end{array}$$

3 strategies

s — study

h — homework

r — review hw solutions

$$\begin{aligned} x + z &\geq 5 \\ y + z &\geq 1.5 \\ z &\geq 1 \\ x, y, z &\geq 0 \end{aligned}$$

$$\min \quad 2x + 6y + 25z.$$

$$r\underset{\substack{\vee\vert \\ 5}}{\boxed{(x+z)}} + \underset{\substack{\vee\vert \\ 1.5}}{(y+z)}\,h + s\underset{\substack{\vee\vert \\ 1}}{z} \le 2x + 6y + 25z$$

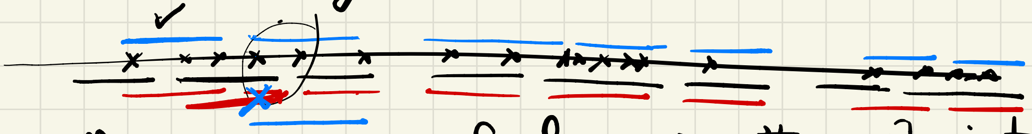
$$r + h \geq 3. \longleftrightarrow \quad -r - h \le -3.$$

$$t \le 0 \longleftrightarrow \quad u = -t \qquad u \geq 0$$

$$z \text{ unconstrained} \longleftrightarrow \quad z = z^+ - z^-$$

$$z^+ \geq 0 \quad z^- \geq 0.$$

Greedy



$r_1, - - -, r_n$ find min # unit intervals that contain all n pts.

start at leftmost pt $[l, l+1]$ throw out all pts in this interval

repeat.

Claim: optimal.

Main idea: Exchange argument.

Assume for contradiction — not optimal. Better solution.

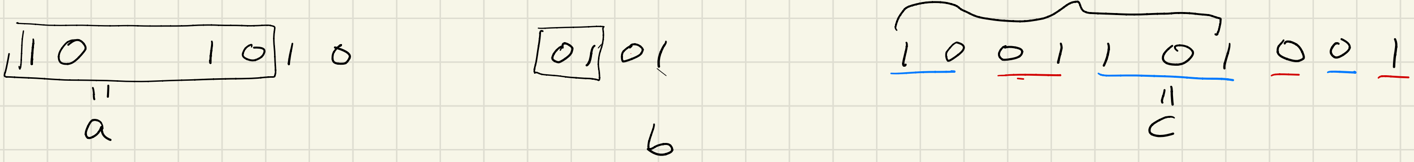
Walk from left to right — find 1st difference } keep doing this
substitute red interval with blue one.

Assume for contradiction — not optimal. Better soln ⟷ pick one with max overlap with greedy

max j : 1st j intervals are the same

DP: Binary strings a, b, c.

Can you interleave a & b to get c.



$S(i,j)$ = True if $a[1,\ldots,i]$ & $b[1,\ldots j]$ can be interleaved to get $c[1,\ldots i+j]$.

$S(i,j) = S(i,j-1)$ & $b(j) = c(i+j)$ ✓

or $S(i-1,j)$ & $a(i) = c(i+j)$ ✓

$S(0,0) =$ true.